

King Saud University
College of Sciences
Mathematics Department

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Final Exam Quantitative Methods in Finance (three pages) Act. 468 (40%)

Monday, January 16, 2016 / Rabi II 18, 1438 (three hours)

Problem 1. (8 marks)

We specify below the basic elements of a financial market with T periods:

- A finite probability space $\Omega = \{\omega_1, \dots, \omega_k\}$ with k elements.
- A probability measure P on Ω , such that $P(\omega) > 0$ for all $\omega \in \Omega$.
- A riskless asset (a saving account) $S_t^0, t \in \{0, 1, 2, \dots, T\}$ such that $S_0^0 = 1$ with a constant interest rate r .
- A d -dimensional price process $S_t, t \in \{0, 1, 2, \dots, T\}$ where $S_t = (S_t^0, S_t^1, \dots, S_t^d)$ and S_t^i stands for the price of the asset i at time t .

Assume that $k = 3, d = 1$, and $T = 2$ and consider the following model

n	S_n^0	S_n^1			
		ω_1	ω_2	ω_3	ω_4
0	1	5	5	5	5
1	$1 + r$	8	8	4	4
2	$(1 + r)^2$	9	6	6	3

(for simplicity of computations take $r = 0$)

1. (1 mark) Find the sigma algebras \mathcal{F}_0 and \mathcal{F}_2 .
2. (1 mark) Find the sigma algebra \mathcal{F}_1 .
3. (1 mark) Let Q be a probability on $\Omega = \{\omega_1, \dots, \omega_4\}$. Calculate $E_Q[S_2^1 \mid \mathcal{F}_1]$
4. (1 mark) Is this model a arbitrage free? explain your answer
5. (1 mark) Is this model complete? explain your answer
6. (1 mark) Consider a put option with strike price 7. Give the payoff of this put. Is it attainable? explain your answer
7. (1 mark) Find the price of this option at times 0 and 1. (+ 1 mark bonus)
8. (1 mark) Find the hedging portfolio (+ 1 mark bonus)

Problem 2. (8 marks)

Consider stock with a current price \$100 and a constant annualized volatility σ of 20%. The stock does not pay dividends. A risk-less asset is worth \$0.95 today and is worth \$1 in one year maturity.

Consider also European and American put options on the stock with a maturity of **two years** and a strike price of \$110.

1. (1 mark) Using the approach discussed in the lectures, construct a **two-step** binomial tree to approximate the stock price dynamics, with each step being **one year**. Build the stock price at each **node** at **one** and **two** years.
2. (1 mark) Compute the risk-neutral probability measure.
3. (1 mark) Give the binomial tree of the European put option.
4. (1 mark) Based on the binomial tree, find the current value of the European put option.
5. (1 mark) Compute the delta of the European put option for the first and the second periods.
6. (1 mark) Give the binomial tree of the American put option.
7. (1 mark) Based on the binomial tree, find the current value of the American put option.
8. (1 mark) Compute the delta of the American put option for the first and the second periods.

Problem 3. (8 marks)

An XYZ asset is currently priced at \$700 per share and an analyst predicts that the stock will hit 1000 at some point in the next $T = 1.5$ years. Assume that the stock price S_t of XYZ is given by the Black-Scholes formula. The stock pays no dividends, and the risk-free interest rate is $r = 6\%$. Take the rate of return of the stock price to be $\mu = 10\%$ and the volatility to be $\sigma = 30\%$.

1. (1 mark) Give the expression of S_T under the historical probability
2. (1 mark) Give the distribution of $\ln(\frac{S_T}{S_0})$.
3. (1 mark) Let $\mathbf{N}(\cdot)$ be the c.d.f. of the S.N.D. $\mathcal{N}(0, 1)$, we know that $\mathbf{N}(x) + \mathbf{N}(-x) = 1$ for all x . Calculate the probability that $S_{1.5} > 1000$ in this model assuming that $\mathbf{N}(0.7463) = 0.7722$.
4. (1 mark) To take advantage of the expected rise in XYZ stock, you decide to buy a call option with strike 1000 on one share of XYZ. The option expires in 1.5 years and implied volatility is 30%.
What are the input parameters for the Black-Scholes formula to value this option?
5. (1 mark) What are their values?
6. (1 mark) Given $\mathbf{N}(-0.5421) = 0.2939$ and $\mathbf{N}(-0.9095) = 0.1815$. What is the Black-Scholes value of your call option at the time of purchase?
7. (1 mark) Find the replicating portfolio from the side of the seller of this option today?
8. (1 mark) Suppose that the **6 months** price of XYZ stock is 1000. Given $\mathbf{N}(0.35) = 0.6368$ and $\mathbf{N}(0.05) = 0.52$ what would the value of your option be then at this time?

Problem 4. (8 marks)

Assume that the market-maker is trading options on an underlying asset with the price process $(S_t)_{t \geq 0}$ with initial price $S_0 = 40$, a volatility of 25% and the risk-free rate $r = 5\%$. Assume that the asset pays a dividend yield $q = 2\%$. The market-maker sells a 42-strike put on $(S_t)_{t \geq 0}$ in three months from now.

1. (1 mark) Give the formula of the Black–Scholes price of the above put ?
2. (1 mark) Given $N(0.3928) = 0.6527$ and $N(0.2678) = 0.6056$, find numerically the put price
3. (1 mark) Give the formula of the delta of the above put?
4. (1 mark) Find a numerical value of the delta
5. (1 mark) The market–maker sells 100 shares of the above put and decides to delta–hedge this position. What investment is required to do so?
6. (1 mark) If the next month price of the stock is 42. Find the one month price of the option assuming that $N(0.002) = 0.5$ and $N(-0.1) = 0.46$.
7. (1 mark) Find the one month delta of 100 shares of the put.
8. (1 mark) Find the hedging strategy of the above position.

Problem 5. (8 marks)

A financial institution has the following portfolio of over–the–counter options on **sterling (the underlying)**

Type of option	Position	Delta of option	Gamma of option	Vega of option
Call of type 1	–1000	0.5	2.2	1.8
Call of type 2	–500	0.8	0.6	0.2
Put of type 3	–2000	–0.4	1.3	0.7
Call of type 4	–500	0.7	1.8	1.4

The sign (–) means the position is short.

A **traded option** (say of type 5) is available in the market with a **delta** of 0.6, a **gamma** of 1.5 and a **vega** of 0.8.

Denote by Π_1 the value of the above portfolio.

1. (1 mark) Calculate the Delta of Π_1 .
2. (1 mark) Calculate the Gamma of Π_1 .
3. (1 mark) Consider a new portfolio Π_2 composed on Π_1 and β position in the **traded option**. Find β in such away that **new portfolio** Π_2 is gamma–neutral.
4. (1 mark) What position in the **traded option** would make the **new portfolio** Π_2 gamma–neutral?
5. (1 mark) Consider a new portfolio Π_3 composed on Π_2 and δ position in the **sterling (the stock)**. Find δ in such away that **new portfolio** Π_3 is delta–neutral.
6. (1 mark) What position in the **traded option** and in **sterling** (the stock) would make the **new portfolio** Π_3 delta–neutral and gamma–neutral?
7. (1 mark) Calculate the Vega of this portfolio.
8. (1 mark) What position in the **traded option** and in **sterling** (the stock) would make the **new portfolio both** vega–neutral and delta–neutral? (+1 mark bonus).