

### Grading scheme for midterm1

Q1 a)  $F'(x) = -\sin x \int_0^{\tan x} (\sqrt{1+t^2}) dt + \cos x \cdot \sqrt{1+(\tan x)^2} (\sec x)^2$  ■

$F'(0) = 1$  ■

b)  $\int \frac{dx}{\sqrt{x}(1+\sqrt{x})^2} = 2 \int \frac{1}{u^2} du, \quad u = \sqrt{x} + 1$  ■

$= \frac{-2}{u} + C = -\frac{2}{\sqrt{x}+1} + C$  ■

c)  $P = \{0,1,2,3,4,5\}$

$T_5 = \frac{5-0}{2 \times 5} \left( 1 + 2 + \frac{8}{3} + 4 + \frac{32}{5} + \frac{32}{6} \right)$  ■

$= 10,7 \quad (0,5)$

Q2 a)  $\int (\ln x + 1) 3^{x \ln x} dx = \int 3^u du, \quad u = x \ln x$  ■

$= \frac{3^u}{\ln 3} + C = \frac{3^{x \ln x}}{\ln 3} + C$  ■

b)

$\ln y = \ln x + 3 \ln(x^2 + 1) - \frac{1}{4} \ln(2x - 1)$  ■

$\frac{y'}{y} = \frac{1}{x} + \frac{6x}{x^2 + 1} - \frac{1}{2(2x - 1)}$  (1)

So  $y' = \left( \frac{1}{x} + \frac{6x}{x^2 + 1} - \frac{1}{2(2x - 1)} \right) \frac{x(x^2 + 1)^3}{\sqrt[4]{2x - 1}}$  ■

$$\begin{aligned} \text{c) } \int \frac{\sec^2 x dx}{\sqrt{9 - (\tan x)^2}} &= \int \frac{du}{\sqrt{9 - u^2}} \quad u = \tan x \quad \blacksquare \\ &= \sin^{-1}\left(\frac{u}{3}\right) + C = \sin^{-1}\left(\frac{\tan x}{3}\right) + C \quad \blacksquare \end{aligned}$$

$$\begin{aligned} \text{Q3) a) } \int \frac{dx}{x\sqrt{16x^4 - 1}} &= \frac{1}{2} \int \frac{du}{u\sqrt{u^2 - 1}}, \quad u = 4x^2 \quad \blacksquare \\ &= \frac{1}{2} \sec^{-1}(|u|) + C = \frac{1}{2} \sec^{-1}(4x^2) + C \quad \blacksquare \end{aligned}$$

$$\begin{aligned} \text{b) } \int \frac{x^2 dx}{\sqrt{x^6 - 25}} &= \frac{1}{3} \int \frac{du}{\sqrt{u^2 - 25}}, \quad u = x^3 \quad \blacksquare \\ &= \frac{1}{3} \cosh^{-1}\left(\frac{u}{5}\right) + C = \frac{1}{3} \cosh^{-1}\left(\frac{x^3}{5}\right) + C \quad \blacksquare \end{aligned}$$

$$\begin{aligned} \text{c) } \int \frac{dx}{x \ln x \sqrt{1 - (\ln x)^4}} &= \frac{1}{2} \int \frac{du}{u\sqrt{1 - u^2}}, \quad u = (\ln x)^2 \quad \blacksquare \\ &= -\frac{1}{2} \operatorname{sech}^{-1}(|u|) + C \\ &= -\frac{1}{2} \operatorname{sech}^{-1}(\ln x)^2 + C \quad \blacksquare \end{aligned}$$