Exercises -1-

Question 1:

A furniture manufacturer makes wooden tables and chairs. The production process involves two types of labor: carpentry and finishing. A **table** requires 2 hours of carpentry and 1 hour of finishing, and a **chair** requires 3 hours of carpentry and 1/2 hour of finishing. The profit is $35 per table and $20 per chair. The manufacturer's employees can supply a maximum of 108 hours of carpentry work and 20 hours of finishing work per day. **How many tables and chairs should be made each day to maximize the profit?**

Answer:

Let x_1 = number of tables made per day

x_2 = number of chairs made per day

Linear programming model:

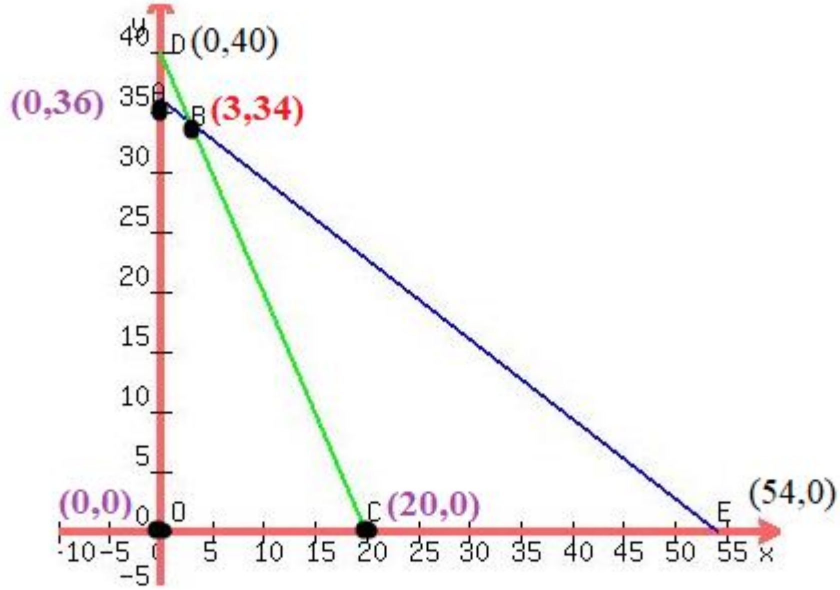
Max $Z = 35\,x_1 + 20\,x_2$

Subject to :

$2x_1 + 3x_2 \leq 108$ (carpentry)

$x_1 + 0.5x_2 \leq 20$ (finishing)

$x_1, x_2 \geq 0$ (nonnegativity)



$$2x_1 + 3x_2 = 108$$
$$-2 * \left(x_1 + \frac{1}{2}x_2 = 20 \right) \quad +$$

$$2x_1 + 3x_2 = 108$$
$$-2x_1 - x_2 = -40 \quad +$$

$$2x_2 = 68$$
$$\boxed{x_2 = 34}$$
$$\Rightarrow \boxed{x_1 = 3}$$

(3,34)

(x_1, x_2)	Objective function $Z = 35\,x_1 + 20\,x_2$
(0,0)	0
(0,36)	720
(20,0)	700
(3,34)	785

There should be [3] tables and [34] chairs made each day to maximize profit, and that would yield a profit of $785.

Question 2:

A small manufacturer employs 5 skilled men and 10 semi - skilled men and makes an article in two qualities , a deluxe model and an ordinary model. The making of a **deluxe model** requires 2 hours work by a skilled man and 2 hours work by a semi - skilled man. The **ordinary model** requires 1 hour by a skilled man and 3 hours by a semi - skilled man . By work rules no man can work more than 8 hours per day. The manufacturers clear profit of the deluxe model is L.E. 10 and of the ordinary model L.E. 8 . **How many of each type should be made in order to maximize his total daily profit.**

Answer:

Let x_1 = deluxe model

x_2 = ordinary model

Linear programming model:

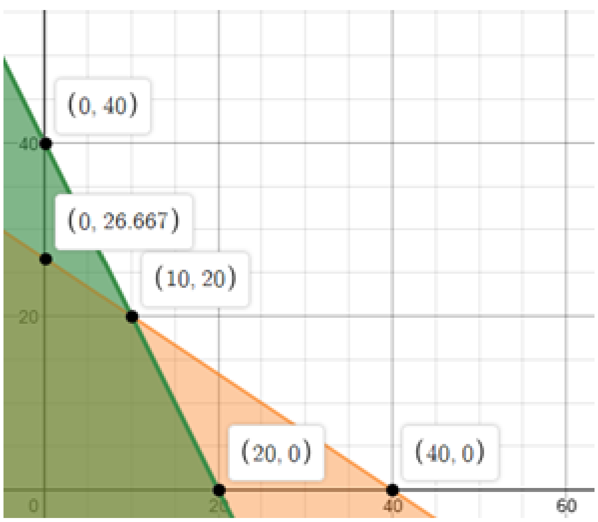
Max Z= 10 x_1 + 8 x_2

Subject to

$2x_1 + 1x_2 \leq 40$ (skilled men)

$2x_1 + 3x_2 \leq 80$ (semi-skilled men)

$x_1, x_2 > 0$ (nonnegativity)



$$2x_1 + x_2 = 40$$

$$-1 * (2x_1 + 3x_2 = 80)$$

$$2x_1 + x_2 = 40$$

$$-2x_1 - 3x_2 = -80$$

$$2x_2 = 40$$

$$x_2 = 20$$

$$x_1 = 10$$

(10, 20)

(x_1, x_2)	Objective function Z=10 x_1 + 8 x_2
(0,0)	0
(0,26.67)	213.36
(20,0)	200
(10,20)	260

Thus, the maximum profit is 260 obtained when 10 units of deluxe model and 20 unit of ordinary model is produced

Question 3:

The Manager of an oil refinery has to decide on the optimal mix of two possible blending processes of which the inputs and outputs per production run as follows:

Process	Input		Output	
	Crude A	Crude B	Gasoline X	Gasoline Y
1	5	3	5	8
2	4	5	4	4

The maximum amount available crude A and B are 200 units and 150 units respectively. The market requirement shows that at least 100 units of gasoline X and 80 units of gasoline Y must be produced. The profit per production run from process 1and process 2 are 3$ and 4$ respectively. **Formulate the problem as linear programming problem**.

Answer:

Maximize $Z = 3x_1 + 4x_2$, subject to: $5x_1 + 4x_2 \leq 200$, $3x_1 + 5x_2 \leq 150$, $5x_1 + 4x_2 \geq 100$, $8x_1 + 4x_2 \geq 80$ and $x_1, x_2 \geq 0$.