King Saud University: Mathematics Department MAth-254
First Semester 1432-33 H Final Examination
Maximum Marks = 50 Time: 180 mins.

Name of the Student:———————————— I.D. No. ————————

Name of the Teacher:———————————— Section No. ————————

Note: Check please the total number of pages are six.
(20 Multiple choice questions and 3 Full questions)

The Answer Tables for Q.1 to Q.20 : Marks: 1.5 for each one ($1.5 \times 20 = 30$)

Ps. : Mark {a, b, c or d} for the correct answer in the box.

Q. No.	1	2	3	4	5	6	7	8	9	10
a,b,c,d										

Q. No.	11	12	13	14	15	16	17	18	19	20
a,b,c,d										

Quest. No.	Marks
Q. 1 to Q. 20	
Q. 21	
Q. 22	
Q. 23	
Total	

Question 1: The number of bisections required to solve the equation $x^3 + x = 1$ in $[0, 1]$ accurate to within 10^{-3} is:

(a) 10 (b) 8 (c) 9 (d) 7

Question 2: Let α be a fixed point of a continuously differentiable function g. Then, the sufficient condition for the convergence of the fixed point iteration $x_{n+1} = g(x_n)$, $n \geq 0$, towards α is:

(a) $-1 < g''(\alpha) < 1$ (b) $g'(\alpha) < 1$ (c) $-1 < g(\alpha) < 1$ (d) $-1 < g'(\alpha) < 1$

Question 3: The order of multiplicity of the root $\alpha = 0$ of the equation $e^x - \dfrac{x^2}{2} = x + 1$ is:

(a) 2 (b) 3 (c) 1 (d) 4

Question 4: Given $x_0 = 0$ and $x_1 = 1$, then the next approximation x_2 of the solution of the equation $x^4 + 2x = 1$ using the Secant method is:

(a) 0.250 (b) 0.500 (c) 0.333 (d) 0.225

Question 5: The rate of convergence of the iterative scheme $x_{n+1} = \dfrac{1}{2}(x_n^2 + 1) - \ln x_n$, $n \geq 0$ to $\alpha = 1$ is:

(a) Order 2 (b) Order 3 (c) Order 4 (d) Order 1

Question 6: The l_∞-norm of the Jacobian matrix of the system $x^2 + y^2 = 1$, $xy = 1$ at the point $(1, 0)$ is:

(a) 3 (b) 1 (c) 2 (d) 4

Question 7: The goal of forward elimination steps in simple Gauss elimination is to reduce the coefficient matrix to a matrix of the form:

(a) Identity (b) Lower-triangular (c) Upper-triangular (d) Diagonal

Question 8: If $\hat{x} = [0.5, 0.0]^T$ is an approximate solution for the system $2x - y = 1$, $x + y = 2$, then the l_∞-norm of the corresponding residual vector is:

(a) 0.5 (b) 1.5 (c) 0.25 (d) 2.5

Question 9: Let $A = \begin{pmatrix} \alpha & 0 \\ 1 & 1 \end{pmatrix}$ and $\alpha > 2$. If the condition number $k(A)$ of the matrix A is 6, then α equals to:

(a) 5 (b) 3 (c) 4 (d) 6

Question 10: If $f(x) = xe^x$, then $f[0, 1, 0]$ equals to:

(a) $e + 1$ (b) $e - 1$ (c) $e - 2$ (d) $e + 2$

Question 11: Using data points: $(0,-2),(0.1,-1),(0.15,1),(0.2,2),(0.3,3)$, the best approximate value of $f(0.11)$ by a linear spline function is:

(a) -0.3 (b) -0.5 (c) -0.4 (d) -0.6

Question 12: Using data points: $(0,-2),(0.1,-1),(0.15,1),(0.2,2),(0.3,3)$,
if $\max_{0\leq x\leq 0.3} f^{(5)}(x)=1$, then the absolute error in approximating $f(0.25)$ by using a fourth degree interpolating polynomial is bounded by:

(a) 5.16×10^{-5} (b) 8.7×10^{-7} (c) 1.56×10^{-6} (d) 7.8×10^{-8}

Question 13: The number of subintervals required to approximate $\int_0^2 \frac{1}{x+4}\,dx$ within the accuracy 10^{-4} by using Composite Simpson's rule is:

(a) 8 (b) 6 (c) 4 (d) 2

Question 14: If $f(0)=3, f(1)=\frac{\alpha}{2}, f(2)=\alpha$, and the Simpson's rule for $\int_0^2 f(x)\,dx$ gives 2, then the value of α is:

(a) 0.5 (b) 2.0 (c) 1.0 (d) 3.0

Question 15: Using data points: $(0,-2),(0.1,-1),(0.15,1),(0.2,2),(0.3,3)$, the best approximate value of the integral $\int_0^{0.3} f(x)\,dx$, using the composite Trapezoidal rule is:

(a) 0.15 (b) 0.3 (c) 0.2 (d) 0.1

Question 16: If $f(-1)=0, f(0)=1, f(1)=2, f(3)=3$, then the best approximate value of $f'(1)$ using 3-point difference formula is:

(a) 0.0 (b) 1.0 (c) 2.0 (d) 3.0

Question 17: Using data points: $(0,-2),(0.1,-1),(0.15,1),(0.2,2),(0.3,3)$, the best approximation of $f'(0.1)$ using 3-point difference formula is:

(a) 20.0 (b) 10.0 (c) 30.0 (d) 15.0

Question 18: Using data points: $(0,-2),(0.1,-1),(0.15,1),(0.2,2),(0.3,3)$, then the worst approximation of $f''(0.15)$ using 3-point difference formula is:

(a) 44.44 (b) -40.00 (c) -40.44 (d) -44.44

Question 19: Given $xy'+y=1,\ y(1)=0$, the approximate value of $y(2)$ using Euler's method when $n=1$ is:

(a) 1.0 (b) 0.0 (c) 2.0 (d) 3.0

Question 20: Given $y'-\frac{1}{3}y=0,\ y(0)=1$, the approximate value of $y(1)$ using Taylor's method of order 2 when $n=1$ is:

(a) 9/13 (b) 25/18 (c) 13/9 (d) 18/25

Question 21: Consider the following system of equations [7 points]

$$\begin{array}{rcrcrcr} 6x_1 & + & 2x_2 & & & = & 1 \\ x_1 & + & 7x_2 & - & 2x_3 & = & 2 \\ 3x_1 & - & 2x_2 & + & 9x_3 & = & -1 \end{array}$$

If the initial approximation is $\mathbf{x}^{(0)} = [0, 0, 0]^T$, then find the second approximation $\mathbf{x}^{(2)}$ and an error estimate for your approximation using Jacobi method. Also, compute the number of iterations required to approximate the solution within accuracy 10^{-4}.

Question 22: Given $f(x) = x^2 + 5$, and $x_0 = 0.0, x_1 = 0.5, x_2 = 1.0, x_3 = 1.5, x_4 = 2.5$. Construct the divided differences table for the function and then use it to find approximation of $f(2.0)$ using quadratic Newton's Divided Difference interpolation formula. Compute an error bound and absolute error for your approximation. [7 points]

Question 23: **(i)** Let $f(x) \in C^2[x_0, x_1]$ and $h = x_1 - x_0$, then show that

$$\int_{x_0}^{x_1} f(x)\, dx \approx \frac{h}{2}[f(x_0) + f(x_1)]$$

(ii) Use suitable numerical integration rule to approximate $\int_0^1 \frac{1}{(5-x)}\, dx$ when $h = 0.2$. Also, compute an error bound for your approximation.

[6 points]