Kingdom of Saudi Arabia Ministry of Education King Saud University College of Science Department of mathematics	المملكة العربية السعودية وزارة التعليم جامعة الملك سعود كلية العلوم قسم الرياضيات

FINAL EXAM

SEMESTER	SECOND TERM	YEAR	2017/2016
	COURSE	ACTU 465	
DATE	10/05/2017	DURATION	3Hours

	رقم الشعبة:		إســم الطالب(ة):
	توقيع الطالب(ة):		الرقم الجامعي للطالب(ة):

INSTRUCTIONS

1) Please check that your exam contains **10 pages** total (including the first page!!), **07 questions and a Bonus question**.
2) **Answer all questions.**
3) No books, No notes and no phones are allowed.
4) A standard no programmable calculator is allowed.
5) Table for most used distributions is included.
6) Z-table is included.

Question	**1**	**2**	**3**	**4**	**5**	**6**	**7**	
Total score	**5**	**6**	**6**	**6**	**6**	**6**	**5**	
Score								

Exercise 1. (3+2=5 marks) Suppose $\Lambda \sim \text{Exponential}(\beta)$ and $X_{|\Lambda}$ has a Weibull distribution with cdf $F_{X|\Lambda}(x|\lambda) = 1 - e^{-\lambda(x^2)}$ for $x > 0$.

a) Compute the cdf and pdf of X.
b) Compute the mean of X.

Bonus question. (2 marks) Suppose $\text{B} \sim \text{Gamma}(1,2)$ and $N_{|\text{B}=b} \sim Negative\_Binomial(1, b)$. Compute the Variance of N.

Exercise 2. (2+2+2=6 marks)
The model for an annual total claim is given as follows:
(i) The number of claims N follows a Poisson distribution with parameter $\lambda = 10$.
(ii) Claim severity Y follows an exponential distribution with parameter $\beta = 2$.
(iii) The number of claims is independent of the severity of claims.
We suppose that aggregate (total) losses are within 10% of expected aggregate (total) losses with 96% probability.

a) Compute the mean and variance of the annual total claim $X = Y_1 + \cdots + Y_N$.
b) Determine the standard of full credibility n_F, measured in terms of the number of observations.
c) Compute the credibility factor based on $n = 0.75\ n_F$ observations.

Exercise 3. (2+2+2=6 marks) Let $X_1, \ldots, X_n$ be past claim numbers. Suppose that $X_i|\Theta$ are independent and identically Poisson distributed with parameter Θ and Θ is Gamma distributed with parameters α and β.
Determine

a) the hypothetical mean, its mean and variance.
b) the process variance and its mean.
c) the Buhlmann estimate for the number of claims.

Exercise 4. (2+2+2=6 marks) Suppose the number of claims $X_i|\Theta \sim Binomial(\Theta, 0.2)$ and the prior distribution of Θ is given as follows:

θ	1	2	3
$P(\Theta = \theta)$	0.2	0.3	0.5

Suppose further that a randomly chosen insured has 3 claims in year 1 and 2 claims in year 2. Calculate

a) the hypothetical mean, its mean and variance.
b) the process variance and its mean.
c) the Buhlmann estimate for the number of claims in year 3.

Exercise 5. (2+2+2=6 marks)

You are given:

(i) The number of claims $N_{|\Lambda=\lambda}$ incurred in a year by any insured has a Poisson distribution with parameter λ.

(ii) The claim frequencies of different insureds are independent.

(iii) The prior distribution Λ is uniform on the interval $(1,10)$.

(iv)

Year	Annual Number of insureds	Annual Number of claims
1	100	10
2	80	8
3	120	?

Calculate the following:

a) the hypothetical mean, its mean and variance.
b) the process variance and its mean.
c) the Buhlmann-Straub credibility estimate of the number of claims in Year 3.

Exercise 6. (2+2+2=6 marks)

You are given total claims for three policyholders:

Policyholder	Year 1	Year 2
(1)	500	700
(2)	550	650
(3)	450	550

Using the nonparametric empirical Bayes method, determine the estimated value of

a) the mean and variance of the hypothetical mean, and the mean of the process variance.

b) the Buhlmann credibility premium for Policyholder (2).

Exercise 7. (5 marks)
The claim payments on a sample of ten policies are: 2 3 3^+ 5 5^+ 6 7 7 9^+ 10, where '+' indicates that the loss exceeded the policy limit.
Using the Kaplan-Meier (Product limit) method, calculate the probability that the loss in a policy exceeds 6.5.

Table A The most frequently used discrete and continuous distributions

Distribution	Density & support	Moments & cumulants	Mgf
Binomial(n,p) $(0<p<1,\ n\in\mathbb{N})$	$\binom{n}{x}p^x(1-p)^{n-x}$, $x=0,1,\ldots,n$	$E=np, \mathrm{Var}=np(1-p)$, $\gamma=\frac{np(1-p)(1-2p)}{\sigma^3}$	$(1-p+pe^t)^n$
Bernoulli(p)	$\equiv$ Binomial$(1,p)$		
Poisson(λ) $(\lambda>0)$	$e^{-\lambda}\frac{\lambda^x}{x!}, x=0,1,\ldots$	$E=\mathrm{Var}=\lambda$, $\gamma=1/\sqrt{\lambda}$, $\kappa_j=\lambda,\ j=1,2,\ldots$	$\exp[\lambda(e^t-1)]$
Negative binomial(r,p) $(r>0,\ 0<p<1)$	$\binom{r+x-1}{x}p^r(1-p)^x$, $x=0,1,2,\ldots$	$E=r(1-p)/p$, $\mathrm{Var}=E/p$, $\gamma=\frac{(2-p)}{p\sigma}$	$\left(\frac{p}{1-(1-p)e^t}\right)^r$
Geometric(p)	$\equiv$ Negative binomial$(1,p)$		
Uniform(a,b) $(a<b)$	$\frac{1}{b-a}; a<x<b$	$E=(a+b)/2$, $\mathrm{Var}=(b-a)^2/12$, $\gamma=0$	$\frac{e^{bt}-e^{at}}{(b-a)t}$
$N(\mu,\sigma^2)$ $(\sigma>0)$	$\frac{1}{\sigma\sqrt{2\pi}}\exp\frac{-(x-\mu)^2}{2\sigma^2}$	$E=\mu$, $\mathrm{Var}=\sigma^2$, $\gamma=0$ $(\kappa_j=0,\ j\geq 3)$	$\exp(\mu t+\frac{1}{2}\sigma^2t^2)$
Gamma(α,β) $(\alpha,\beta>0)$	$\frac{\beta^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x}, x>0$	$E=\alpha/\beta$, $\mathrm{Var}=\alpha/\beta^2$, $\gamma=2/\sqrt{\alpha}$	$\left(\frac{\beta}{\beta-t}\right)^\alpha$ $(t<\beta)$
Exponential(β)	$\equiv$ gamma$(1,\beta)$		
$\chi^2(k)$ $(k\in\mathbb{N})$	$\equiv$ gamma$(k/2,1/2)$		
Inverse Gaussian(α,β) $(\alpha>0,\beta>0)$	$\frac{\alpha x^{-3/2}}{\sqrt{2\pi\beta}}\exp\left(\frac{-(\alpha-\beta x)^2}{2\beta x}\right)$	$E=\alpha/\beta$, $\mathrm{Var}=\alpha/\beta^2$, $\gamma=3/\sqrt{\alpha}$	$e^{\alpha(1-\sqrt{1-2t/\beta})}$ $(t\leq\beta/2)$

$$F(x)=\Phi\left(\frac{-\alpha}{\sqrt{\beta x}}+\sqrt{\beta x}\right)+e^{2\alpha}\Phi\left(\frac{-\alpha}{\sqrt{\beta x}}-\sqrt{\beta x}\right),\quad x>0$$

Standard Normal Probabilities

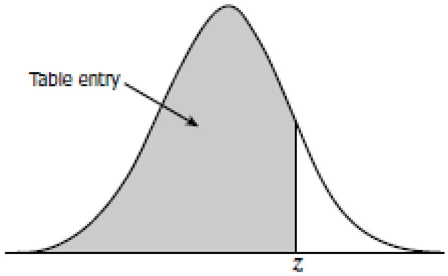

Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998