كلية العلوم
قسم الإحصاء وبحوث العمليات

Final Exam

استعن بالله وكن على يقين بأن كل ما ورد في هذه الورقة تعرفه جيدا وقد تدربت عليه بما فيه الكفاية

Question #1:

Consider the following LCG generator: $X_n = (7\, X_{n-1} + 5)\ mod\ (12)$, $X_0 = 2$

Answer the following:

a) Show by that because that the **three conditions** are not satisfied, the LCG does not have full period.

b) Find all different stream of the LCG above.

Question #2:

Consider the following set of pseudo-random numbers.

1	0.7551
2	0.8469
3	0.2268
4	0.6964
5	0.842
6	0.8075
7	0.5480
8	0.4047
9	0.6545
10	0.3427

α	$\chi^2_{(\alpha,3)}$	$\chi^2_{(\alpha,4)}$	$\chi^2_{(\alpha,9)}$
0.10	6.251	7.779	4.865
0.05	7.815	9.488	3.940
0.025	9.348	11.143	3.247
0.01	11.345	13.277	2.558

Test the hypothesis that these numbers are drawn from a U (0, 1) at a 99% confidence level using the Chi-squared goodness of fit test using **4 intervals**.

Question #3:

Suppose that the service time for a patient consists of two distributions. There is a 25% chance that the service time is uniformly distributed with minimum of 20 minutes and a maximum of 25 minutes, and a 75% chance that the time is distributed according to a Weibull distribution with shape of 2 and a scale of 4.5. Using the table of U(0,1) random numbers below, generate the service time for two patients.

n	1	2	3	4	5
$U_n(0,1)$	0.943	0.398	0.372	0.943	0.204

Question #4:

An electronics store sells smartphones. Number of smartphones the store sell per day is a random variable between 0 and 4. The owner can model the sold smartphones in a day as a Binomial distribution with p=0.65 and $n = \overset{4}{\cancel{5}}$.

a) Find the inverse transform for number of smartphones sold in a day.

b) Generate ***three*** random numbers from $f(x)$ using the table of U(0,1) random numbers below.

n	1	2	3
$U_n(0,1)$	0.305	0.696	0.171

Question #5:

Consider the following probability function:

$$f(x) = \begin{cases} \frac{4}{80}x^3 \; ; & 1 \le x \le 3 \\ 0 \quad ; & otherwise \end{cases}$$

n	1	2	3	4	5	6	7	8	9	10
$U_n(0,1)$	0.3045	0.6964	0.1709	0.3387	0.9804	0.1246	0.842	0.6557	0.3415	0.6254

a) Find the inverse transform of the probability and **generate 5 random** numbers from $f(x)$ using the table of U(0,1) random numbers above for n=1,2,3,4,5.

b) Use acceptance/rejection method to **generate 2 random numbers** from $f(x)$ using the table of U(0,1) random numbers above for n=6,7,8,9,10.

$u_1 \rightarrow 1-5$

$u_2 \rightarrow 6-10$

Question #6:

Customers arrive to a minimarket according to a random process with arrival rate that is assumed to be constant. After the customer finishes shopping, the arriving customers proceeds to a single server checkout counter. The checkout sever takes a random amount of time to finish the checkout for a customer. Data collected for customers entered the minimarket in the last 40 mints as follows.

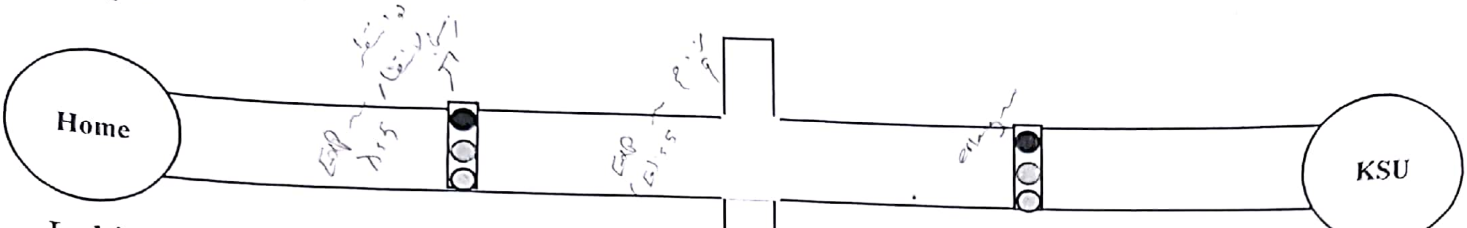
Cust. #	Arrival time (min)	Service time (min)	Service start (min)	Cust. WAIT?	Wait Time (min)	Dep. time (min)	Idle Time (min)	Money Spent (SR)
1	0.24	0.33	0.24	0	0	0.58	0.24	30
2	0.93	2.10	0.93	0	0.00	3.03	0.36	20
3	1.76	9.51	3.03	1	1.27	12.54	0.00	30
4	9.39	4.27	12.54	1	3.15	16.81	0.00	20
5	12.58	4.22	16.81	1	4.23	21.03	0.00	20
6	14.26	1.42	21.03	1	6.77	22.45	0.00	20
7	19.29	0.49	22.45	1	3.16	22.94	0.00	30
8	22.52	1.53	22.94	1	0.42	24.47	0.00	10
9	27.94	2.25	27.94	0	0.00	30.19	3.48	20
10	37.96	1.61	37.96	0	0.00	39.56	7.77	30
	146.87	27.73		6				

Answer The following

1. What is the expected service time?
2. What is the expected number of arrivals in ***one hour***?
3. What is the average waiting time?
4. What is the probability that the cashier is BUSY serving customers during the simulation time?
5. On average what is the expected time that customers spend in the minimarket from the time they enter until the time the leave the minimarket?

Question #7:

A student takes a rout to the university that takes an integer uniform time between 20 min and 30 min if there is nothing stopes him in his way to the university. However, there are two traffic lights and an intersection that may be congested as shown in the graph.



In his way to the university:

- If the student stops at the 1st traffic light (with probability 0.6), then he will spend a random time that follows an exponential distribution with mean 5 min.
- If the intersection is congested (with probability 0.4) then he will spend a random time that follows an exponential distribution with mean 10 min.
- If the student stops at the 2nd traffic light (with probability 0.3), then he will spend a random time that follows the Erlang distribution with parameters $\alpha = 2$ and $\beta = 0.25$.

1. Write the steps of the simulation for this student.
2. Make a simulation for 5 days for this students to give the data for:
   - Time to the university
   - Waiting time at the 1st traffic light, if the student stops.
   - Delay at the intersection if there is congestion.
   - Waiting time at the 2nd traffic light, if the student stops.

Day 1	Day 2	Day 3	Day 4	Day 5
0.909	0.635	0.077	0.309	0.114
0.228	0.809	0.456	0.590	0.767
0.787	0.724	0.458	0.254	0.127
0.140	0.135	0.153	0.536	0.126
0.277	0.887	0.698	0.394	0.823
0.063	0.099	0.116	0.270	0.882
0.272	0.707	0.013	0.611	0.577
0.271	0.362	0.179	0.934	0.316