Exercise

Q: A Company has 2 production facilities S1 and S2 with production capacity of 100 and 110 units per week of a product, respectively. These units are to be shipped to 3 warehouses D1, D2 and D3 with requirement of 80,70 and 60 units per week, respectively. The transportation costs (in $) per unit between factories to warehouses are given in the table below.

A)

Destination / Sources	D_1	D_2	D_3	Supply
S_1	1	2	3	100
S_2	4	1	5	110
Demand	80	70	60	

Find initial basic feasible solution (IBFS) to the following transportation problem using NWCM, then optimize the solution using MODI method (Modified Distribution Method or (u - v) method).

Answer:

$$\text{Min } Z = x_{11} + 2x_{12} + 3x_{13} + 4x_{21} + x_{22} + 5x_{23}$$
$$x_{11} + x_{12} + x_{13} \le 100$$
$$x_{21} + x_{22} + x_{23} \le 110$$
$$x_{11} + x_{21} \ge 80$$
$$x_{12} + x_{22} \ge 70$$
$$x_{13} + x_{23} \ge 60$$

Min $Z = \sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij} x_{ij}$

s.t

$\sum_{j=1}^{m} x_{ij} \le s_i$

$\sum_{i=1}^{n} x_{ij} \le d_j$

$\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j = 210$, so we don't need dummy demand or dummy supply.
$min(S_1 = 100, D_1 = 80) = \mathbf{80}$, This satisfies the complete demand of D_1 and leaves 100 - 80 = 20 units with S_1.
$min(S_1 = 20, D_1 = 70) = \mathbf{20}$, This exhausts the capacity of S_1 and leaves 70 - 20 = 50 units with D_2.
$min(S_2 = 110, D_2 = 50) = \mathbf{50}$, This satisfies the complete demand of D_2 and leaves 110 - 50 = 60 units with S_2.
$min(S_2 = 60, D_3 = 60) = \mathbf{60}$, This satisfies S_2 and D_3.

Destination / Sources	D_1	D_2	D_3	Supply		
S_1	1 80	2 20	3	100	20	0
S_2	4	1 50	5 60	110	60	0
Demand	80	70	60			
	0	50 0	0			

Initial feasible solution (IBFS) is:

$$X_{11} = 80,\ X_{12} = 20,\ X_{22} = 50,\ X_{23} = 60$$

The minimum total transportation cost:

$TTC = Z = 80 * 1 + 20 * 2 + 50 * 1 + 60 * 5 = \mathbf{470\$}$

Here, the number of allocated cells = 4 is equal to m + n - 1 = 3 + 2 - 1 = 4

Optimality test using MODI method...

$$\boldsymbol{\delta_{kj}} = v_j + u_i - \boldsymbol{C_{kj}},$$

1. Find u_i and v_j for all occupied cells (i, j), where $v_j + u_i = C_{ij}$
   - Substituting, u_1=0, we get
   - $c_{11} = u_1 + v_1 \Rightarrow v_1 = c_{11} - u_1 \Rightarrow v_1 = 1 - 0 \Rightarrow v_1 = 1$
   - $c12 = u1 + v2 \Rightarrow v2 = c12 - u1 \Rightarrow v2 = 2 - 0 \Rightarrow v2 = 2$
   - $c22 = u2 + v2 \Rightarrow u2 = c22 - v2 \Rightarrow u2 = 1 - 2 \Rightarrow u2 = -1$
   - $c_{23} = u_2 + v_3 \Rightarrow v_3 = c_{23} - u_2 \Rightarrow v_3 = 5 + 1 \Rightarrow v_3 = 6$
2. Find $\boldsymbol{\delta_{kl}} = \boldsymbol{v_l} + \boldsymbol{u_k} - \boldsymbol{C_{kl}}$ for all *unoccupied* cells (k, l). IF all $\boldsymbol{\delta_{kl}} \leq 0$, the solution is optimal solution.
3. Now choose the maximum positive value from all $\boldsymbol{\delta_{kj}}$ **(opportunity cost)** = $\boldsymbol{\delta_{13} = 3}$ and draw a closed path $\boldsymbol{S}1\boldsymbol{D}3 \to \boldsymbol{S}1\boldsymbol{D}2 \to \boldsymbol{S}2\boldsymbol{D}2 \to \boldsymbol{S}2\boldsymbol{D}3$ with plus/minus sign allocation.

   Minimum allocated value among all negative position (-) on closed path $\theta = 20$ Subtract 20 from all (-) and Add it to all (+).

		$V_1=1$	$V_2=2$	$V_3=6$	
	Destination / Sources	D_1	D_2	D_3	Supply
$U_1=0$	S_1	1 80	- 2 20	3 + δ_{13}= 3	100
$U_2=-1$	S_2	4 δ_{21}= - 4	+ 1 50	- 5 60	110
	Demand	80	70	60	

4. Repeat the step 1 to 4, until an optimal solution is obtained.

		$V_1= 1$	$V_2= -1$	$V_3= 3$	
	Destination / Sources	D_1	D_2	D_3	Supply
$U_1= 0$	S_1	1 80	2 δ_{12}=-3	3 20	100
$U_2= 2$	S_2	4 δ_{21}= -1	1 70	5 40	110
	Demand	80	70	60	

We note that all $\boldsymbol{\delta_{kj}} \leq 0$, so final optimal solution is arrived

Therefore, the optimal solution $\boldsymbol{X_{11} = 80},\ \boldsymbol{X_{13} = 20},\ \boldsymbol{X_{22} = 70},\ \boldsymbol{X_{23} = 40}$

$$\boldsymbol{And\ Z = 80 * 1 + 20 * 3 + 70 * 1 + 40 * 5 = 410\$}$$

B) same previous example (A) but change S2 to 130 rather than 110.

Answer:

Destination / Sources	D_1	D_2	D_3	Supply
S_1	1	2	3	100
S_2	4	1	5	130
Demand	80	70	60	230 / 210

Here Total Demand = 210 is less than Total Supply = 230. So, we add a **dummy demand** constraint with 0 unit cost and with allocation 20.

Destination / Sources	D_1	D_2	D_3	D_4(Dummy)	Supply
S_1	1	2	3	0	100
S_2	4	1	5	0	130
Demand	80	70	60	20	230=230

		$V_1=1$	$V_2=2$	$V_3=6$	$V_4=-1$		
	Destination / Sources	D_1	D_2	D_3	D_4(Dummy)	Supply	
$U_1=0$	S_1	1 80	- 2 20	+ 3 $\delta_{13}=3$	0 $\delta_{14}=1$	100	20 0
$U_2=-1$	S_2	4 $\delta_{21}=-4$	+ 1 50	- 5 60	0 20	130	80 20 0
	Demand	80	70	60	20		
		0	50 0	0	0		

We note that not all $\boldsymbol{\delta_{kj}} \leq 0$, so we don't reach to optimal solution yet.

Initial feasible solution (IBFS) is:

$$X_{11} = 80,\ X_{12} = 20,\ X_{22} = 50,\ \ X_{23} = 60,\ \ X_{24} = 20$$

The minimum total transportation cost:

$$TTC = Z = 80 * 1 + 20 * 2 + 50 * 1 + 60 * 5 + 20 * 0 = 470$$

Here, the number of allocated cells = 5 is equal to m + n - 1 = 2 + 4 - 1 = 5

		V_1= 1	V_2= -1	V_3= 3	V_4= -2	
	Destination / Sources	D_1	D_2	D_3	D_4(Dummy)	Supply
U_1= 0	S_1	1 80	2 δ_{12}= -3	3 20	0 δ_{14}=-2	100
U_2= 2	S_2	4 δ_{21}= -1	1 70	5 40	0 20	110
	Demand	80	70	60	20	

We note that all $\delta_{kj} \leq 0$, so final optimal solution is arrived

C) same previous example in part (B) but change D1, D2 and D3 to 90,80 and 100 units per week, respectively.

Answer:

Destination / Sources	D_1	D_2	D_3	Supply
S_1	1	2	3	100
S_2	4	1	5	130
Demand	90	80	100	230 / 270

Here Total Demand = 270 is greater than Total Supply = 230. So, we add a dummy supply constraint with 0 unit cost and with allocation 40.

		V_1= 1	V_2= 2	V_3= 6			
	Destination / Sources	D_1	D_2	D_3	Supply		
U_1= 0	S_1	1 90	- 2 10	+ 3 δ_{13}= 3	100	10	0
U_2= -1	S_2	4 δ_{21}= - 4	+ 1 70	- 5 60	130	60	0
U_3= -6	S_3 (Dummy)	0 δ_{12}= -5	0 δ_{12}= - 4	0 40	40	0	
	Demand	90	80	100	270 / 270		
		0	70	40			
			0	0			

H.W Example: The ICARE Company has three factors located throughout a state with production capacity 40, 15 and 40 gallons. Each day the firm must furnish its four retail shops D1, D2, D3 with at least 25, 55 , and 20 gallons respectively. The transportation costs (in $.) are given below.

Destination / Sources	D_1	D_2	D_3	Supply
S_1	10	7	8	40
S_2	15	12	9	15
S_3	7	8	12	40
Demand	25	55	20	95 / 100

Q: Find the **optimum** transportation schedule and minimum total cost of transportation.

Answer:

The minimum total transportation cost $=7\times40+9\times15+7\times25+8\times15+0\times5=710$