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| **King Saud University Second Semester** |
| **College of Science 1432/1433** |
| **Mathematics department(Girls section)** |

**Final Exam Math 385**

**Question1**

**Prove or disprove the following statements**:

a- is no where analytic.

b- If f has an essential singularity at and g has a pole at

,then f+g has an essential singularity at

c-There exist a power series ,which is convergent at z=2+3i and divergent at z=3-i.

d**-** has a zero at z=0 of order 2**.**

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**Question2**

a-If u is a harmonic function in a domain D, show that is analytic in D.

b-Find a branch of which is analytic at z=-5 and has a value there.

c-If f is an entire function and satisfies ,show that f is a constant.

**Question3**

a- a-If is a sequence of analytic functions in a simply connected region D and converging uniformly to f in D, show that f is analytic in D.

b-Let Show that ( converges uniformly to f(z)=z on every closed disk

c- Find the radius of convergence for the series .

**Question4**

a- Show that the zeros of a nonconstant analytic function are isolated.

b-Let where the functions p and q are both analytic at ,and q has a simple zero at ,while p(Prove that

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c-Classify the singularities of i- ii-

**Question5**

a- Find the Laurent series for

in the regions: i- ii-1.

b-Evaluate the following integrals:

i- ii-

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**Bonus Question**

Suppose that f is analytic and has a zero of order m at the point Show that the function has a simple pole at with

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