A4 $\displaystyle\int_0^\infty \frac{\lambda \sin(\lambda x)}{\beta^2+\lambda^2}\, d\lambda = \frac{\pi}{2} e^{-\beta x}, \quad x>0,\ \beta>0$

Define $f(x) = \begin{cases} e^{-\beta x}, & x>0 \\ -e^{-\beta x}, & x<0 \end{cases}$

f is an odd $\Rightarrow A(\lambda) = \displaystyle\int_{-\infty}^{\infty} \underbrace{f(x)\cos(\lambda x)}_{\text{odd function}}\, dx = 0$

$B(\lambda) = 2\displaystyle\int_0^\infty e^{-\beta x}\sin(\lambda x)\, dx = \frac{2\lambda}{\beta^2+\lambda^2}$

$\Rightarrow f(x) = \dfrac{1}{\pi}\displaystyle\int_0^\infty B(\lambda)\sin(\lambda x)\, d\lambda$

$e^{-x} = \dfrac{1}{\pi}\displaystyle\int_0^\infty \frac{2\lambda}{\beta^2+\lambda^2}\sin(\lambda x)\, d\lambda$

$\dfrac{\pi}{2} e^{-\beta x} = \displaystyle\int_0^\infty \frac{\lambda \sin(\lambda x)}{\beta^2+\lambda^2}\, d\lambda$

A3

ممكن تضيف فرع آخر للسؤال

$= \begin{cases} \frac{\pi}{2}, & 0<x<\pi \\ \frac{\pi}{4}, & x=\pi \\ 0, & x>\pi \end{cases}$

نعرف الدالة $f(x) = \begin{cases} 1, & 0<x<\pi \\ 0; & x>\pi \end{cases}$ كدالة فردية لكل $\mathbb{R}$

وبما أن $A(\lambda)=0$ ونوجد $B(\lambda)$:

$B(\lambda) = 2\displaystyle\int_0^\infty f(x)\sin(\lambda x)\, dx = 2\int_0^\infty 1\cdot\sin(\lambda x)\, dx = -\frac{1}{\lambda}\cos(\lambda x)\Big]_0^\pi$

$= \dfrac{2}{\lambda}\left[1-\cos(\lambda\pi)\right]$

$\Rightarrow f(x) = \dfrac{1}{\pi}\displaystyle\int_0^\infty B(\lambda)\sin(\lambda x)\, d\lambda$

$\begin{cases} 1; & 0<x<\pi \\ 0; & x>\pi \end{cases} = \dfrac{1}{\pi}\displaystyle\int_0^\infty \frac{2}{\lambda}\left[1-\cos(\lambda\pi)\right]\sin(\lambda x)\, d\lambda$

$\begin{cases} \frac{\pi}{2}; & 0<x<\pi \\ 0; & x>\pi \end{cases} = \displaystyle\int_0^\infty \frac{1-\cos(\lambda\pi)}{\lambda}\sin(\lambda x)\, d\lambda$