TIME: 3hours
M - 107

# KING SAUD UNIVERSITY

## DEPARTMENT OF MATHEMATICS

(SEMESTER II, 1431-1432) FINAL

FULL MARKS :100

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**Question: 1(a) For the given system of linear equations:**

$$5x + y - 2z = 1$$
$$-2x + 3y + 4z = 3$$
$$3x + 2y + 6z = 2$$

i. **Write the system of equation in the form AX=B,**
ii. **Use elementary matrix method to find $A^{-1}$, if exists, and**
iii. **Use $A^{-1}$ to solve the given system.**

**(b) Given A and B be two 4x4 matrices with det A = 2 and det B = -2.**

Evaluate $\det(A^{-2}.B^{3}.A^{t})$

**Question:2. (a)**

i. **Find equation of the line passing through points A ( 2,4,-3) and B(3, -1, 1),**
ii. **Where does this line intersect the xy – plane?**
iii. **Is this line parallel to line $L_2$** $x=2-t,\ y=4+5t,\ z=-4t$ **?**
iv. **Is this line parallel to line $L_3$** $x=0,\ y=4t,\ z=1+4t$ **?**

**(b)**

I. **Find equation of plane P containing points P(1, 3, 2), Q(3 , -1 ,6) and R(5, 2, 0)**

II. **Find a and b such that plane $P_1$** $ax+a^2y+2az+b=0$ **is orthogonal to plane P.**

**(b) Find the point at which the line**

$$x=2+3t,\quad y=-4t,\quad z=5+t,\quad t\in R$$

**intersects the plane** $4x+5y-2z=18$.

**(c) Find the direction cosines and direction angles of the vector**

$$\boldsymbol{a} = 2\boldsymbol{i} + 3\boldsymbol{j} - 6\boldsymbol{k}$$

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**Question: 3 . (a) Show that** $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$ **does not exist.**

**[6+8+6] (b) A particle starts from position** $r(1) = i + j$ **. Its velocity is** $v(t) = 2ti + 3t^2 j + \sqrt{t}k$ **. Find its position at time t.**

**(c) Find the parametric equations of the tangent line to the curve with parametric equations** $x = t^5, y = t^4, z = t^3$, **at the point ( 1, 1, 1).**

**Question: 4 .(a) Show that the function** $f(x,y) = \ln(x^2+y^2) \quad x \neq 0, y \neq 0$

**[6+6+8] satisfies the Laplace equation** $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$

**(b) If** $3xy^3 + 3x^2y - 6y + 9xy = 8$ **find** $\frac{dy}{dx}$

**(c) Find the tangential and normal components of the acceleration of a particle moving along th curve** $x = t^3, y = t+2, z = 4, t \in R$ **at** $t = 1$**. Also find the curvature at** $t = 1$**.**

**Question: 5 . (a) The base radius and the height of the right circular cone are measured to be 10cm and 25cm respectively, with a possible error in measurement of 0.1cm in each. Use differentials to estimate the maximum error in the calculated volume of the cone.** $\left[V = \frac{1}{3}\pi r^2 h\right]$

**[6+8+6] (b) Find the directional derivative of** $f(x,y) = xy$ **at the point P( 3, 2 ) in the direction of <1, 2>. In which direction is the direction of the derivative maximum? What is the maximum value of the derivative?**

**(c) Find the minimum distance between the origin and the point on the surface** $y^2 = x^2 z + 1$.

**NOTE: For solution of the paper visit**
**http://faculty.ksu.edu.sa/khawaja/default.aspx**