

**Exercise -5-**

**Solve the following LPP using simplex method:**

**1- Max  $Z = 3X_1 + 4X_2$**

Subject to

$15X_1 + 10X_2 \leq 300$

$2.5X_1 + 5X_2 \leq 110$

$X_1 \geq 0, X_2 \geq 0$

**Solution: (we have canonical form)**

The standard form of LPP

$Max Z - 3X_1 - 4X_2 = 0$

Subject to

$15X_1 + 10X_2 + S_1 = 300$

$2.5X_1 + 5X_2 + S_2 = 110$

$X_1 \geq 0, X_2 \geq 0, S_1 \geq 0, S_2 \geq 0$

We have  $m=2$  and  $n=4$ , thus  $n-m=2$  ( Non-basic variable which equal zero)

Entering  
Variable  
(pivot Column)

Iteration 1

Basic Variables	$x_1$	$x_2$	$S_1$	$S_2$	Solution	Ratio
Z	-3	-4	0	0	0	
$S_1$	15	10	1	0	300	300/10=30
$S_2$	2.5	5	0	1	110	110/5=22

Leaving Variable

*Row 3  
pivot element*

Basic Variables	$x_1$	$x_2$	$S_1$	$S_2$	Solution	Ratio
Z	-3	-4	0	0	0	
$S_1$	15	10	1	0	300	300/10=30
$x_2$	0.5	1	0	0.2	22	110/5=22

Row 2  $- (10) \text{ Row 3} =$   
new Row2  
  
Row 1  $- (-4) \text{ Row 3} =$   
new Row1

Iteration 2

Basic Variables	$x_1$	$x_2$	$S_1$	$S_2$	Solution	Ratio
Z	-1	0	0	4/5	88	
$S_1$	10	0	1	-2	80	80/10=8
$x_2$	0.5	1	0	0.2	22	22/0.5=44

$-(-1) \text{ (Row2 / 10)} +$   
Row 1 = new Row1  
  
 $-(0.5) \text{ (Row2/10)} +$   
Row 3 = new Row3

Basic Variables	$x_1$	$x_2$	$S_1$	$S_2$	Solution
Z	0	0	0.1	0.6	96
$x_1$	1	0	0.1	-0.2	8
$x_2$	0	1	-0.05	0.3	18

The optimal solution:  $x_1 = 8, x_2 = 18, Z = 96$

**2- Min  $Z = -3X_1 + X_2$**

Subject to

$X_1 + X_2 \leq 5$

$2X_1 + X_2 \leq 8$

$X_1 \geq 0, X_2 \geq 0$

**Solution:**

The standard form of LPP

**Min  $Z + 3X_1 - X_2 = 0$**

Subject to

$X_1 + X_2 + S_1 = 5$

$2X_1 + X_2 + S_2 = 8$

$X_1 \geq 0, X_2 \geq 0, S_1 \geq 0, S_2 \geq 0$

We have  $m= 2$  and  $n= 4$ , thus  $n-m=2$  ( Non-basic variable which equal zero)

Basic Variables	$x_1$	$x_2$	$S_1$	$S_2$	Solution	Ratio
Z	3	-1	0	0	0	
$S_1$	1	1	1	0	5	5/1
$S_2$	2	1	0	1	8	8/2=4

Basic Variables	$x_1$	$x_2$	$S_1$	$S_2$	Solution	Ratio
Z	0	-5/2	0	-3/2	-12	
$S_1$	0	1/2	1	-1/2	1	
$x_1$	1	1/2	0	1/2	4	

We note all coefficient of objective function are non-positive values. Thus, the optimal solution is  $x_1 = 4, S_1 = 1, x_2 = 0, S_2 = 0, Z = -12$

**3- Max  $Z = 200X_1 + 140X_2$**

Subject to

$3X_1 \leq 6000$

$2.9X_2 \leq 8000$

$2.5X_1 + 2X_2 \leq 7500$

$1.3X_1 + 1.5X_2 \leq 5000$

$X_1 \geq 0, X_2 \geq 0$

**Solution: (we have canonical form)**

The standard form of LPP

$\text{Max } Z - 200X_1 - 140X_2 = 0$

Subject to

$3X_1 + S_1 = 6000$

$2.9X_2 + S_2 = 8000$

$2.5X_1 + 2X_2 + S_3 = 7500$

$1.3X_1 + 1.5X_2 + S_4 = 5000$

$X_1 \geq 0, X_2 \geq 0, S_1 \geq 0, S_2 \geq 0, S_3 \geq 0, S_4 \geq 0$

We have  $m = 4$  and  $n = 6$ , thus  $n - m = 2$  ( Non-basic variable which equal zero)

Iteration 1								
Basic Variables	$x_1$	$x_2$	$S_1$	$S_2$	$S_3$	$S_4$	Solution	Ratio
Z	-200	-140	0	0	0	0	0	
$S_1$	3	0	1	0	0	0	6000	6000/3=2000
$S_2$	0	2.9	0	1	0	0	8000	-----
$S_3$	2.5	2	0	0	1	0	7500	7500/2.5=3000
$S_4$	1.3	1.5	0	0	0	1	5000	5000/1.3=3846

*New pivot row = current pivot row / pivot element*  
*All other rows*  
*New row = (current row) - (pivot column coefficient) (New pivot row)*

$[-200 \ -140 \ 0 \ 0 \ 0 \ 0 \ 0]$ - (-200)*	$[0 \ 2.9 \ 0 \ 1 \ 0 \ 0 \ 8000]$ - (0)*	$[2.5 \ 2 \ 0 \ 0 \ 1 \ 0 \ 7500]$ - (2.5)*	$[1.3 \ 1.5 \ 0 \ 0 \ 0 \ 1 \ 5000]$ - (1.3)*
$[1 \ 0 \ 1/3 \ 0 \ 0 \ 0 \ 2000]$ = $[0 \ -140 \ 200/3 \ 0 \ 0 \ 0 \ 400000]$	$[1 \ 0 \ 1/3 \ 0 \ 0 \ 0 \ 2000]$ = $[0 \ 2.9 \ 0 \ 1 \ 0 \ 0 \ 8000]$	$[1 \ 0 \ 1/3 \ 0 \ 0 \ 0 \ 2000]$ = $[0 \ 2 \ -2.5/3 \ 0 \ 1 \ 0 \ 2500]$	$[1 \ 0 \ 1/3 \ 0 \ 0 \ 0 \ 2000]$ = $[0 \ 1.5 \ -1.3/3 \ 0 \ 0 \ 1 \ 2400]$

Iteration 2								
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Basic Variables	$x_1$	$x_2$	$S_1$	$S_2$	$S_3$	$S_4$	Solution	Ratio
Z	0	-140	200/3	0	0	0	400000	
$x_1$	1	0	1/3	0	0	0	2000	----
$S_2$	0	2.9	0	1	0	0	8000	8000/2.9=2758.62
$S_3$	0	2	-2.5/3	0	1	0	2500	2500/2=1250
$S_4$	0	1.5	-1.3/3	0	0	1	2400	2400/1.5=1600

[ 0 -140 200/3 0 0 0 400000 ] -(-140)* [0 1 -2.5/6 0 0.5 0 ] = [0 0 25/3 0 70 0 575000]	[1 0 1/3 0 0 0 2000] -(0)* [0 1 -2.5/6 0 0.5 0 ] = [1 0 1/3 0 0 0 2000]	[0 2.9 0 1 0 0 8000 ] -(2.9)* [0 1 -2.5/6 0 0.5 0 ] = [0 0 7.25/6 0 -2.9/2 0 4375]	[0 1.5 -1.3/3 0 0 1 2400 ] -(1.5)* [0 1 -2.5/6 0 0.5 0 ] = [0 0 1.15/6 0 -1.5/2 1 ]
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Iteration 3							
Basic Variables	$x_1$	$x_2$	$S_1$	$S_2$	$S_3$	$S_4$	Solution
Z	0	0	25/3	0	70	0	575000
$x_1$	1	0	1/3	0	0	0	2000
$S_2$	0	0	7.25/6	1	-2.9/2	0	4375
$x_2$	0	1	-2.5/6	0	1/2	0	1250
$S_4$	0	0	1.15/6	0	-1.5/2	1	525

**The optimal solution:**

$$x_1 = 2000, S_2 = 4375, x_2 = 1250, S_4 = 525, Z=575000$$

**H.W 3- Max Z = 30X<sub>1</sub> + 20X<sub>2</sub> + 5 X<sub>3</sub>**

Subject to

$$2X_1 + X_2 + X_3 \leq 8$$

$$X_1 + 3X_2 - 4X_3 \leq 8$$

$$X_1 \geq 0, X_2 \geq 0, X_3 \geq 0$$

**H.W 4- Max Z = 2X<sub>1</sub> - X<sub>2</sub> + X<sub>3</sub>**

Subject to

$$2X_1 + X_2 \leq 10$$

$$X_1 + 2X_2 - 2X_3 \leq 20$$

$$X_2 + 2X_3 \leq 5$$

$$X_1 \geq 0, X_2 \geq 0, X_3 \geq 0$$

The optimal solution:  $Z = \frac{25}{2}, x_1 = 5, x_2 = 0, x_3 = \frac{5}{2}, s_2 = 20, s_1 = 0, s_3 = 0$