Capter6: Partial derivatives

6.1-Functions of several variables

Q1) Determine the domain of f and the value of at the indicated points.

A)Compute the indicated expression if it is defined:

1-$f(x,y) = 2x - y^2$; $(-2,5)$; $(5,-2)$; $(0,-2)$

2- $f(x,y) = \frac{y+2}{x}$; $(3,1)$; $(1,3)$; $(2,0)$

3- $f(u,v) = \frac{uv}{u-2v}$; $(2,3)$; $(-1,4)$; $(0,1)$

4- $f(x,y,z) = \sqrt{25 - x^2 - y^2 - z^2}$; $(1,-2,2)$; $(-3,0,2)$

5- $f(x,y,z) = 2 + \tan x + y\, sin\, z$; $(1,-2,2)$; $(-3,0,2)$

6.2-partial derivatives

I)Find the partial derivatives of f:

1)$f(x,y) = 2x^4y^3 - xy^2 + 3y + 1$

2)$f(x,y) = (x^3 - y^2)^2$

3)$f(x,y) = \frac{x}{y} - \frac{y}{x}$

4)$f(x,y) = e^x \ln y$

5)$f(x,y) = x\, e^y + y \sin x$

6)$f(x,y) = \arctan(\frac{x}{y})$

7)$f(x,y) = x \cos\left(\frac{x}{y}\right)$

8)$f(x,y) = \ln\sqrt{\frac{x+y}{x-y}}$

9)$f(x,y) = \sqrt{x^2 + y^2}$

10)$f(x,y) = (2x + y)^{\cos x}$

II)Find the partial derivatives of f:

1-$f(x,y,z) = \sqrt{4x^2 - y^2}\, \sec(x+z)$

2- $f(x,y,z) = x^2 e^{2y} \cos z$;

3- $f(x,y,z) = \frac{x^2 - z^2}{1 + \sin(3y)}$;

4-$f(x,y,z) = (y^2 + z^2)^x$

5- $f(x,y,z) = x\, e^z - y\, e^x + z\, e^{-y}$;

6- $f(x,y,z) = xyz\, e^{xyz}$

III)Verify that $f_{xy} = f_{yx}$

1-$f(x,y) = xy^4 - 2x^2y^3 + 4x^2 - 3y$

2- $f(x,y) = \frac{x^2}{x+y}$

3- $f(x,y) = x^3e^{-2y} + y^{-2}\cos x$

4- $f(x,y) = y^2e^{x^2} + \frac{1}{x^2y^3}$

5- $f(x,y) = \sqrt{x^2 + y^2}$

IV)Solve the following:

1)if $w = 3x^2y^3z + 2xy^4z^2 - yz$, Find w_{xyz}

2) if $w = u^4vt^2 - 3uv^2t^3$, Find w_{tuv}

3)if $v = y\ ln(x^2 + z^4)$, Find v_{zzy}

4)if $w = \sin(xyz)$, find $\frac{\partial^3 w}{\partial z\, \partial y\, \partial x}$

5)if $w = \frac{x^2}{y^2+z^2}$, Find $\frac{\partial^3 w}{\partial z\, \partial y^2}$

6)if $f(x,y) = \ln\sqrt{x^2 + y^2}$, show that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$

7) if $f(x,y) = e^{-x}\cos y + e^{-y}\cos x$ show that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$

8) if $w = \cos(x - y) + \ln(x + y)$, show that $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = 0$

9)if$w = (y - 2x)^3 - \sqrt{y - 2x}$, show that $w_{xx} - 4w_{yy} = 0$

10)if $u(x,y) = x^2 - y^2, v(x,y) = 2xy$, show that $u_x = v_y$ and $u_y = -v_x$

11)if $u(x,y) = \frac{y}{x^2+y^2}, v(x,y) = \frac{x}{x^2+y^2}$, show that $u_x = v_y$ and $u_y = -v_x$

<u>6.3-Chain Rules</u>

<u>I)Solve:</u>

1)$w = u\sin v\,,\;\; u = x^2 + y^2\,, v = xy$. Find $\frac{\partial w}{\partial x}\,,\;\; \frac{\partial w}{\partial y}$

2) $w = u\,v + v^2\;\;,\;\; u = x\sin y\,, v = y\sin x$. Find $\frac{\partial w}{\partial x}\,,\;\; \frac{\partial w}{\partial y}$

3) $w = u^2 + 2uv\,, u = r\ln s\,,\;\; v = 2\,r + s$. Find $\frac{\partial w}{\partial r}\,,\;\; \frac{\partial w}{\partial s}$

4) $w = e^{tv}\,,\; t = r + s,\;\; v = rs$. Find $\frac{\partial w}{\partial r}\,,\;\; \frac{\partial w}{\partial s}$

5)$z = r^3 + s + v^2\,,\;\;\; r = xe^y,\;\; s = ye^x,\;\;\; v = x^2y$. Find $\frac{\partial z}{\partial x},\; \frac{\partial z}{\partial y}$

6)$z = pq + qw,\;\;\; p = 2x - y,\;\; q = x - 2y,\;\;\; w = -2x + 2y$. Find $\frac{\partial z}{\partial x},\;\; \frac{\partial z}{\partial y}$

7)$r = x\ln y,\;\;\;\; x = 3u + vt,\;\;\; y = uvt$. Find $\frac{\partial r}{\partial u},\; \frac{\partial r}{\partial v}\,,\; \frac{\partial r}{\partial t}$

8) $r = w^2\cos z\,,\; w = u^2vt,\;\; z = ut^2$. Find $\frac{\partial r}{\partial u},\; \frac{\partial r}{\partial v}\,,\; \frac{\partial r}{\partial t}$

9)$p = u^2 + 3v^2 - 4w^2,\;\; u = x - 3y + 2r - s,\;\; v = 2x + y - r + 2s,\qquad w = -x + 2y + r + s$. Find $\frac{\partial p}{\partial r}$

10)$s = tr + ue^v,\;\; t = xy^2z,\;\; r = x^2yz,\;\; u = xyz^2,\;\; v = xyz$. Find $\frac{\partial s}{\partial z}$

11)$w = x^3 - y^3,\;\; x = \frac{1}{t+1},\;\; y = \frac{t}{t+1}$. Find $\frac{dw}{dt}$

12)$w = \ln(u + v),\;\;\;\; u = e^{-2t},\;\;\; v = t^3 - t^2.\; Find\;\; \frac{dw}{dt}$

<u>6.4-Implicit differentiation</u>

I)If $y = f(x)$, find y'

1) $2x^3 + yx^2 + y^3 = 1$

2) $x^4 + 2x^2y^2 - 3xy^3 + 2x = 0$

3)$6x + \sqrt{xy} = 3y - 4$

4)$x^{\frac{2}{3}} + y^{\frac{2}{3}} = 4$

II)If $z = f(x, y)$, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$

1)$2xz^3 - 3yz^2 + x^2y^2 + 4z = 0$

2)$xz^2 + 2x^2y - 4y^2z + 3y - 2 = 0$

3)$xe^{yz} - 2ye^{xz} + 3ze^{xy} = 1$

4)$yx^2 + z^2 + cos(xyz) = 4$

III) if $w = f(x, y), x = r\cos\theta,\ y = r\sin\theta$, show that

$$\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2 = \left(\frac{\partial w}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial w}{\partial \theta}\right)^2.$$

*Iv)*if $v = f(x - at) + g(x + at)$, show that $\frac{\partial^2 v}{\partial t^2} = a^2\,\frac{\partial^2 y}{\partial x^2}$

V)if $w = \cos(x + y) + \cos(x - y)$, show that $w_{xx} = w_{yy}$