|  |
| --- |
| **King Saud University Second Semester** |
| **College of Science 1432/1433** |
| **Mathematics department(Girls section)** |

**Final Exam Math 385**

**Question1**

**Prove or disprove the following statements**:

a- $\left|z^{2}-z\right|$is no where analytic.

b- If f has an essential singularity at $z\_{0}$ and g has a pole at $z\_{0}$

,then f+g has an essential singularity at $z\_{0}.$

c-There exist a power series $\sum\_{j=0}^{\infty }a\_{j}z^{j}$ ,which is convergent at z=2+3i and divergent at z=3-i.

d**-** $f\left(z\right)=z^{2}\sinh(z)$has a zero at z=0 of order 2**.**

------------------------------------------------------------------------------------------------

**Question2**

a-If u is a harmonic function in a domain D, show that $f=u\_{x}-iu\_{y}$ is analytic in D.

b-Find a branch of $log⁡(z+4)$ which is analytic at z=-5 and has a value $7πi $there.

c-If f is an entire function and satisfies $\left|f(z)\right|\geq 1,∀zϵC$,show that f is a constant.

**Question3**

a- a-If is a sequence of analytic functions in a simply connected region D and converging uniformly to f in D, show that f is analytic in D.

b-Let $f\_{n}\left(z\right)=\frac{nz}{n+1}+\frac{3}{n},nϵN.$ Show that ($f\_{n})$ converges uniformly to f(z)=z on every closed disk $\left|z\right|\leq R.$

c- Find the radius of convergence for the series $\sum\_{n=0}^{\infty }\frac{(z-2i)^{n}}{(1+i)^{n}}$ .

**Question4**

a- Show that the zeros of a nonconstant analytic function are isolated.

b-Let $f\left(z\right)=\frac{p(z)}{q(z)}$ where the functions p and q are both analytic at $z\_{0}$,and q has a simple zero at $z\_{0}$ ,while p($z\_{0})\ne 0.$Prove that

 $Res\left(f, z\_{0}\right)=\frac{p(z\_{0})}{q^{'}(z\_{0})}$.

c-Classify the singularities of i- $f\left(z\right)=\frac{1}{z(e^{z}-1)}$ ii-$g\left(z\right)=tanz.$

**Question5**

a- Find the Laurent series for $f\left(z\right)=\frac{1}{z+z^{2}}$

in the regions: i-$0<\left|z\right|<1$ ii-1$<\left|z+1\right|$.

b-Evaluate the following integrals:

 i-$∮\_{\left|z\right|=1}^{}e^{\frac{1}{z}}cos⁡(\frac{1}{z})dz$ ii-$∮\_{\left|z\right|=\frac{1}{2}}^{}\frac{sin⁡(z)}{z(z-1)}dz.$

----------------------------------------------------------------------------------------------

**Bonus Question**

Suppose that f is analytic and has a zero of order m at the point $z\_{0}.$ Show that the function $g\left(z\right)=\frac{f^{'}(z)}{f(z)}$ has a simple pole at $z\_{0}$ with

Res(g,$ z\_{0})=m.$