King Saud University: Math. Dept. Math-254
First Semester 1430-31 H Final Exam. Time: 3 hours

Name: ……………………… Student ID #:…………… Section #: …………
Instructor's name: …………………

For questions 1 t0 20, Please write your answer in the following table:

Qu.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
An.																				

The total marks for Qs. 1-20 is: $\overline{}$ 30

Questions 1 – 20	
Question 21	
Question 22	
Question 23	
Question 24	
Total	

For questions 1 to 20: Only one answer must be chosen. Zero mark will be given for any question with more than one answer.

1. Consider using the bisection method for solving the nonlinear equation $x^4 - x^3 - 8 = 0$ in $[1,3]$. The first approximation x_1 satisfies:

(a) $f(x_1)f(3) > 0$ (b) $f(x_1)f(1) < 0$ (c) $f(x_1)f(3) = 0$ (d) $f(x_1)f(1) > 0$

2. When using Newton's method with $x_0 = 1$ for solving $x^2 - 4 = 0$, then x_1 is:

(a) 2.5 (b) -2.5 (c) -1.5 (d) 2

3. When using secant method with $x_0 = 0$ and $x_1 = 1$ for solving $x^2 - 4 = 0$, then x_2 is:

(a) 2.5 (b) 4 (c) 3.5 (d) -2.5

4. The sufficient condition for the uniqueness of the fixed point for $x = g(x)$ is:

(a) $g'(x) < 1$ (b) $|g(x)| < 1$ (c) $|g'(x)| < 1$ (d) $|g'(x)| > 1$

5. We say that α is a multiple root of multiplicity m for $f(x) = 0$ if:

(a) $f^{(m)}(\alpha) \neq 0$ (b) $f^{(m)}(\alpha) = 0$ (c) $f^{(m-1)}(\alpha) \neq 0$ (d) $f^{(m+1)}(\alpha) \neq 0$

6. The order of convergence of $x_{n+1} = g(x_n), \quad n \geq 0$ is four if :

(a) $g^{(4)}(\alpha) = 0$ (b) $g^{(3)}(\alpha) = 0$ (c) $g^{(5)}(\alpha) \neq 0$ (d) $g^{(4)}(\alpha) \neq 0$

7. If the iterative scheme $x_{n+1} = \frac{c}{3}(x_n - 1) + \frac{2}{x_n} - 1, \quad n \geq 0$ converges quadratically to $\alpha = 1$, then the value of c is:

(a) 4 (b) 6 (c)3 (d)5

8. The iterative scheme for the Newton's method for approximating $\sqrt[3]{A}$, where $A > 0$ can be written in the form:

(a) $x_{n+1} = \frac{1}{3}(2x_n + \frac{A}{x_n^3}), \quad n \geq 0$ (b) $x_{n+1} = \frac{1}{3}(2x_n + \frac{A}{x_n}), \quad n \geq 0$

(c) $x_{n+1} = \frac{1}{3}(2x_n + \frac{A}{x_n^2}), \quad n \geq 0$ (d) $x_{n+1} = \frac{1}{3}(2x_n - \frac{A}{x_n^3}), \quad n \geq 0$

9. Using Gaussian elimination the exact solution for the linear system $\mathbf{Ax} = \mathbf{b}$, where $\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ is:

(a) $\begin{bmatrix} 0.5 \\ 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 \\ 0.5 \end{bmatrix}$ (c) $\begin{bmatrix} 0.5 \\ 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$

10. Let $\tilde{\mathbf{x}} = [1,2]'$ be an approximate solution for the linear system given in question 9. The value of $\| \mathbf{r} \|_\infty$ is:

(a) 2 (b) 3 (c) 0 (d) 4

where $\mathbf{r}$ is the residual vector with respect to $\tilde{\mathbf{x}}$.

11. The error $\| \mathbf{x} - \tilde{\mathbf{x}} \|_\infty$, where $\mathbf{x}$ is the exact solution of the linear system given in question 9, is bounded by:

(a) 4 (b) 5 (c) 4.5 (d) 3

Questions 12 to 17 are concerned with the data

x :	0.9	1.3	1.7	1.9	2.1	2.3
$f(x)$:	2.3	2.05	1.8	1.65	1.4	1.15

12. The function $L_0(x)$ of the linear Lagrange polynomial which gives the best approximation for $f(1.8)$ is given by

(a) $-5(x-1.7)$ (b) $-5(x-1.9)$ (c) $-4(x-1.7)$ (d) $-5(x-2.1)$

13. Consider using Newton divided difference formula to write the linear polynomial which give the best possible approximation for $f(2)$ the value of the first divided difference ($f[x_0, x_1]$) is

(a) -1.5 (b) -1.35 (c) -1.75 (d) -1.25

14. Using a suitable 2-point formula the worst possible computed approximation for $f'(1.7)$ is

(a) -0.625 (b) -0.825 (c) -0.725 (d) -0.925

15. Using a suitable 3-point formula the best possible computed approximation for $f'(1.9)$ is

(a) -2 (b) -1 (c) -1.5 (d) -3

16. Using a suitable 3-point formula the worst possible computed approximation for $f'(1.5)$ is

(a) -0.5 (b) -1.5 (c) -1 (d) -0.75

17. Using a suitable numerical integration formula the best possible approximation for the integral of $f(x)$ from 0.9 to 2.1 ($\int_{0.9}^{2.1} f(x)dx$) is

(a) 2.28 (b) 1.28 (c) 3.28 (d) 0.28

18. The upper bound for the error in approximating $\int_0^1 \frac{1}{x-2} dx$ using the composite trapezoidal rule with n=5 is

(a) $\frac{1}{100}$ (b) $\frac{1}{150}$ (c) $\frac{1}{170}$ (d) $\frac{1}{200}$

19. The local truncation error for the Taylor's method of order 3 is

(a) $\frac{h}{2} y''(\eta_i)$ (b) $\frac{h^2}{2} y''(\eta_i)$ (c) $\frac{h^3}{6} y'''(\eta_i)$ (d) $\frac{h^4}{24} y^{(4)}(\eta_i)$

20. When using Euler's method with $h = 0.3$ for solving the initial value problem $y' = t + y$, $y(0) = 1$, the computed approximation for $y(0.3)$ is

(a)1.2 (b) 1.3 (c) 1.02 (d) 1.1

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Please give a complete answer for each of the following questions:

21. (6 marks = 3+3)

(a) Show that the iterative scheme of Newton's method for approximating $\frac{1}{\mu}$ where $\mu \neq 0$ is a real number, can be written in the following form

$$x_{n+1} = 2x_n - \mu x_n^2, \qquad n \geq 0$$

then use this scheme with $x_0 = 0.25$ to compute approximation for $\frac{1}{3}$ with accuracy 5×10^{-2}.

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(b) Find the order of convergence for the iterative scheme given in part (a).

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22. Use Newton's method with $x_0 = 1$ and $y_0 = 0$ to solve the nonlinear system

$$f_1(x, y) = x^4 + y - 2 = 0$$
$$f_2(x, y) = x + 2y - 3 = 0$$

compute the first approximation only. (5 marks)

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23. Consider the linear system $\mathbf{Ax} = \mathbf{b}$, where $\mathbf{A} = \mathbf{B} - \mathbf{C}$ and (4 marks)

$$\mathbf{B} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{b} \in \Re^3$$

Show that the convergence of the iterative scheme $\mathbf{x}^{(k+1)} = \mathbf{B}^{-1}\mathbf{C}\mathbf{x}^{(k)} + \mathbf{B}^{-1}\mathbf{b},\ k \geq 0$ for solving this linear system is guaranteed.

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24. Consider the initial value problem (5 marks)

$$y' = y - t + 1$$
$$y(1) = 2$$

Use a Runge-Kutta method of order 2 with h=0.2 to compute approximation for $y(1.4)$.