Exercise #5

Q1: A medical research team wishes to assess the usefulness of a certain symptom (call is S) in the diagnosis of a particular disease. In a random sample of 775 patients with the disease. 744 reported having the symptom. In an independent random sample of 1380 subjects without the disease, 21 reported having that they had the symptom.

	With Disease (D)	Without Disease (D^C)	Total
Positive (T)	744	21	765
Negative (T^C)	31	1359	1390
Total	775	1380	2155

1. **what is false positive?**

(A) Probability that result of the test is positive given that patient has disease.
(B) **Probability** that result of the test is negative given that patient has disease.
(C) Probability that result of the test is positive given that patient doesn't have disease.
(D) Probability that result of the test is negative given that patient doesn't have disease.

2. **What is false negative?**

(E) Probability that result of the test is positive given that patient has disease.
(F) **Probability** that result of the test is negative given that patient has disease.
(G) Probability that result of the test is positive given that patient doesn't have disease.
(H) Probability that result of the test is negative given that patient doesn't have disease.

3. **Compute the sensitivity of the symptom?**

$$P(T|D) = \frac{n(T\cap D)}{n(D)} = \frac{744}{775} = 0.96$$

4. **Compute the specificity of the symptom?**

$$P(\bar{T}|\bar{D}) = \frac{n(\bar{T}\cap\bar{D})}{n(\bar{D})} = \frac{1359}{1380} = 0.9848$$

5. **Suppose it is know that the rate of the disease in the general population is 0.001. What is the predictive value negative of the symptom?**

$$P(\bar{D}) = 1 - 0.001 = 0.999$$

$$P(\bar{D}|\,\bar{T}) = \frac{Specificity * P(\bar{D})}{Specificity * P(\bar{D}) + (1 - Sensitivity) * P(D)}$$

$$= \frac{(0.9848)(0.999)}{(0.9848)(0.999) + (1 - 0.96\,)(0.001)} \quad = 0.999$$

6. **What is the predictive value positive of the symptom?**

$$P(D|T) = \frac{sensitivity\ \ P(D)}{sensitivity\ \ P(D) + (1 - Specificity)P(\bar{D})}$$

$$= \frac{(0.96)(\,0.001)}{(0.96)(\,0.001) + (1 - 0.9848)(0.999)} = 0.0595$$

H.W :Find the predictive value positive and the predictive value negative for the symptom for the following hypothetical disease 0.0001, 0.01, and 0.1?

	Predictive value positive	**Predictive value negative**
P(D)=0.1	**0.8753**	**0.9955**
P(D)=0.01	**0.3895**	**0.9996**
P(D)=0.0001	**0.006277**	**0.999996**

Q2: In article entitled "Bucket-Handle Meniscal Tears of the Knee: Sensitivity and Specificity of MRI signs" Dorsay and Helms (A-6) performed a retrospective study of 72 knees scanned by MRI. One of the indicators they examined was the absence of the "bow tie sign" in the MRI as evidence of a bucket-handle or "bucket-handle type" tar of the meniscal. In the study, surgery confirmed that 43 of the 73 cases were bucket-handle tears. The cases may be cross-classified by "bow tie sign" status and surgical results as follows:

	Tears Surgically Confirmed (D)	Tears Surgically confirmed as not Present ($\dot{D}^C$)	Total
Positive Test (absent bow tie sign) (T)	38	10	48
Negative Test (bow tie sign Present) ($\dot{T}^C$)	5	18	23
Total	43	28	71

1. what is false negative?

(A) Probability that result of the test is positive given that patient has disease.
(B) **Probability** that result of the test is negative given that patient has disease.
(C) Probability that result of the test is positive given that patient doesn't have disease.
(D) Probability that result of the test is negative given that patient doesn't have disease.

2. What is false positive?

(A) Probability that result of the test is positive given that patient has disease.
(B) **Probability** that result of the test is negative given that patient has disease.
(C) Probability that result of the test is positive given that patient doesn't have disease.
(D) Probability that result of the test is negative given that patient doesn't have disease.

3. Compute the sensitivity of the symptom?

$$P(T|D) = \frac{n(T \cap D)}{n(D)} = \frac{38}{43} = 0.8837$$

4. **Compute the specificity of the symptom?**

$$\boldsymbol{P}(\bar{T}|\bar{D}\,) = \frac{n(\,\bar{T} \cap \bar{D})}{n(\overline{D}\,)} = \frac{\mathbf{18}}{\mathbf{28}} = \mathbf{0.6429}$$

5. **Suppose it is known that the rate of the disease in the general population is 0.001. What is the predictive value positive of the symptom?**

$$\boldsymbol{P}(\boldsymbol{D}) = \mathbf{0.001}\,; \quad \boldsymbol{P}(\overline{\boldsymbol{D}}) = \mathbf{1 - 0.001 = 0.999}$$

$$P(D|T) = \frac{sensitivity\ \ P(D)}{sensitivity\ \ P(D) + (1 - Specificity)P(\bar{D})}$$

$$= \frac{(\,\mathbf{0.8837})(\,0.001)}{(\,\mathbf{0.8837})(\,0.001) + (1 - \mathbf{0.6429})(0.999)} = 2.47 \times 10^{-3} = 0.00247$$

6. **What is the predictive value negative of the symptom?**

$$P(\bar{D}|\,\bar{T}) = \frac{Specificity * P(\bar{D})}{Specificity * P(\bar{D}) + (1 - Sensitivity) * P(D)}$$

$$= \frac{(\mathbf{0.6429})(\mathbf{0.999})}{(\mathbf{0.6429})(\mathbf{0.999}) + (\mathbf{1 -\ 0.8837}\,)(\mathbf{0.001})} \quad = \mathbf{0.9998}$$

<u>H.W</u>

Repeat exercise question 2 at disease rate 0.01 P(D) = 0.01

Predictive value positive=0.0244

Predictive value negative=099

Chapter (4) :

Q1: For the following probability distribution

x	0	1	2	3	4	5
f(x)	0.05	0.15	K=0.15	0.25	0.3	0.1

1. The value of K is ……k=1-(0.05+0.15+0.25+0.3+0.1)=0.15………
2. The value of x with the highest probability is ……x=4…….
3. P(X< 3) = P(X=0) + P(X=1) + P(X=2)

   = 0.05+0.15 +0.15 = 0.35
4. P(1 ≤ X < 4) = P(X=1) + P(X=2)+ P(X=3)
   = 0.15 + 0.15 +0.25 = 0.55
5. Mean of x is $= \sum_x x\, f(x) = 2.9$

x	0	1	2	3	4	5	
$f(x)$	0.05	0.15	0.15	0.25	0.3	0.1	
$x.f(x)$	0	0.15	0.3	0.75	1.2	0.5	The total= 2.9

Then

6. Standard deviation of X is = 1.3748
7. Variance is $= 1.3748^2$ =1.89

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Q2: For a population of families, Let

X = the number of children in primary school.

We randomly choose one and the cumulative distributed is given below

X	P(X≤x)	$P(X = x)$
0	0.12	0.12
1	0.36	0.36-0.12= 0.24
2	0.72	0.72-0.36=0.36
3	0.95	0.95-0.72= 0.23
5	1	1-0.95= 0.05

1. P(X=2) = 0.36
2. P(X =4) = 0
3. P(1.5 ≤ X ≤2) = P(X=2) =0.36
4. P(X > 2) = 1- $P(X \le 2) = 1 - 0.72$ =0.28

Or

P(X > 2) = P(X=3)+ P(X=5)= 0.23+0.05= 0.28

5. **Mean is** $=\sum_x x\, f(x)$ **=1.9**

X	$P(X = x)$	$x.f(x)$
0	**0.12**	**0**
1	**0.36-0.12= 0.24**	**0.24**
2	**0.72-0.36=0.36**	**0.72**
3	**0.95-0.72= 0.23**	**0.69**
5	**1-0.95= 0.05**	**0.25**
		The total =1.9

6. **Variance is =** $(1.17898)^2$ **= 1.39**

<u>Q3</u>: Given the following probability distribution of a discrete random variable X representing the number of defective teeth of the patient visiting a certain dental clinic

X	P(X= x)
1	**0.25**
2	**0.35**
3	**0.20**
4	**0.15**
5	**K= 0.05**

1. **The value of the k is …0.05**
k= 1- (0.25+0.35+0.20+0.15) = 0.05

2. **P (x < 3) = P(X=1)+ P(X=2)**
= 0.25+0.35 = 0.6

3. **P(X ≤ 3) = P(X=1)+ P(X=2)+ P(X=3)**
= 0.25+0.35+0.20 = 0.8

4. **P(X < 6) = P(X=1)+ P(X=2)+ P(X=3)+ P(X=4)+ P(X=5)**
= 1

5. **P(X = 3.5) =0**

6. **Probability that the patient has at least 4 defective Teeth ……0.20…….**
P(X ≥ 4) = P(X=4)+ P(X=5)= 0.05+0.15= 0.20

7. **Probability that the patient has at most 2 defective Teeth ……0.6…….**
P(X ≤ 2) = P(X=1)+ P(X=2)= 0.25+0.35 = 0.6

8. **The expected number of defective teeth (Mean) = 2.4**

9. **The variance of X is ……1.34…**