King Saud University
First quizz Math 106 - 12/2/2020
Time Limit: 30 Minutes

Name : Answer

Assistant prof: *Dr Borhen Halouani*

1. (10 points) Evaluate the following integrals:

- $I = \int (x^2+1)^2 \, dx.$

$$I = \int (x^2+1)^2 \, dx = \int [x^4 + 2x^2 + 1] \, dx$$
$$= \frac{x^5}{5} + 2\frac{x^3}{3} + x + c, \quad c \in \mathbb{R}.$$

1

- $J = \int \frac{x^3 - x^2 + 3x - 1}{\sqrt{x}} \, dx.$

$$J = \int (x^3 - x^2 + 3x - 1)\, x^{-1/2} \, dx$$
$$= \int (x^{5/2} - x^{3/2} + 3x^{1/2} - x^{-1/2}) \, dx$$
$$= \frac{2}{7} x^{7/2} - \frac{2}{5} x^{5/2} + 2x^{3/2} - 2x^{1/2} + c, \quad c \in \mathbb{R}$$

1,5

- $K = \int \frac{x^2-1}{(x^3-3x+1)^6} \, dx.$

$$K = \int (x^3 - 3x + 1)^{-6} (x^2 - 1) \, dx$$

put $u = x^3 - 3x + 1$ then $du = (3x^2 - 3)\,dx = 3(x^2-1)\,dx$

so $(x^2-1)\,dx = \frac{1}{3}\,du$

1,5

$$K = \frac{1}{3} \int u^{-6} \, du = \frac{1}{3} \frac{u^{-5}}{-5} + c = -\frac{1}{15} \frac{1}{(x^3-3x+1)^5} + c$$

- $L = \int x\sqrt{7+6x^2}\,dx.$

put $u = 7+6x^2$ then $du = 12x\,dx$ so $x\,dx = \frac{1}{12}du$

1,5

$$L = \frac{1}{12}\int u^{1/2}\,du = \frac{1}{12}\cdot\frac{2}{3}u^{3/2} + c = \frac{1}{18}(7+6x^2)^{3/2} + c$$

- $M = \int \left(1+\frac{4}{x}\right)^3 \frac{1}{x^2}\,dx.$

put $u = 1+\frac{4}{x}$ then $du = -\frac{4}{x^2}dx$ So $\frac{dx}{x^2} = -\frac{1}{4}du$

1,5

$$M = -\frac{1}{4}\int u^3\,du = -\frac{1}{4}\cdot\frac{u^4}{4} + c = -\frac{1}{16}\left(1+\frac{4}{x}\right)^4 + c$$

- $N = \int \frac{1}{\cos^2 \pi x}\,dx.$

$$N = \int\left(\frac{1}{\cos(\pi x)}\right)^2 dx = \int \sec^2(\pi x)\,dx$$

$u = \pi x$ then $u = \pi\,dx$ so $dx = \frac{1}{\pi}du$

1,5

$$N = \frac{1}{\pi}\int \sec^2 u\,du = \frac{1}{\pi}\tan u + c = \frac{1}{\pi}\tan(\pi x) + c$$

- $P = \int_0^2 |x-1|\,dx.$

x	0	1	2
$\lvert x-1\rvert$	$1-x$	ϕ	$x-1$

1,5

$$P = \int_0^2 |x-1|\,dx = \int_0^1 (1-x)\,dx + \int_1^2 (x-1)\,dx$$

$$= \left[x - \frac{x^2}{2}\right]_0^1 + \left[\frac{x^2}{2} - x\right]_1^2$$

$$= (1 - 1/2) + \left[(2-2) - (1/2 - 1)\right]$$

$$= 1/2 + 1/2 = 1$$