

## 106Midterm1grading scheme(Sem1-40/41)

Question1 a)  $\sum_{11}^{30} k^2 = \sum_1^{30} k^2 - \sum_1^{10} k^2 = 20c$  (1)

$$9455 - 385 = 20c \text{ so } c = \frac{907}{2} = 453.5 \text{ (1)}$$

b)  $S_6 = \frac{2\pi-0}{3.8} (1 + 4\left(\frac{\sqrt{2}}{2}\right)^4 + 2.0 + 4\left(\frac{\sqrt{2}}{2}\right)^4 + 2(-1)^4 + 4\left(\frac{\sqrt{2}}{2}\right)^4 + 2.0$   
 $+ 4\left(\frac{\sqrt{2}}{2}\right)^4 + 1)$  (2)

$$= \frac{2\pi}{3} \approx 2.09439 \text{ (1)}$$

c)  $P = \{0, \frac{1}{n}, \frac{2}{n}, \dots, 1\}$   $x_k = \frac{k}{n}$  and  $\Delta x_k = \frac{1}{n}$ , take  $u_k = x_k$   $1 \leq k \leq n$  (1).

$$R_P = \sum_1^n \left(\frac{k}{n}\right)^3 \frac{1}{n} = \frac{1}{n^4} \left(\frac{n(n+1)}{2}\right)^2 \rightarrow \frac{1}{4} \text{ as } n \rightarrow \infty \text{ so } \int_0^1 x^3 dx = \frac{1}{4} \text{ (2)}$$

Question2 a)  $\int \frac{5^{\tan x}}{(\cos x)^2} dx = \int 5^u du$   $u = \tan x$  (2)

$$= \frac{1}{\ln 5} 5^{\tan x} + C \text{ (1)}$$

b)  $y' = 2 \ln 2 \sin x \cos x 2^{(\sin x)^2} + \pi x^{\pi-1} \pi^x + \ln \pi x^\pi \pi^x$

$$(1) + \frac{1}{2} + \left(\frac{1}{2}\right)$$

c)  $\int \frac{dx}{\sqrt{x}(2+x)} = 2 \int \frac{du}{2+u^2}$   $u = \sqrt{x}$  (2)

$$= \sqrt{2} \tan^{-1}(\sqrt{x/2}) + C \text{ (1)}$$

Question 3 a)  $\int \frac{\ln x + 1}{\sqrt{16(x \ln x)^2 - 9}} dx = \frac{1}{4} \int \frac{du}{\sqrt{u^2 - 9}} \quad u = 4x \ln x \quad (2)$

$$= \frac{1}{4} \cosh^{-1}\left(\frac{4x \ln x}{3}\right) + C \quad (1)$$

b)  $\int \frac{dx}{x\sqrt{x^5 - 4}} = \frac{2}{5} \int \frac{du}{u\sqrt{u^2 - 4}} \quad u = x^{5/2} \quad (2)$

$$= \frac{2}{10} \sec^{-1}\left(\frac{x^{5/2}}{2}\right) + C \quad (1)$$

c)  $\int \frac{2e^{-3x}}{1 - e^{-6x}} dx = \frac{-2}{3} \int \frac{du}{1 - u^2} \quad u = e^{-3x} \quad (2)$

$$= \frac{-2}{3} \tanh^{-1}(e^{-3x}) + C \quad (1)$$

Note: the last answer implicitly assumes  $x > 0$ . Also accept the answer

$$\frac{-2}{3} \coth^{-1}(e^{-3x}) + C$$