Question 1. a) [3] Determine the largest region for which the following initial value problem admits a unique solution
\[ \ln(x-2) \frac{dy}{dx} = \sqrt{y-2}, \quad y\left(\frac{5}{2}\right) = 4. \]

b) [3]. Solve the linear first order differential equation:
\[ (y - x + xy \cot x)dx + xdy = 0, \quad x \in (0, \pi). \]

Question 2. a) [4]. Verify that \( \mu(x, y) = xy^2 \) is an integrating factor for the equation
\[ (4x^2y + 2y^2)dx + (3x^3 + 4xy)dy = 0, \]
and hence solve it.

b) [4]. Find the family of orthogonal trajectories for the family of curves:
\( x^2 - y^2 = C \). Which curve of the orthogonal family passes through \((0, 0)\).

Question 3. a) [4]. Find the general solution of the differential equation
\[ y'' - 2y' + y = e^x, \quad x > 0. \]

b) [4]. Write down the general form of the particular solution \( y_p \) for the differential equation
\[ y^{(4)} - y'' = x + xe^x + xe^{-x}. \]

Question 4. a) [4] Solve the initial value problem:
\[ x^2y'' + 3xy' + 2y = 0, \quad y(1) = 0, \quad y'(1) = 1, \quad x > 0. \]

b) [4]. Solve the following differential equation by using the method of power series about \( x = 0 \).
\[ y'' - 2x^2y' + 8y = 0 \]

Question 5. a) [5]. Expand in Fourier series the function \( f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \pi - x, & 0 < x < \pi \end{cases} \)
and deduce that \( \sum_{n=1}^{\infty} \frac{1}{(2n+1)^2} = \frac{\pi^2}{8} \).

b) [5]. Find the Fourier integral representation of the function
\[ g(x) = \begin{cases} 0, & -\infty < x < -1 \\ 2, & -1 < x < 1 \\ 0, & 1 < x < \infty \end{cases} \]
and deduce that \( \int_{-\infty}^{\infty} \frac{\sin \lambda}{\lambda} d\lambda = \frac{\pi}{2} \).