Fourier-Legendre Series

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Because the Legendre polynomials form a complete orthogonal system over the interval $[-1, 1]$ with respect to the weighting function $w(x) = 1$, any function $f(x)$ may be expanded in terms of them as

$$f(x) = \sum_{n=0}^{\infty} a_n P_n(x). \quad (1)$$

To obtain the coefficients a_n in the expansion, multiply both sides by $P_m(x)$ and integrate

$$\int_{-1}^{1} P_m(x) f(x)\,dx = \sum_{n=0}^{\infty} a_n \int_{-1}^{1} P_n(x) P_m(x)\,dx. \quad (2)$$

But the Legendre polynomials obey the orthogonality relationship

$$\int_{-1}^{1} P_n(x) P_m(x)\,dx = \frac{2}{2m+1}\delta_{mn}, \quad (3)$$

where δ_{mn} is the Kronecker delta, so

$$\int_{-1}^{1} P_m(x) f(x)\,dx = \sum_{n=0}^{\infty} a_n \frac{2}{2m+1}\delta_{mn} \quad (4)$$

$$= \frac{2}{2m+1} a_m \quad (5)$$

and

$$a_m = \frac{2m+1}{2}\int_{-1}^{1} P_m(x) f(x)\,dx. \quad (6)$$

For example, for $f(x) = \sin(\pi x)$, the first few terms of the Fourier-Legendre series are

$$f(x) = \frac{3}{\pi} P_1(x) + \frac{7(\pi^2 - 15)}{\pi^3} P_3(x) + \frac{11(\pi^4 - 105\pi^2 + 945)}{\pi^5} P_5(x) + \ldots. \quad (7)$$

SEE ALSO:
Fourier-Bessel Series, Fourier Series, Generalized Fourier Series, Jackson's Theorem, Laplace Series, Legendre Polynomial, Picone's Theorem

REFERENCES:
Kaplan, W. "Fourier-Legendre Series." §7.14 in *Advanced Calculus, 4th ed.* Reading, MA: Addison-Wesley, pp. 508-512, 1992.

Referenced on Wolfram|Alpha: Fourier-Legendre Series

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