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Calculus and Analysis > Series > Fourier Series >

## Fourier-Legendre Series

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Because the Legendre polynomials form a complete orthogonal system over the interval  $[-1, 1]$  with respect to the weighting function  $w(x) = 1$ , any function  $f(x)$  may be expanded in terms of them as

$$f(x) = \sum_{n=0}^{\infty} a_n P_n(x). \tag{1}$$

To obtain the coefficients  $a_n$  in the expansion, multiply both sides by  $P_m(x)$  and integrate

$$\int_{-1}^1 P_m(x) f(x) dx = \sum_{n=0}^{\infty} a_n \int_{-1}^1 P_n(x) P_m(x) dx. \tag{2}$$

But the Legendre polynomials obey the orthogonality relationship

$$\int_{-1}^1 P_n(x) P_m(x) dx = \frac{2}{2m+1} \delta_{mn}, \tag{3}$$

where  $\delta_{mn}$  is the Kronecker delta, so

$$\int_{-1}^1 P_m(x) f(x) dx = \sum_{n=0}^{\infty} a_n \frac{2}{2m+1} \delta_{mn} \tag{4}$$

$$= \frac{2}{2m+1} a_m \tag{5}$$

and

$$a_m = \frac{2m+1}{2} \int_{-1}^1 P_m(x) f(x) dx. \tag{6}$$

For example, for  $f(x) = \sin(\pi x)$ , the first few terms of the Fourier-Legendre series are

$$f(x) = \frac{3}{\pi} P_1(x) + \frac{7(\pi^2 - 15)}{\pi^3} P_3(x) + \frac{11(\pi^4 - 105\pi^2 + 945)}{\pi^5} P_5(x) + \dots \tag{7}$$

**SEE ALSO:**

[Fourier-Bessel Series](#), [Fourier Series](#), [Generalized Fourier Series](#), [Jackson's Theorem](#), [Laplace Series](#), [Legendre Polynomial](#), [Picone's Theorem](#)

**REFERENCES:**

Kaplan, W. "Fourier-Legendre Series." §7.14 in *Advanced Calculus, 4th ed.* Reading, MA: Addison-Wesley, pp. 508-512, 1992.

Referenced on Wolfram|Alpha: [Fourier-Legendre Series](#)

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1+2+3+...+10

THINGS TO TRY:

- = 1+2+3+...+10
- = exponential fit
- 0.783,0.552,0.383,0.245,0.165,0.09
- = integrate x^2 sin y dx dy, x=0..1, y=0..pi

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