Kingdom of Saudi Arabia King Saud University Faculty of Sciences Final Exam - ACTU 464 1438- 1st Semester Duration: 3 hours	المملكة العربية السعودية جامعة الملك سعود كلية العلوم قسم الرياضيات

الإســـم:		رقم الشعبة:	
الرقم الجامعي:		التوقيع:	

INSTRUCTIONS

1- Please check that your exam contains **12 pages** total (including the first page!!) **9 questions and a Bonus question**.
2- **Answer all questions.**
3- No books, No notes and no phones are allowed.
4- A standard no programmable calculator is allowed.
5- Also included in this paper exam:
   - A table for most used distributions.
   - A Z-table.
   - Some formulas are included.

Question	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***
Score										

Total score

Question 1. *(3 marks)*

Suppose Z *is the* q*-mixture of* X *and* Y, *with* $X \sim Negative - Binomial(r, p)$ *and* $Y \sim Uniform(a, b)$. *Compute the mean and the mgf of* Z.

Question 2. *(2 marks)*

Let consider the random variable X *having the following cumulative distribution function:*

$$F(x) = \begin{cases} 0 & if \quad x < -1 \\ (x+1)/4 & if \quad -1 \leq x < 1 \\ 1 & if \quad x \geq 1 \end{cases}$$

Compute the mean of X.

Question 3. *(2+2+2=6 marks)*

Suppose $X_1, X_2, \ldots X_n$ *are iid with common distribution* $X \sim Gamma(\alpha. 2)$*.*

a) *Determine the distribution of* $S = X_1 + \cdots + X_n$*.*
b) *Compute the mean and variance of* S*.*
c) *Using the normal approximation, compute* $P(S < 205)$ *for* $\alpha = 4$ *and* $n = 100$*.*

Question 4. *(3+2=5 marks)*

A total claim amount S *has a* $\chi^2(20)$ *distribution. Calculate*

a) *The mean, the variance and the skewness of* S.

b) $P(S \geq 30)$, *using the NP approximation.*

Question 5. *(3+2+3=8 marks)*

Let S *be a compound random variable with claim* X *and claim number* N. *Suppose further that* $f_X(1) = 0.5$, $f_X(2) = 0.1$, $f_X(3) = 0.4$ *and* $N \sim Poisson(4)$.

a) *Compute the mean, the variance and the mgf of* S.
b) *Compute, using Sparse algorithm, the mass function of* S *at* $s = 0,1,2,3$.
c) *Conclude the stop-loss premium* $\pi(3) = E(S-3)_+$.

Question 6. *(4 marks)*

Let S be an aggregate loss random variable with a discrete frequency distribution N (number of claims) defined by the table below.

n	*0*	*1*	*2*
$P(N = n)$	*0.3*	*0.5*	*0.2*

The severity claim X has a geometric distribution with parameter $p = 0.2$. Find the mass function of S using the convolution formula.

***Question 7**. (2+2=4 marks)*

The frequency distribution of an aggregate loss S *follows a binomial distribution with parameters* $n = 4$ *and* $p = 0.3$*. Loss amount has the mass distribution function:* $f_X(0) = 0.2$*,* $f_X(1) = 0.7$ *and* $f_X(2) = 0.1$*. Using the Panjer's recursive formula for the binomial distribution with* $a = -3/7$ *and* $b = 12/7$*,*

a) *Compute the mass function* f *of* S *at* $s = 1,2$*.*
b) *Compute the mass function* f *of* S *for all* $s \geq 3$*.*

Question 8. *(3 marks)*
In some ruin process, the individual claims have a $Gamma(2,1)$ *distribution.*
Determine the adjustment coefficient R *for a loading factor* $\theta = 1$.

Question 9. *(2+3=5 marks)*
Let $(S_t, t \geq 0)$ *be a compound Poisson process with parameter* λ *and exponentially distributed claims with parameter* β.
Compute $E(S_s S_t)$ *and* $E(e^{2S_t} | S_s)$ *for* $s < t$.

Bonus Question. *(3 marks)*
The frequency distribution of an aggregate loss S *follows a negative-binomial distribution with parameters* $r = 4$ *and* $p = 0.2$. *Loss amount has the mass distribution function: f(0) = 0.2; f(1) = 0.7; f(2) = 0.1. Calculate the mass function of* S *at* $s = 1,2$.

Good Luck

- **NP approximation formulas.**

Let a random variable S with mean μ, finite variance σ^2 and skewness γ. For $x \geq 1$ we have

$$P\left(\frac{S-\mu}{\sigma} \leq x\right) \approx \phi\left(\sqrt{\frac{9}{\gamma^2} + \frac{6x}{\gamma} + 1} - \frac{3}{\gamma}\right)$$

and for $s \geq 1$ we have

$$P\left(\frac{S-\mu}{\sigma} \leq s + \frac{\gamma}{6}(s^2 - 1)\right) \approx \phi(s),$$

where ϕ is the standard normal cdf.

- **Translated Gamma approximation formula.**

The translated gamma approximation can then be formulated as follows:

$$F_S(s) \approx G(s - x_0; \alpha, \beta),$$
$$\text{where } G(x; \alpha, \beta) = \frac{1}{\Gamma(\alpha)} \int_0^x y^{\alpha-1} \beta^\alpha \mathrm{e}^{-\beta y} \mathrm{d}y, \quad x \geq 0. \tag{2.56}$$

Here $G(x; \alpha, \beta)$ is the gamma cdf. We choose α, β and x_0 such that the first three moments are the same, hence $\mu = x_0 + \frac{\alpha}{\beta}$, $\sigma^2 = \frac{\alpha}{\beta^2}$ and $\gamma = \frac{2}{\sqrt{\alpha}}$ (see Table A), so

$$\alpha = \frac{4}{\gamma^2}, \quad \beta = \frac{2}{\gamma\sigma} \quad \text{and} \quad x_0 = \mu - \frac{2\sigma}{\gamma}. \tag{2.57}$$

- **Panjer's recursive formula.**

Theorem 3.5.1 (Panjer's recursion)
Consider a compound distribution with integer-valued non-negative claims with pdf $p(x)$, $x = 0, 1, 2, \ldots$, for which, for some real a and b, the probability q_n of having n claims satisfies the following recursion relation

$$q_n = \left(a + \frac{b}{n}\right) q_{n-1}, \quad n = 1, 2, \ldots \tag{3.26}$$

Then the following relations for the probability of a total claim equal to s hold:

$$f(0) = \begin{cases} \Pr[N = 0] & \text{if} \quad p(0) = 0; \\ \mathrm{m}_N(\log p(0)) & \text{if} \quad p(0) > 0; \end{cases}$$
$$f(s) = \frac{1}{1 - ap(0)} \sum_{h=1}^{s} \left(a + \frac{bh}{s}\right) p(h) f(s-h), \quad s = 1, 2, \ldots \tag{3.27}$$

Table A The most frequently used discrete and continuous distributions

Distribution	Density & support	Moments & cumulants	Mgf
Binomial(n,p) $(0<p<1,\ n\in\mathbb{N})$	$\binom{n}{x}p^x(1-p)^{n-x}$, $x=0,1,\ldots,n$	$\mathrm{E}=np, \mathrm{Var}=np(1-p)$, $\gamma=\dfrac{np(1-p)(1-2p)}{\sigma^3}$	$(1-p+pe^t)^n$
Bernoulli(p)	$\equiv$ Binomial$(1,p)$		
Poisson(λ) $(\lambda>0)$	$e^{-\lambda}\dfrac{\lambda^x}{x!}, x=0,1,\ldots$	$\mathrm{E}=\mathrm{Var}=\lambda$, $\gamma=1/\sqrt{\lambda}$, $\kappa_j=\lambda,\ j=1,2,\ldots$	$\exp[\lambda(e^t-1)]$
Negative binomial(r,p) $(r>0,\ 0<p<1)$	$\binom{r+x-1}{x}p^r(1-p)^x$, $x=0,1,2,\ldots$	$\mathrm{E}=r(1-p)/p$, $\mathrm{Var}=\mathrm{E}/p$, $\gamma=\frac{(2-p)}{p\sigma}$	$\left(\dfrac{p}{1-(1-p)e^t}\right)^r$
Geometric(p)	$\equiv$ Negative binomial$(1,p)$		
Uniform(a,b) $(a<b)$	$\dfrac{1}{b-a}$; $a<x<b$	$\mathrm{E}=(a+b)/2$, $\mathrm{Var}=(b-a)^2/12$, $\gamma=0$	$\dfrac{e^{bt}-e^{at}}{(b-a)t}$
$\mathrm{N}(\mu,\sigma^2)$ $(\sigma>0)$	$\dfrac{1}{\sigma\sqrt{2\pi}}\exp\dfrac{-(x-\mu)^2}{2\sigma^2}$	$\mathrm{E}=\mu$, $\mathrm{Var}=\sigma^2$, $\gamma=0$ $(\kappa_j=0,\ j\geq 3)$	$\exp(\mu t+\frac{1}{2}\sigma^2t^2)$
Gamma(α,β) $(\alpha,\beta>0)$	$\dfrac{\beta^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x}, x>0$	$\mathrm{E}=\alpha/\beta$, $\mathrm{Var}=\alpha/\beta^2$, $\gamma=2/\sqrt{\alpha}$	$\left(\dfrac{\beta}{\beta-t}\right)^\alpha$ $(t<\beta)$
Exponential(β)	$\equiv$ gamma$(1,\beta)$		
$\chi^2(k)$ $(k\in\mathbb{N})$	$\equiv$ gamma$(k/2,1/2)$		
Inverse Gaussian(α,β) $(\alpha>0,\beta>0)$	$\dfrac{\alpha x^{-3/2}}{\sqrt{2\pi\beta}}\exp\left(\dfrac{-(\alpha-\beta x)^2}{2\beta x}\right)$	$\mathrm{E}=\alpha/\beta$, $\mathrm{Var}=\alpha/\beta^2$, $\gamma=3/\sqrt{\alpha}$	$e^{\alpha(1-\sqrt{1-2t/\beta})}$ $(t\leq\beta/2)$

$$F(x)=\Phi\left(\frac{-\alpha}{\sqrt{\beta x}}+\sqrt{\beta x}\right)+e^{2\alpha}\Phi\left(\frac{-\alpha}{\sqrt{\beta x}}-\sqrt{\beta x}\right),\quad x>0$$

Standard Normal Probabilities

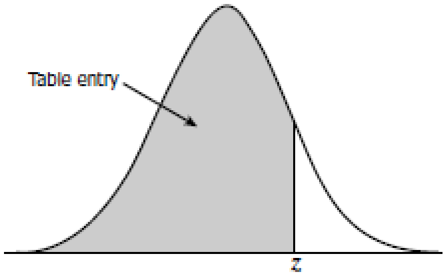

Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998