OPER 441: Modeling and Simulation
Exercises Sheet #3

Question1:

Consider the multiplicative congruential generator with (a = 13, m = 64, and seeds X_0 = 1,2,3,4)

a) Does this generator achieve its maximum period for these parameters?

b) Generate a period's worth of uniform random variables from each of the supplied seeds.

Question2:

Consider the multiplicative congruential generator with (a = 11, m = 64, and seeds X_0 = 1,2,3,4)

a) Does this generator achieve its maximum period for these parameters? Why? justify your answer.

b) Generate a period's worth of uniform random variables from each of the supplied seeds.

Question 3:

Analyze the following LCG: $X_i = (11X_{i1} + 5)(\mathrm{mod}(16))$, $X_0 = 1$ using Theo- rem 2.1.

a) What is the maximum possible period length for this generator? Does this generator achieve the maximum possible period length? Justify your answer.

b) Generate 2 pseudo-random uniform numbers for this generator.

Question4:

Analyze the following LCG generator: $X_i = (13X_{i1} + 13)(\mathrm{mod}(16))$, $X_0 = 37$ using Theorem 2.1.

a) What is the maximum possible period length for this generator? Does this generator achieve the maximum possible period length? Justify your answer.

b) Generate 2 pseudo-random uniform numbers for this generator.

Question 5:

Analyze the following LCG generator: $X_i = (4X_{i1} + 3)(\mathrm{mod}(16))$, $X_0 = 11$ using Theorem 2.1.

a) What is the maximum possible period length for this generator? Does this generator achieve the maximum possible period length? Justify your answer.

b) Generate 2 pseudo-random uniform numbers for this generator.

Question 6:

Analyze the following LCG generator: $X_i = (8X_{i1} + 1)(\mathrm{mod}(10))$, $X_0 = 3$ using Theorem 2.1.

a) What is the maximum possible period length for this generator? Does this generator achieve the maximum possible period length? Justify your answer.

b) Generate 2 pseudo-random uniform numbers for this generator.

Question 7:

The following results are from a random sample of 100 uniform(0,1) numbers.

n	100
$\bar{x}$	0.4615
s	0.2915
minimum	0.0102
1st Quartile	0.1841
median	0.4609
3rd Quartile	0.7039
maximum	0.9687
D^+	0.090733
D^{--}	0.080733

a) Form a 95% confidence interval for the mean. State any assumptions you need in order to make this confidence interval.

b) What sample size would be necessary to estimate the mean to within ±0.01 with 95% confidence?

c) What would you conclude based on the Kolmogorov-Smirnov Test results at the $\alpha = 0.01$ level? Justify your answer using statistics.

Question 8:

Consider the following sequence of (0,1) random numbers:

0.943	0.398	0.372	0.943	0.204	0.794
0.498	0.528	0.272	0.899	0.294	0.156
0.102	0.057	0.409	0.398	0.400	0.997

Question 9:

Consider the following set of pseudo-random numbers.

0.2379	0.7551	0.2989	0.247	0.3237
0.2972	0.8469	0.4566	0.6146	0.6723
0.9496	0.2268	0.8699	0.9084	0.5649
0.3045	0.6964	0.1709	0.3387	0.9804
0.1246	0.842	0.6557	0.9672	0.3356
0.3525	0.8075	0.9462	0.9583	0.3807
0.1489	0.5480	0.9537	0.9376	0.8364
0.5095	0.4047	0.9058	0.3795	0.6242
0.5195	0.6545	0.1117	0.3258	0.8589
0.6536	0.3427	0.6653	0.7864	0.5824

a) Test the hypothesis that these numbers are drawn from a U (0, 1) at a 95% confidence level using the Chi-squared goodness of fit test using 10 intervals.
b) Test the hypothesis that these numbers are drawn from a U (0, 1) at a 95% confidence level using K-S Test.

c) Test the hypothesis that these numbers are uniformly distributed within the unit square, {(x, y) : x 2 (0, 1), y 2 (0, 1)} using the 2-D Chi-Squared Test at a 95% confidence level. Use 4 intervals for each of the dimensions.

d) Test the hypothesis that these numbers have a lag-1 correlation of zero. Make an autocorrelation plot of the numbers.

Question 10:

Consider the following discrete distribution of the random variable X whose probability mass function is *p(x)*.

x	0	1	2	3	4
$p(x)$	0.3	0.2	0.2	0.1	0.2

a) Determine the CDF F(x) for the random variable, X.
b) Create a graphical summary of the CDF.
c) Create a look-up table that can be used to determine a sample from the discrete distribution, p(x).
d) Generate 3 values of X using the sequence of (0,1) random numbers in **Question 9**

(starting with the first row, reading across).

Question 11:

Consider the following uniformly distributed random numbers:

U_1	U_2	U_3	U_4	U_5	U_6	U_7	U_8
0.9396	0.1694	0.7487	0.3830	0.5137	0.0083	0.6028	0.8727

a) Generate an exponentially distributed random number with a mean of 10 using the 1st random number.

b) Generate a random variate from a (12,22) discrete uniform distribution using the 2nd random number.