King Saud University
Department of Mathematics

Final Home Assignment **280-Math** **2Semester (1440/1441)**

Question1 (2+1). **(a)** Decide whether the series $\sum_{n=1}^{\infty} 2^{(-1)^n-n}$ is convergent or divergent.

(b) Find $\lim_{n\to\infty} x_n$ if $x_n = 2^{(-1)^n-n}$ or show that it DNE.

Question2 (2+2). (a). Show that if $f(x)$ is a continuous function on $[a,b]$ and

$f(x) > 0 \quad \forall x \in [a,b]$, then $\exists \alpha \in \Re$ such that $\alpha > 0$ *and* $f(x) \geq \alpha \quad \forall x \in [a,b]$.

(b) Show that the part (a) maybe not true in that case when the interval $[a,b]$ is open.

Question3 (2+2). (a) Let a and $b \in \Re$. Show that the function $f(x) = x^2$ is uniformly

continuous on $[a,b]$

(b) show that the function $f(x) = x^2$ is not uniformly continuous on $\Re$.

Question4 (3). Calculate $\lim_{n\to\infty} x_n$ of the number sequence $x_n = \int_0^2 \frac{n^2x^2 + \sin^2 nx}{n^2}\, dx$.

(With explanation of each step).

Question5 (2+2+2). (a) Find the power series representation of the function $f(x) = \ln(2+x)$

(b) find the interval of convergence of the resulting power series.

(c) Use parts (a) and (b) to get the following equality

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} = \ln 2$$