Kingdom of Saudi Arabia
Ministry of Education
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المملكة العربية السعودية
وزارة التعليم
جامعة الملك سعود
كلية العلوم
قسم الرياضيات

FINAL EXAM

SEMESTER	SECOND TERM	YEAR	2017/2016
	COURSE	ACTU 466	
DATE	18/05/2017	DURATION	3Hours

	رقم الشعبة:		إســـم الطالب(ة):
	توقيع الطالب(ة):		الرقم الجامعي للطالب(ة):

INSTRUCTIONS

1) Please check that your exam contains **08 pages** total (including the first page!!), and **07 questions**.
2) **Answer all questions.**
3) No books, No notes and no phones are allowed.
4) A standard no programmable calculator is allowed.
5) Table for most used distributions is included.
6) Modifications for distributions table is included.

Question	**1**	**2**	**3**	**4**	**5**	**6**	**7**	
Total score	**5**	**3**	**4**	**8**	**6**	**8**	**6**	
Score								

1) (5 marks) The cdf of a loss random variable L is given by:

$$F(x) = \begin{cases} 0 & if \quad x < 0 \\ x^2/4 & if \quad 0 \leq x > 2 \\ 1 & if \quad x \geq 2 \end{cases}$$

Express $VaR_p(L)$ and $TVaR_p(L)$ for all $0 < p < 1$.

2) (3 marks) Show that the normal distribution with parameters μ and $\sigma^2 = 1$, belongs to the linear exponential family with respect to the parameter $\theta = \mu$.

3) (2+2=4 marks) Let $N \sim Geometric(p)$ and let N^M be the zero-modified random variable associated to N with $p_0^M = \alpha$.

a) Compute:

- $p^M(k)$ for $k \geq 1$.
- the mean of N^M.

b) Based on a sample $N_1^M, N_2^M, \ldots N_n^M$, compute an estimate of the parameter α, using the method of moments.

4) (2+2+4=8 marks) Suppose a random variable X has an exponential distribution with parameter θ. We define the random variable $Y = X^{1/\tau}$ for a positive parameter τ.

a) Find the cdf and pdf of Y.

b) Find the expected value of Y.

c) The 20th and 80th percentiles of a sample are given by 5 and 12. Using the percentile matching method, estimate the two parameters θ and τ.

5) (6 marks) You are given:

(i) Losses follow a single parameter Pareto distribution with pdf:
$f_X(x) = \frac{\alpha}{x^{\alpha+1}}$ for $x > 1$ and $0 < \alpha < \infty$.

(ii) A random sample of size five produced three losses with values 3, 6 and 14, and two losses exceeding 25.

Determine the maximum likelihood estimate of α.

6) (2+2+2+2=8 marks) An insurance policy is subject to an ordinary deductible of d. The loss amount X has an exponential distribution with parameter θ.

a) Compute the cdf and pdf for Y^L.

b) Compute the cdf and pdf for Y^P.

c) Compute the mean of Y^L and Y^P.

d) Compute the loss elimination ratio.

7) (6 marks) A policy has an ordinary deductible of 100. You observe the following 5 payments: 15, 50, 170, 216, 400.

Given that the ground-up loss follows an exponential distribution with parameter θ, determine the maximum likelihood estimate of θ.

		CDF	PDF
Ordinary deductible $Y^L = (x-d)_+$	Y^L	$\begin{cases} 0 & ; y < 0 \\ F_X(y+d) & ; y \geq 0 \end{cases}$	$\begin{cases} 0 & ; y < 0 \\ F_X(d) & ; y = 0 \\ f_X(y+d) & ; y \geq 0 \end{cases}$
$Y^P = (Y^L \mid X > d)$	Y^P	$\begin{cases} 0 & ; y < 0 \\ \dfrac{F_X(y+d) - F_X(d)}{1 - F_X(d)} & ; y \geq 0 \end{cases}$	$\begin{cases} 0 & ; y < 0 \\ \dfrac{f_X(y+d)}{1 - F_X(d)} & ; y \geq 0 \end{cases}$
Franchise deductible $Y^L = \begin{cases} 0 & ; X \leq d \\ X & ; X \geq d \end{cases}$ $Y^P = (Y^L \mid X > d)$	Y^L	$\begin{cases} 0 & ; y < 0 \\ F_X(d) & ; 0 \leq y < d \\ F_X(y) & ; y \geq d \end{cases}$	$\begin{cases} 0 & ; y < 0 \\ F_X(d) & ; y = 0 \\ 0 & ; 0 < y < d \\ f_X(y) & ; y \geq d \end{cases}$
	Y^P	$\begin{cases} 0 & ; y < d \\ \dfrac{F_X(y) - F_X(d)}{1 - F_X(d)} & ; y \geq d \end{cases}$	$\begin{cases} 0 & ; y < d \\ \dfrac{f_X(y)}{1 - F_X(d)} & ; y \geq d \end{cases}$
Policy limit $Y = X \wedge u$		$\begin{cases} F_X(y) & ; 0 \leq y < u \\ 1 & ; y \geq u \end{cases}$	$\begin{cases} f_X(y) & ; 0 \leq y < u \\ 1 - F_X(u) & ; y = u \\ 0 & ; y > u \\ 0 & ; y < 0 \end{cases}$
coinsurance $Y = \alpha X$		$= F_X\left(\frac{y}{\alpha}\right)$	$= \frac{1}{\alpha} f_X\left(\frac{y}{\alpha}\right)$
Inflation effect with coefficient r $Y = (1+r)X$		$= F_X\left(\frac{y}{1+r}\right)$	$= \frac{1}{1+r} f_X\left(\frac{y}{1+r}\right)$
Combination $Y^L = \alpha((1+r)X \wedge u - (1+r)X \wedge d)$ $\begin{cases} 0 & ; x < \dfrac{d}{1+r} \\ \alpha((1+r)x - d); & \dfrac{d}{1+r} \leq x < \dfrac{u}{1+r} \\ \alpha(u-d) & ; x \geq \dfrac{u}{1+r} \end{cases}$	Y^L	$\begin{cases} 0 & ; y < 0 \\ F_X\left(\dfrac{y/\alpha + d}{1+r}\right) & ; 0 \leq y < \alpha(u-d) \\ 1 & ; y \geq \alpha(u-d) \end{cases}$	$\begin{cases} \dfrac{1}{\alpha(1+r)} f_X\left(\dfrac{y/\alpha + d}{1+r}\right) & ; 0 \leq y < \alpha(u - d \\ 1 - F_X\left(\dfrac{u}{1+r}\right) & ; y = \alpha(u-d) \\ F_X\left(\dfrac{d}{1+r}\right) & ; y = 0 \\ 0 & ; y < 0 \;\; y > \alpha(u-d) \end{cases}$
	Y^P	$\begin{cases} 0 & ; y < 0 \\ \dfrac{F_X\left(\dfrac{\frac{y}{\alpha} + d}{1+r}\right) - F_X\left(\dfrac{d}{1+r}\right)}{1 - F_X\left(\dfrac{d}{1+r}\right)} & ; 0 \leq y < \alpha(u - \\ 1 & ; y \geq \alpha(u-d) \end{cases}$	$\begin{cases} 0 & ; y < 0 \\ \dfrac{\frac{1}{\alpha(1+r)} f_X\left(\frac{\frac{y}{\alpha}+d}{1+r}\right)}{1 - F_X\left(\frac{d}{1+r}\right)} & ; 0 \leq y < \alpha(u-d) \\ 0 & ; y \geq \alpha(u-d) \end{cases}$

Table A The most frequently used discrete and continuous distributions

Distribution	Density & support	Moments & cumulants	Mgf
Binomial(n,p) $(0<p<1,\ n\in\mathbb{N})$	$\binom{n}{x}p^x(1-p)^{n-x}$, $x=0,1,\ldots,n$	$\mathrm{E}=np, \mathrm{Var}=np(1-p)$, $\gamma=\frac{np(1-p)(1-2p)}{\sigma^3}$	$(1-p+pe^t)^n$
Bernoulli(p)	$\equiv$ Binomial$(1,p)$		
Poisson(λ) $(\lambda>0)$	$e^{-\lambda}\frac{\lambda^x}{x!}, x=0,1,\ldots$	$\mathrm{E}=\mathrm{Var}=\lambda$, $\gamma=1/\sqrt{\lambda}$, $\kappa_j=\lambda, j=1,2,\ldots$	$\exp[\lambda(e^t-1)]$
Negative binomial(r,p) $(r>0,\ 0<p<1)$	$\binom{r+x-1}{x}p^r(1-p)^x$, $x=0,1,2,\ldots$	$\mathrm{E}=r(1-p)/p$, $\mathrm{Var}=\mathrm{E}/p$, $\gamma=\frac{(2-p)}{p\sigma}$	$\left(\frac{p}{1-(1-p)e^t}\right)^r$
Geometric(p)	$\equiv$ Negative binomial$(1,p)$		
Uniform(a,b) $(a<b)$	$\frac{1}{b-a}; a<x<b$	$\mathrm{E}=(a+b)/2$, $\mathrm{Var}=(b-a)^2/12$, $\gamma=0$	$\frac{e^{bt}-e^{at}}{(b-a)t}$
$\mathrm{N}(\mu,\sigma^2)$ $(\sigma>0)$	$\frac{1}{\sigma\sqrt{2\pi}}\exp\frac{-(x-\mu)^2}{2\sigma^2}$	$\mathrm{E}=\mu$, $\mathrm{Var}=\sigma^2$, $\gamma=0$ $(\kappa_j=0,\ j\geq 3)$	$\exp(\mu t+\frac{1}{2}\sigma^2t^2)$
Gamma(α,β) $(\alpha,\beta>0)$	$\frac{\beta^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x}, x>0$	$\mathrm{E}=\alpha/\beta$, $\mathrm{Var}=\alpha/\beta^2$, $\gamma=2/\sqrt{\alpha}$	$\left(\frac{\beta}{\beta-t}\right)^\alpha$ $(t<\beta)$
Exponential(β)	$\equiv$ gamma$(1,\beta)$		
$\chi^2(k)$ $(k\in\mathbb{N})$	$\equiv$ gamma$(k/2,1/2)$		
Inverse Gaussian(α,β) $(\alpha>0,\beta>0)$	$\frac{\alpha x^{-3/2}}{\sqrt{2\pi\beta}}\exp\left(\frac{-(\alpha-\beta x)^2}{2\beta x}\right)$	$\mathrm{E}=\alpha/\beta$, $\mathrm{Var}=\alpha/\beta^2$, $\gamma=3/\sqrt{\alpha}$	$e^{\alpha(1-\sqrt{1-2t/\beta})}$ $(t\leq\beta/2)$

$$F(x)=\Phi\left(\frac{-\alpha}{\sqrt{\beta x}}+\sqrt{\beta x}\right)+e^{2\alpha}\Phi\left(\frac{-\alpha}{\sqrt{\beta x}}-\sqrt{\beta x}\right), \quad x>0$$