Capter2: Matrices and determinants

1-Matrecis

Q1)Let $A=\begin{bmatrix}3 & 0\\ -1 & 2\\ 1 & 1\end{bmatrix}, B=\begin{bmatrix}4 & -1\\ 0 & 2\end{bmatrix}, C=\begin{bmatrix}1 & 4 & 2\\ 3 & 1 & 5\end{bmatrix}, D=\begin{bmatrix}1 & 5 & 2\\ -1 & 0 & 1\\ 3 & 2 & 4\end{bmatrix},$

$$E=\begin{bmatrix}6 & 1 & 3\\ -1 & 1 & 2\\ 4 & 1 & 3\end{bmatrix}$$

A)Compute the indicated expression if it is defined:

a)D+E	b)D-E	c)5A	d)-7C
e)2B-C	f)4E-2D	g)-3(D+2E)	h)A-A
i)tr(D)	j)tr(D-3E)	k)4tr(7B)	

B)Compute the indicated expression if it is defined:

a) $2A^T+C$	b)D^T-E^T	c)$(D-E)^T$	d)B^T+5C^T
e)$2E^T-3D^T$	f)$(2E^T-3D^T)^T$	g)(CD)E	h)C(BA)
i)$tr(DE^T)$	j)AB	k)A(BC)	l)BA
m)$(C^TB)A^T$	n)$(2D^T-E)A$	s)(4B)C+2B	

Q2)Let $A=\begin{bmatrix}3 & -1\\ 2 & 4\end{bmatrix}, B=\begin{bmatrix}0 & 2\\ 1 & -4\end{bmatrix}, C=\begin{bmatrix}4 & 1\\ -3 & -2\end{bmatrix}, a=4$

A)Verify that

a)(AB)C=A(BC)	*b)*A(B+C)=AB+AC	c) (A+B)C=AC+BC
d)$(A^T)^T=A$	e)a(B+C)=aB+aC	f)B(C-A)=BC-BA
g)$(A+B)^T=A^T+B^T$	h)a(A-B)=aA-aB	i$(ABC)^T=C^TB^TA^T$

*Q*3)Find the diagonal sets of the following matrices:

$a)A=\begin{bmatrix}3 & 2 & 6\\ 0 & 1 & -2\\ 0 & 0 & -1\end{bmatrix}$	$b)B=\begin{bmatrix}-1 & 2 & 7\\ 0 & 5 & 3\\ 0 & 0 & 6\end{bmatrix}$
$c)C=\begin{bmatrix}4 & 0 & 0\\ -2 & 0 & 0\\ -3 & 0 & 7\end{bmatrix}$	$d)D=\begin{bmatrix}6 & 0 & 0\\ 1 & 5 & 0\\ 3 & 2 & 6\end{bmatrix}$

Q4)Find x, such that $A = \begin{bmatrix} x-1 & x^2 & x^4 \\ 0 & x+2 & x^3 \\ 0 & 0 & x-4 \end{bmatrix}$ *is a diagonal matrix*.

Q5)Find a, such that $A = \begin{bmatrix} 4 & -3 \\ a+5 & -1 \end{bmatrix}$ is upper triangular.

Q6)Find all 3×3 diagonal matrices A that satisfy $A^2 - 3A - 4I = 0$.

Q7)Find an upper triangular matrix that satisfies $A^3 = \begin{bmatrix} 1 & 30 \\ 0 & -8 \end{bmatrix}$.

Q8)Find a,b,c such that $A = \begin{bmatrix} 2 & a-2b+2c & 2a+b+c \\ 3 & 5 & a+c \\ 0 & -2 & 7 \end{bmatrix}$

is lower triangular.

2-Determinants

$Q1$)Compute the determinants of the following matrices:

$a)\begin{bmatrix} 3 & 5 \\ -2 & 4 \end{bmatrix}$	$b)\begin{bmatrix} 4 & 1 \\ 8 & 2 \end{bmatrix}$	c)$\begin{bmatrix} -5 & 7 \\ -7 & -2 \end{bmatrix}$
d)$\begin{bmatrix} \sqrt{2} & \sqrt{6} \\ 4 & \sqrt{3} \end{bmatrix}$	$e)\begin{bmatrix} a-3 & 5 \\ -3 & a-2 \end{bmatrix}$	$f)\begin{bmatrix} 2 & 7 & 6 \\ 5 & 1 & -2 \\ 3 & 8 & 4 \end{bmatrix}$
$g)\begin{bmatrix} 3 & 0 & 0 \\ 2 & -1 & 5 \\ 1 & 9 & -4 \end{bmatrix}$	$h)\begin{bmatrix} -1 & 1 & 2 \\ 3 & 0 & -5 \\ 1 & 7 & 2 \end{bmatrix}$	$i)\begin{bmatrix} c & -4 & 3 \\ 2 & 1 & c^2 \\ 4 & c-1 & 2 \end{bmatrix}$
$j)\begin{bmatrix} 3 & 3 & 0 & 5 \\ 2 & 2 & 0 & -2 \\ 4 & 1 & -3 & 0 \\ 2 & 10 & 3 & 2 \end{bmatrix}$	$k)\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$	l)$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 9 & 16 \\ 1 & 8 & 27 & 64 \\ 1 & 16 & 81 & 256 \end{bmatrix}$

Q2)Find λ ,for which $\det(A) = 0$.

a)$A = \begin{bmatrix} \lambda-2 & 1 \\ -5 & \lambda+4 \end{bmatrix}$, b)A=$\begin{bmatrix} \lambda-4 & 0 & 0 \\ 0 & \lambda & 2 \\ 0 & 3 & \lambda-1 \end{bmatrix}$.

Q3)Show that det(A)=$\frac{1}{2}\begin{vmatrix} tr(A) & 1 \\ tr(A^2) & tr(A) \end{vmatrix}$for every 2×2 matrix A.

Q4)Evaluate the following determinants:

a)$\begin{vmatrix} 3 & -6 & 9 \\ -2 & 7 & -2 \\ 0 & 1 & 5 \end{vmatrix}$	b)$\begin{vmatrix} 3 & 6 & -9 \\ 0 & 0 & -2 \\ -2 & 1 & 5 \end{vmatrix}$
c)$\begin{vmatrix} 2 & 1 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{vmatrix}$	d)$\begin{vmatrix} 1 & -2 & 3 & 1 \\ 5 & -9 & 6 & 3 \\ -1 & 2 & -6 & -2 \\ 2 & 8 & 6 & 1 \end{vmatrix}$

Q5)Prove that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (b-a)(c-a)(c-b)$.