Chapter (6)
Discrete Probability Distributions
Examples

Example (1)

Two balanced dice are rolled. Let X be the sum of the two dice. Obtain the probability distribution of X.

Solution

When the two balanced dice are rolled, there are 36 equally likely possible outcomes as shown below:

$$S = \begin{Bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{Bmatrix}$$

- The possible values of X are: 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12.
- The discrete probability distribution of X is given by

X	P(X)
2	1 / 36
3	2/36
4	3/36
5	4/36
6	5/36
7	6/36
8	5 / 36
9	4/36
10	3/36
11	2/36
12	1 / 36
Total	36/36 =1

Example (2)

The number of persons *X*, in Al Riyadh family chosen at random has the following probability distribution:

X	1	2	3	4	5	6	7	8	Total
P(X)	0.34	0.44	0.11	0.06	0.02	0.01	0.01	0.01	1

1/ Find the average family size { E(X)}

2/ Find the variance of probability distribution

Solution

X	P(X)	XP(X)	$(X-\mu)$	$(X-\mu)^2$	$P(X)*(X-\mu)^2$
1	0.34	0.34	-1.1	1.21	0.4114
2	0.44	0.88	-0.1	0.01	0.0044
3	0.11	0.33	0.9	0.81	0.0891
4	0.06	0.24	1.9	3.61	0.2166
5	0.02	0.1	2.9	8.41	0.1682
6	0.01	0.06	3.9	15.21	0.1521
7	0.01	0.07	4.9	24.01	0.2401
8	0.01	0.08	5.9	34.81	0.3481
		2.1			1.63

$\mu = x\,P(x) = E(x)$ = 2.1 = 3 person

$\sigma^2 = (x-\mu)^2\,P(x)$ = 1.63

Example (3)

John Ragsdale sells new cars for Pelican Ford. John usually sells the largest number of cars on Saturday. He has developed the following probability distribution for the number of cars he expects to sell on a particular Saturday.

x	0	1	2	3	4	**Total**
$p(x)$	0.10	0.20	0.30	0.30	0.10	$\sum p(x) = 1$

Find:

1. Expected value of x(The mean of probability distribution)
2. σ^2 (The variance of probability distribution)

Solution:

$\mu = x\,P(x) = E(x)$

$\sigma^2 = (x-\mu)^2 P(x)$

x	$p(x)$	$x\,P(x)$	$(x-\mu)$	$(x-\mu)^2$	$(x-\mu)^2\ p(x)$
0	0.10	0	-2.1	4.41	0.441
1	0.20	0.20	-1.1	1.21	0.242
2	0.30	0.60	-0.1	0.01	0.003
3	0.30	0.90	0.9	0.81	0.243
4	0.10	0.40	1.9	3.61	0.361
	1	2.1			1.29

$$\mu = \sum x\,P(x) = E(x) = 0\times 0.10 + 1\times 0.20 + 2\times 0.30 + 3\times 0.30 + 4\times 0.10 = 2.1$$

$$(x-\mu)^2 P(x) = 4.41\times 0.10 + 1.21\times 0.20 + 0.01\times 0.30 + 0.81\times 3 + 3.61\times 4$$

$$= 0.441 + 0.242 + 0.003 + 0.243 + 0.361 = 1.29$$

Example (4)

Which one of these tables is actually a probability distribution?

x	P(x)
5	0.3
10	0.3
15	0.2
20	0.4

A

x	P(x)
5	0.1
10	0.3
15	0.2
20	0.4

B

x	P(x)
5	0.5
10	0.3
15	-0.2
20	0.4

C

Binomial distribution

Example (5)

If the experiment is tossing a coin 6 times, what is the probability of:

1. Getting two heads.
2. Getting at least 4 heads.
3. Getting at most one head.
4. Getting at least two heads
5. Find mean, variance and deviation

Solution:

$$n = 6 \qquad \pi = \frac{1}{2} \qquad 1-\pi = 1-\frac{1}{2} = \frac{1}{2}$$

$$P(x) = {}_nC_x \pi^x (1-\pi)^{n-x} = \frac{n!}{x!(n-x)!}\pi^x(1-\pi)^{n-x}$$ **, x = 0, 1, … , n**

1.
$$P(x=2) = {}_6C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{6-2} = \frac{6!}{2!(6-2)!}\left(\frac{1}{4}\right)\left(\frac{1}{16}\right) = \frac{(6)(5)4!}{(2)\,4!}\left(\frac{1}{64}\right) = \frac{15}{64} = 0.23$$

2. $P(x \ge 4) = P(x=4) + P(x=5) + P(x=6)$

$$P(x \ge 4) = {}_6C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^{6-4} + {}_6C_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^{6-5} + {}_6C_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^{6-6}$$

$$= \frac{6!}{4!(6-4)!}\left(\frac{1}{16}\right)\left(\frac{1}{4}\right) + \frac{6!}{5!(6-5)!}\left(\frac{1}{32}\right)\left(\frac{1}{2}\right) + \frac{6!}{6!(6-6)!}\left(\frac{1}{64}\right)\left(\frac{1}{2}\right)^0$$

$$= 15\left(\frac{1}{64}\right) + 6\left(\frac{1}{64}\right) + \left(\frac{1}{64}\right) = \frac{22}{64} = 0.34$$

3. $P(x \le 1) = P(x=0) + P(x=1)$

$$= {}_6C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{6-0} + {}_6C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{6-1} = \left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^6 + 6\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{6-1}$$

$$= \left(\frac{1}{64}\right) + 6\left(\frac{1}{64}\right) = \frac{7}{64} = 0.11$$

4. $P(x \ge 2) = 1 - p(x \le 1) = 1 - 0.11 = 0.89$

5. $\mu = n\pi = 6\left(\frac{1}{2}\right) = \frac{6}{2} = 3$

$$\mathrm{var}\,iance(\sigma^2) = n\pi(1-\pi) = 6\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{6}{4} = 1.5$$

$$\sigma = \sqrt{1.5} = 1.22$$

Example (6)

Over a long period of time it has been observed that a given marksman can hit a target on a single trial with probability equal to 0.8. Suppose he fires four shots at the target. Answer the following:

1. What is the probability that he will hit the target exactly two times?
2. What is the probability that he will hit the target at least once?
3. Find mean, variance and standard deviation.

Solution:

$n = 4 \qquad \pi = 0.8 \qquad 1-\pi = 1-0.8 = 0.2$

1.

$$P(x=2) = {}_4C_2\,(0.8)^2(0.2)^{4-2} = \frac{4!}{2!(4-2)!}(0.8)^2(0.2)^2$$

$$= \frac{24}{4}(0.64)(0.04) = 6(0.64)(0.04) = 0.15$$

2.

$$P(x \ge 1) = 1 - P(x<1) = 1 - P(x=0) = 1 - \left[\,{}_4C_0\,(0.8)^0(0.2)^{4-0}\right]$$

$$= 1 - \left[\frac{4!}{0!(4-0)!}(0.8)^0(0.2)^4\right] = 1 - [(1)(0.0016)] = 1 - 0.0016 = 0.99$$

3. $\mu = n\pi = 4(0.8) = 3.2$

$$\sigma^2 = n\pi(1-\pi) = 4(0.8)(0.2) = 0.64$$

$$\sigma = \sqrt{0.64} = 0.8$$

Example (7)

Find the probability of guessing correctly exactly 6 of the 10 answers on a true-false examination.

Solution:

$n = 10 \qquad \pi = \frac{1}{2} \qquad 1-\pi = \frac{1}{2}$

$$P(x=6) = {}_{10}C_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^{10-6} = 210\left(\frac{1}{64}\right)\left(\frac{1}{16}\right) = \frac{210}{1024} = 0.21$$

Hypergeometric Distribution

Example (8)

Horwege Discount Brokers plans to hire 5 new financial analysts this year. There is a pool of 12 approved applicants, and George Horwege, the owner, decides to randomly select those who will be hired .There are 8 men and 4 women among the approved applicants. What is the probability that 3 of the 5 hired are men?

Solution:

$N = 12 \quad , n = 5 \quad , S = 8 \ , \quad X = 3$

$\because \frac{n}{N} > 0.05$

$\frac{8}{12} = 0.67 > 0.05$

$$p(X = 3)\frac{(sC_X)(_{N-S}C_{n-x})}{_NC_n} = \frac{(8C_3)(_{12-8}C_{5-3})}{_{12}C_5} = 0.4242$$

Example (9)

Suppose we randomly select 5 cards without replacement from an ordinary deck of playing cards.

1- What is the probability of getting exactly 2 red cards (i.e., hearts or diamonds)?
2- What is the probability of obtaining 2 or fewer hearts?

Solution:

1- This is a hypergeometric experiment in which we know the following:

- N = 52; since there are 52 cards in a deck.
- S = 26; since there are 26 red cards in a deck.
- n = 5; since we randomly select 5 cards from the deck.
- x = 2; since 2 of the cards we select are red.

We plug these values into the hypergeometric formula as follows:

$$p(X = 2)\frac{(sC_X)(_{N-S}C_{n-x})}{_NC_n} = \frac{(26C2)(_{52-26}C_{5-2})}{52C5} = 0.3251$$

Thus, the probability of randomly selecting 2 red cards is 0.3251.

2-

N = 52; since there are 52 cards in a deck.

S = 13; since there are 13 hearts in a deck.

n = 5; since we randomly select 5 cards from the deck.

x = 0 to 2; since our selection includes 0, 1, or 2 hearts.

$$P(X \le 2) = P(2) + P(1) + P(0)$$
$$= \frac{(13C2)(52-13C_{5-2})}{52C_5} + \frac{(13C1)(52-13C_{5-1})}{52C_5} + \frac{(13C0)(52-13C_{5-0})}{52C_5} =$$

$$0.2743 + 0.4114 + 0.2215 = 0.9072$$

Poisson distribution

Example (10)

The average number of traffic accidents on a certain section of highway is two per week assume that the number of accidents follows a Poisson distribution with $\mu = 2$.

1. Find the probability of no accidents on this section of highway during a 1-week period.
2. Find the probability of at most three accidents on this section of highway during a 1-week.
3. Find the probability of at least four accidents during a 1-week
4. Find variance and standard deviation.

Solution:

$\mu = 2$

1. $$P(x=0) = \frac{\mu^x}{e^{\mu}\, x!} = \frac{2^0}{(2.718)^2\, 0!} = \frac{1}{7.387524} = 0.1353$$

2. $P(x \le 3) = P(x=0) + P(x=1) + P(x=2) + P(x=3)$

$$= \frac{2^0}{(2.718)^2 0^!} + \frac{2^1}{(2.718)^2 1^!} + \frac{2^2}{(2.718)^2 2!} + \frac{2^3}{(2.718)^2 3^!}$$

$$\frac{1}{7.387524} + \frac{2}{7.387524} + \frac{4}{14.7750} + \frac{8}{44.3251}$$

$$= 0.1354 + 0.2707 + 0.2707 + 0.1805 = 0.8573$$

3.. $P(x \ge 4) = 1 - P(x \le 3) = 1 - 0.8573 = 0.1427$

4. $Var(x) = \mu = 2$

$\sigma = \sqrt{2} = 1.414$

Example (11)

Suppose a life insurance company insures the lives of 3000 men aged 42. If actuarial studies show the probability that an 42-year-old man will die in a given year to be o.oo1, find the exact probability that the company will have to pay x = 4 claims during a given year.

Solution:

$u = n\pi = 3000(0.001) = 3$

$$P(x=4) = \frac{3^4}{(2.718)^3 4^!} = \frac{81}{(20.0792)(24)} = \frac{81}{481.9008} = 0168$$

Example (12)

The number of people arriving to a movie theater in a 30-minute time period is best modeled using which of the following distributions?

A) Normal	B) Binomial	C) Hypergeometric	D) Poisson

A manufacturing process produces defective items 5% of the time. A sample of 20 items is taken for quality control. If you wanted to determine the probability that exactly 2 of the 20 items are defective, which distribution should be used?

A) Uniform	B) Binomial	C) Hypergeometric	D) Poisson

Which of the following is not a requirement of a binomial distribution?

A) A constant probability of success.
B) Only two possible outcomes.
C) A fixed number of trials.
D) Equally likely outcomes