Example 1

In a study conducted by the Department of Mechanical Engineering at Virginia Tech, the steel rods supplied by two diﬀerent companies were compared. Ten sample springs were made out of the steel rods supplied by each company, and a measure of ﬂexibility was recorded for each. The data are as follows:

|  |  |
| --- | --- |
| Company A | Company B |
| 9.3 | 11 |
| 8.8 | 9.8 |
| 6.8 | 9.9 |
| 8.7 | 10.2 |
| 8.5 | 10.1 |
| 6.7 | 9.7 |
| 8 | 11 |
| 6.5 | 11.1 |
| 9.2 | 10.2 |
| 7 | 9.6 |

Assume the two populations to be normally distributed

Test $H\_{0}:μ\_{1}=μ\_{2}$ vs $H\_{1}: μ\_{1}\ne μ\_{2}$

|  |
| --- |
| **Independent Samples Test** |
|  | Levene's Test for Equality of Variances | t-test for Equality of Means |
| F | Sig. | t | df | Sig. (2-tailed) | Mean Difference | Std. Error Difference | 95% Confidence Interval of the Difference |
| Lower | Upper |
| ﬂexibility | Equal variances assumed | 9.149 | .007 | -5.902 | 18 | .000 | -2.31000 | .39142 | -3.13235 | -1.48765 |
| Equal variances not assumed |  |  | -5.902 | 13.517 | .000 | -2.31000 | .39142 | -3.15234 | -1.46766 |

Example 3

Suppose that an engineer is interested in testing the bias in a pH meter. Data are collected on a neutral substance (pH= 7.0). A sample of the measurements were taken with the data as follows:

7.07 7.00 7.10 6.97 7.00 7.03 7.01 7.01 6.98 7.08

It is, then, of interest to test $H\_{0}:μ=7$ vs $H\_{1}: μ\ne 7$

Assume the population to be normally distributed

|  |
| --- |
| **One-Sample Test** |
|  | Test Value = 7 |
| t | df | Sig. (2-tailed) | Mean Difference | 95% Confidence Interval of the Difference |
| Lower | Upper |
| VAR00001 | 1.725 | 8 | .123 | .02667 | -.0090 | .0623 |

Example 6:

A clinic provides a program to help their clients lose weight and asks a consumer agency to investigate the effectiveness of the program. The agency takes a sample of 15 people, weighing each person in the sample before the program begins and 3 months later to produce the table below

|  |  |  |
| --- | --- | --- |
| Person | Before | After |
| 1 | 210 | 197 |
| 2 | 205 | 195 |
| 3 | 193 | 191 |
| 4 | 182 | 174 |
| 5 | 259 | 236 |
| 6 | 239 | 226 |
| 7 | 164 | 157 |
| 8 | 197 | 196 |
| 9 | 222 | 201 |
| 10 | 211 | 196 |
| 11 | 187 | 181 |
| 12 | 175 | 164 |
| 13 | 186 | 181 |
| 14 | 243 | 229 |
| 15 | 246 | 231 |

Determine whether the program is effective?

|  |
| --- |
| **Paired Samples Statistics** |
|  | Mean | N | Std. Deviation | Std. Error Mean |
| Pair 1 | Before | 207.9333 | 15 | 28.56188 | 7.37465 |
| After | 197.0000 | 15 | 24.39262 | 6.29815 |

|  |
| --- |
| **Paired Samples Correlations** |
|  | N | Correlation | Sig. |
| Pair 1 | Before & After | 15 | .984 | .000 |

|  |
| --- |
| **Paired Samples Test** |
|  | Paired Differences | t | df | Sig. (2-tailed) |
| Mean | Std. Deviation | Std. Error Mean | 95% Confidence Interval of the Difference |
| Lower | Upper |
| Pair 1 | Before - After | 10.93333 | 6.32982 | 1.63435 | 7.42799 | 14.43867 | 6.690 | 14 | .000 |

Example 9:

Is gender independent of education level? A random sample of 395 people were surveyed and each person was asked to report the highest education level they obtained. The data that resulted from the survey is summarized in the following table:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|   | High School  | Bachelors | Masters  | Ph.d |
| Female | 60 | 54 | 46 | 41 |
| Male | 40 | 44 | 53 | 57 |

Question: Are gender and education level dependent at 5% level of significance? In other words, given the data collected above, is there a relationship between the gender of an individual and the level of education that they have obtained?

|  |
| --- |
| **Gender \* Education Crosstabulation** |
|  | Education | Total |
| High School | Bechelors | Master | Ph.d |
| Gender | Male | Count | 40 | 44 | 53 | 57 | 194 |
| Expected Count | 49.1 | 48.1 | 48.6 | 48.1 | 194.0 |
| Female | Count | 60 | 54 | 46 | 41 | 201 |
| Expected Count | 50.9 | 49.9 | 50.4 | 49.9 | 201.0 |
| Total | Count | 100 | 98 | 99 | 98 | 395 |
| Expected Count | 100.0 | 98.0 | 99.0 | 98.0 | 395.0 |

|  |
| --- |
| **Chi-Square Tests** |
|  | Value | df | Asymptotic Significance (2-sided) |
| Pearson Chi-Square | 8.006a | 3 | .046 |
| Likelihood Ratio | 8.045 | 3 | .045 |
| Linear-by-Linear Association | 7.867 | 1 | .005 |
| N of Valid Cases | 395 |  |  |
| a. 0 cells (0.0%) have expected count less than 5. The minimum expected count is 48.13. |

Example 10:

The following data represents the income and expenditure of 10 families.  Estimate the simple linear regression model of the data and interpret the  results.

|  |  |
| --- | --- |
| Expenditure  | Income  |
| 2400  | 4120  |
| 2650  | 5010  |
| 2350  | 5200  |
| 4950  | 6600  |
| 3100  | 4450  |
| 2500  | 3770  |
| 5106  | 7350  |
| 3100  | 3750  |
| 2900  | 5670  |
| 1750  | 3560  |

|  |
| --- |
| **Correlations** |
|  | Expenditure  | Income |
| Expenditure  | Pearson Correlation | 1 | .840\*\* |
| Sig. (2-tailed) |  | .002 |
| N | 10 | 10 |
| Income | Pearson Correlation | .840\*\* | 1 |
| Sig. (2-tailed) | .002 |  |
| N | 10 | 10 |
| \*\*. Correlation is significant at the 0.01 level (2-tailed). |

|  |
| --- |
| **ANOVAa** |
| Model | Sum of Squares | df | Mean Square | F | Sig. |
| 1 | Regression | 7713004.857 | 1 | 7713004.857 | 19.239 | .002b |
| Residual | 3207267.543 | 8 | 400908.443 |  |  |
| Total | 10920272.400 | 9 |  |  |  |
| a. Dependent Variable: Expenditure  |
| b. Predictors: (Constant), Income |

|  |
| --- |
| **Coefficientsa** |
| Model | Unstandardized Coefficients | Standardized Coefficients | t | Sig. |
| B | Std. Error | Beta |
| 1 | (Constant) | -494.976 | 839.416 |  | -.590 | .572 |
| Income | .723 | .165 | .840 | 4.386 | .002 |
| a. Dependent Variable: Expenditure  |

Example 11:

**One Way ANOVA**

Suppose in an industrial experiment that an engineer is interested in how the mean absorption of moisture in concrete varies among 5 different concrete aggregates. The samples are exposed to moisture for 48 hours. It is decided that 6 samples are to be tested for each aggregate, requiring a total of 30 samples to be tested. The data are recorded in Table 13.1.

The model for this situation may be set up as follows. There are 6 observations taken from each of 5 populations with means *μ*1*, μ*2*, . . . , μ*5, respectively. We may wish to test

*H*0: *μ*1 = *μ*2 = *· · ·* = *μ*5*,*

*H*1: At least two of the means are not equal*.*

Table 13.1: Absorption of Moisture in Concrete Aggregates

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Aggregate | 1 | 2 | 3 | 4 | 5 |
|  | 551 | 595 | 639 | 417 | 563 |
|  | 457 | 580 | 615 | 449 | 631 |
|  | 450 | 508 | 511 | 517 | 522 |
|  | 731 | 583 | 573 | 438 | 613 |
|  | 499 | 633 | 648 | 415 | 656 |
|  | 632 | 517 | 677 | 555 | 679 |

|  |
| --- |
| **ANOVA** |
| VAR00001  |
|  | Sum of Squares | df | Mean Square | F | Sig. |
| Between Groups | 85356.467 | 4 | 21339.117 | 4.302 | .009 |
| Within Groups | 124020.333 | 25 | 4960.813 |  |  |
| Total | 209376.800 | 29 |  |  |  |
| **Multiple Comparisons** |
| Dependent Variable: VAR00001  |
| LSD  |
| (I) VAR00002 | (J) VAR00002 | Mean Difference (I-J) | Std. Error | Sig. | 95% Confidence Interval |
| Lower Bound | Upper Bound |
| 1.00 | 2.00 | -16.00000 | 40.66454 | .697 | -99.7502 | 67.7502 |
| 3.00 | -57.16667 | 40.66454 | .172 | -140.9168 | 26.5835 |
| 4.00 | 88.16667\* | 40.66454 | .040 | 4.4165 | 171.9168 |
| 5.00 | -57.33333 | 40.66454 | .171 | -141.0835 | 26.4168 |
| 2.00 | 1.00 | 16.00000 | 40.66454 | .697 | -67.7502 | 99.7502 |
| 3.00 | -41.16667 | 40.66454 | .321 | -124.9168 | 42.5835 |
| 4.00 | 104.16667\* | 40.66454 | .017 | 20.4165 | 187.9168 |
| 5.00 | -41.33333 | 40.66454 | .319 | -125.0835 | 42.4168 |
| 3.00 | 1.00 | 57.16667 | 40.66454 | .172 | -26.5835 | 140.9168 |
| 2.00 | 41.16667 | 40.66454 | .321 | -42.5835 | 124.9168 |
| 4.00 | 145.33333\* | 40.66454 | .001 | 61.5832 | 229.0835 |
| 5.00 | -.16667 | 40.66454 | .997 | -83.9168 | 83.5835 |
| 4.00 | 1.00 | -88.16667\* | 40.66454 | .040 | -171.9168 | -4.4165 |
| 2.00 | -104.16667\* | 40.66454 | .017 | -187.9168 | -20.4165 |
| 3.00 | -145.33333\* | 40.66454 | .001 | -229.0835 | -61.5832 |
| 5.00 | -145.50000\* | 40.66454 | .001 | -229.2502 | -61.7498 |
| 5.00 | 1.00 | 57.33333 | 40.66454 | .171 | -26.4168 | 141.0835 |
| 2.00 | 41.33333 | 40.66454 | .319 | -42.4168 | 125.0835 |
| 3.00 | .16667 | 40.66454 | .997 | -83.5835 | 83.9168 |
| 4.00 | 145.50000\* | 40.66454 | .001 | 61.7498 | 229.2502 |
| \*. The mean difference is significant at the 0.05 level. |