Frequency Spectral Radiation Fractal Dimension for Characterizing Shajara Reservoirs of the Permo-Carboniferous Shajara Formation, Saudi Arabia

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Abstract

The quality and assessment of a reservoir can be documented in detail by the application of frequency spectral radiation. This research aims to calculate fractal dimension from the relationship among frequency spectral radiation, maximum frequency spectral radiation and wetting phase saturation and to approve it by the fractal dimension derived from the relationship among capillary pressure and wetting phase saturation. Two equations for calculating the fractal dimensions have been employed. The first one describes the functional relationship between wetting phase saturation, frequency spectral radiation, maximum frequency spectral radiation and fractal dimension. The second equation implies to the wetting phase saturation as a function of capillary pressure and the fractal dimension. Two procedures for obtaining the fractal dimension have been utilized. The first procedure was done by plotting the logarithm of the ratio between frequency spectral radiation and maximum frequency spectral radiation versus logarithm wetting phase saturation. The slope of the first procedure = 3 - Df (fractal dimension). The second procedure for obtaining the fractal dimension was determined by plotting the logarithm of capillary pressure versus the logarithm of wetting phase saturation. The slope of the second procedure = Df - 3. On the basis of the obtained results of the fabricated stratigraphic column and the attained values of the fractal dimension, the sandstones of the Shajara reservoirs of the Shajara Formation were divided here into three units.

Keywords: Shajara Reservoirs, Shajara Formation, Frequency spectral radiation fractal dimension, Capillary pressure fractal dimension

Introduction

Seismo electric effects related to electro kinetic potential, dielectric permittivity, pressure gradient, fluid viscosity, and electric conductivity was first reported by [1]. Capillary pressure follows the scaling law at low wetting phase saturation was reported by [2]. Seismo electric phenomenon by considering electro kinetic coupling coefficient as a function of effective charge density, permeability, fluid viscosity and electric conductivity was reported by [3]. The magnitude of seismo electric current depends on porosity, pore size, zeta potential of the pore surfaces, and elastic properties of the matrix was investigated by [4]. The tangent of the ratio of converted electric field to pressure is approximately in inverse proportion to permeability was studied by [5]. Permeability inversion from seismo electric log at low frequency was studied by [6]. They reported that, the tangent of the ratio among electric excitation intensity and pressure field is a function of porosity, fluid viscosity, frequency, tortuosity, fluid density and Darcy permeability. A decrease of seismo electric frequencies with increasing water content was reported by [7]. An increase of seismo electric transfer function with increasing water saturation was studied by [8]. An increase of dynamic seismo electric transfer function with decreasing fluid conductivity was described by [9]. The amplitude of seismo electric signal increases with increasing permeability which means that the seismo electric effects are directly related to the permeability and can be used to study the permeability of the reservoir was illustrated by [10]. Seismo electric coupling is frequency dependent and decreases exponentially when frequency increases were demonstrated by [11]. An increase of permeability with increasing pressure head and bubble pressure fractal dimension was reported.
by [12,13]. An increase of geometric relaxation time of induced polarization fractal dimension with permeability increasing and grain size was described by [14,15].

**Materials and Methods**

Sandstone samples were collected from the surface type section of the Permo-Carboniferous Shajara Formation, latitude 26° 52' 17.4", longitude 43° 36' 18". (Figure 1). Porosity was measured on collected samples using mercury intrusion Porosimetry and permeability was derived from capillary pressure data. The purpose of this paper is to obtain frequency spectral radiation fractal dimension and to confirm it by capillary pressure fractal dimension. The fractal dimension of the first procedure is determined from the positive slope of the plot of logarithm of the ratio of frequency spectral radiation to maximum frequency spectral radiation log \( \frac{B_{vT}(\nu,T)}{B_{vT}(\nu,T)_{\text{max}}} \) versus log wetting phase saturation (logSw). Whereas the fractal dimension of the second procedure is determined from the negative slope of the plot of logarithm of log capillary pressure (log Pc) versus logarithm of wetting phase saturation (log Sw).

The frequency spectral radiation can be scaled as

\[
S_n = \left[ \frac{B_{vT}(\nu,T)^{\frac{1}{4}}}{B_{vT}(\nu,T)_{\text{max}}^{\frac{1}{4}}} \right]^{3-4Y}
\]

Where Sw the water saturation, \( B_{vT}(\nu,T) \) the frequency spectral radiation in watt/square radian * square meter * hertz, \( B_{vT}(\nu,T)_{\text{max}} \) the maximum frequency spectral radiation in watt/square radian * square meter * hertz.

Equation 1 can be proofed from

\[
B_{vT}(\nu,T) = \left[ \frac{2\times h \times v^3}{c^2} \times \frac{1}{\frac{h v}{k B T} \times e^{\frac{k B T}{h v}} - 1} \right]
\]

Where h the Planck constant in Joule/second, v the frequency in hertz, c the speed of light in meter/second, \( k B \) the Boltzmann constant in Joule/kelvin, T the temperature in kelvin.

The Planck constant can be scaled as

\[
h = \left[ \frac{U}{t} \right]
\]

Where h the Planck constant in Joule/second, U the energy in Joule, T the time in second.

Insert equation 3 into equation 2

\[
B_{vT}(\nu,T) = \left[ \frac{2\times h \times v^3}{c^2 \times t} \times \frac{1}{\frac{h v}{k B T} \times e^{\frac{k B T}{h v}} - 1} \right]
\]

The energy can be scaled as

\[ U = V \times q \]

Where U the energy in Joule, V the electric potential in volt, q the electric charge in coulomb.

Insert equation 5 into equation 4

\[
B_{vT}(\nu,T) = \left[ \frac{2\times V \times q \times v^3}{c^2 \times t} \times \frac{1}{\frac{h v}{k B T} \times e^{\frac{k B T}{h v}} - 1} \right]
\]

The electric potential can be scaled as

\[ V = E \times d \]

Where V the electric potential in volt, E the electric field in volt/meter, d the distance in meter.

Insert equation 7 into equation 6

\[
B_{vT}(\nu,T) = \left[ \frac{2\times E \times d \times q \times v^3}{c^2 \times t} \times \frac{1}{\frac{h v}{k B T} \times e^{\frac{k B T}{h v}} - 1} \right]
\]

The electric field can be scaled as

\[ E = \frac{Vol}{CEK} \]

Where E the electric field in volt/meter, Vol the velocity in meter/second, CEK the electro kinetic coefficient in ampere/pascal * meter.

Insert equation 9 into equation 8

\[
B_{vT}(\nu,T) = \left[ \frac{2\times Vol \times d \times q \times v^3}{c^2 \times t \times CEK} \times \frac{1}{\frac{h v}{k B T} \times e^{\frac{k B T}{h v}} - 1} \right]
\]

The velocity can be scaled as

\[ Vol = \frac{Q}{A} \]

Where Vol the velocity in meter/second, Q the flow rate in cubic meter/second, A the area in square meter.

Insert equation 11 into equation 10

\[
B_{vT}(\nu,T) = \left[ \frac{2\times Q \times d \times q \times v^3}{c^2 \times t \times CEK \times A} \times \frac{1}{\frac{h v}{k B T} \times e^{\frac{k B T}{h v}} - 1} \right]
\]

The flow rate can be scaled as

\[ Q = \frac{3.14 \times r^4 \times \Delta P}{8 \times \mu \times L} \]

Where Q the flow rate in cubic meter/second, r the pore radius in meter, \( \Delta P \) the differential pressure in pascal, \( \mu \) the fluid viscosity in pascal * second, L the capillary length in meter.

Insert equation 13 into equation 12
Furthermore, their frequency spectral radiation fractal dimensions were chosen for capillary measurements as described in Table 1. Three of which named as SJ7, SJ8, and SJ9 as illuminated in Table 1. It was nominated by four samples (Figure 1), from the Lower Shajara reservoir by an unconformity surface. Three units from bottom to top are: Lower Shajara Reservoir, Middle Shajara reservoir, and Upper Shajara Reservoir. Their attained results of the frequency spectral radiation fractal dimension and capillary pressure fractal dimension are shown in Table 1. Based on the achieved results it was found that the frequency spectral radiation fractal dimension is equal to the capillary pressure fractal dimension. The maximum value of the fractal dimension was found to be 2.7872 allocated to sample SJ13 from the Upper Shajara Reservoir as verified in Table 1. Whereas the minimum value of the fractal dimension 2.4379 was reported from sample SJ3 from the Lower Shajara reservoir as shown in Table 1. The frequency spectral radiation fractal dimension and capillary pressure fractional dimension were detected to increase with increasing permeability as proved in Table 1 owing to the possibility of having interconnected channels.

The Lower Shajara reservoir was symbolized by six sandstone samples (Figure 1), four of which label as SJ1, SJ2, SJ3 and SJ4 were carefully chosen for capillary pressure measurement as proven in Table 1. Their positive slopes of the first procedure log of the frequency spectral radiation to maximum frequency spectral radiation versus log wetting phase saturation (Sw) and negative slopes of the second procedure log capillary pressure (Pc) versus log wetting phase saturation (Sw) are clarified in (Figure 2-5) (Table 1). Their frequency spectral radiation fractal dimension and capillary pressure fractional dimension values are revealed in Table 1. As we proceed from sample SJ2 to SJ3 a pronounced reduction in permeability due to compaction was described from 1955 md to 56 md which reflects decrease in frequency spectral radiation fractal dimension from 2.7748 to 2.4379 as quantified in Table 1. Again, an increase in grain size and permeability was proved from sample SJ4 whose frequency spectral radiation fractal dimension and capillary pressure fractional dimension was found to be 2.6843 as described in Table 1.

In contrast, the Middle Shajara reservoir which is separated from the Lower Shajara reservoir by an unconformity surface as revealed in Figure 1. It was nominated by four samples (Figure 1), three of which named as SJ7, SJ8, and SJ9 as illuminated in Table 1 were chosen for capillary measurements as described in Table 1. Their positive slopes of the first procedure and negative slopes of the second procedure are shown in Figure 6-8 and Table 1. Furthermore, their frequency spectral radiation fractal dimensions

Results and Discussion

Table 1. The attained results of the frequency spectral radiation fractal dimension and capillary pressure fractal dimension are displayed in Table 1. Based on the achieved results it was found that the frequency spectral radiation fractal dimension is equal to the capillary pressure fractal dimension. The maximum value of the fractal dimension was found to be 2.7872 allocated to sample SJ13 from the Upper Shajara Reservoir as verified in Table 1. Whereas the minimum value of the fractal dimension 2.4379 was reported from sample SJ3 from the Lower Shajara reservoir as shown in Table 1. The frequency spectral radiation fractal dimension and capillary pressure fractional dimension were detected to increase with increasing permeability as proved in Table 1 owing to the possibility of having interconnected channels.

The Lower Shajara reservoir was symbolized by six sandstone samples (Figure 1), four of which label as SJ1, SJ2, SJ3 and SJ4 were carefully chosen for capillary pressure measurement as proven in Table 1. Their positive slopes of the first procedure log of the frequency spectral radiation to maximum frequency spectral radiation versus log wetting phase saturation (Sw) and negative slopes of the second procedure log capillary pressure (Pc) versus log wetting phase saturation (Sw) are clarified in (Figure 2-5) (Table 1). Their frequency spectral radiation fractal dimension and capillary pressure fractional dimension values are revealed in Table 1. As we proceed from sample SJ2 to SJ3 a pronounced reduction in permeability due to compaction was described from 1955 md to 56 md which reflects decrease in frequency spectral radiation fractal dimension from 2.7748 to 2.4379 as quantified in Table 1. Again, an increase in grain size and permeability was proved from sample SJ4 whose frequency spectral radiation fractal dimension and capillary pressure fractional dimension was found to be 2.6843 as described in Table 1.
and capillary pressure fractal dimensions show similarities as defined in Table 1. Their fractal dimensions are higher than those of samples SJ3 and SJ4 from the Lower Shajara Reservoir due to an increase in their permeability as explained in Table 1.

<table>
<thead>
<tr>
<th>AGE</th>
<th>Fm.</th>
<th>Mbr.</th>
<th>unit</th>
<th>LITHOLOGY</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Late Permian</td>
<td>Khuff</td>
<td>Huay1</td>
<td>Member</td>
<td>Limestone: Cream, dense, burrowed, thickness 6.56'</td>
<td>Sub-Khuff unconformity.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Mudstone: Yellow, thickness 17.7’</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Upper</td>
<td>Shajara</td>
<td>Sandstone: Light brown, cross-beded, course-grained, poorly sorted, porous, friable, thickness 6.5'</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Shajara</td>
<td>Member</td>
<td>Sub-Khuff unconformity.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Mudstone: Yellow-green, thickness 11.8’</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Upper</td>
<td>Shajara</td>
<td>Sandstone: Yellow, medium-grained, very coarse-grained, poorly, moderately sorted, porous, friable, thickness 13.1’</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Shajara</td>
<td>Member</td>
<td>Mudstone: Yellow, thickness 13.3’</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Mudstone: Brown, thickness 4.5’</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Upper</td>
<td>Shajara</td>
<td>Sandstone: Light brown, medium-grained, moderately sorted, porous, friable, thickness 3.6’</td>
<td></td>
</tr>
<tr>
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<td></td>
<td>Shajara</td>
<td>Member</td>
<td>Sub-Khuff unconformity.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Sandstone: Yellow, medium-grained, well sorted, porous, friable, thickness 13.4’</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Upper</td>
<td>Shajara</td>
<td>Sandstone: Red, coarse-grained, medium-grained, moderately well sorted, porous, friable, thickness 13.4’</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Shajara</td>
<td>Member</td>
<td>Sandstone: Red, medium-grained, well sorted, porous, friable, thickness 13.4’</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Sandstone: Red, medium-grained, well sorted, porous, friable, thickness 11.8’</td>
<td></td>
</tr>
<tr>
<td>Early Devonian</td>
<td>Tasul</td>
<td>Member</td>
<td></td>
<td>Sub-Unayaz unconformity.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Sandstone: White, fine-grained.</td>
<td></td>
</tr>
</tbody>
</table>

Figure 1: Surface type section of the Shajara Reservoirs of the Permo-Carboniferous Shajara Formation at latitude 26° 52’ 17.4” longitude 43° 36’ 18”.
Figure 2: Log (Bν₁/₄ / Bν₁/₄ₘₐₓ) & log pc versus log Sw for sample SJ1.

Figure 3: Log (Bν₁/₄ / Bν₁/₄ₘₐₓ) & log pc versus log Sw for sample SJ2.

Figure 4: Log (Bν₁/₄ / Bν₁/₄ₘₐₓ) & log pc versus log Sw for sample SJ3.
Figure 5: Log ($Bv^{1/4}/Bv_{\text{max}}^{1/4}$) & log pc versus log Sw for sample SJ4.

Figure 6: Log ($Bv^{1/4}/Bv_{\text{max}}^{1/4}$) & log pc versus log Sw for sample SJ7.

Figure 7: Log ($Bv^{1/4}/Bv_{\text{max}}^{1/4}$) & log pc versus log Sw for sample SJ8.
Figure 8: Log \( \frac{Bv^{1/4}}{Bv^{1/4}_{\text{max}}} \) & log pc versus log Sw for sample SJ9.

Figure 9: Log \( \frac{Bv^{1/4}}{Bv^{1/4}_{\text{max}}} \) & log pc versus log Sw for sample SJ11.

Figure 10: Log \( \frac{Bv^{1/4}}{Bv^{1/4}_{\text{max}}} \) & log pc versus log Sw for sample SJ12.
Figure 11: Log ($Bv^{1/4}/Bv^{1/4\max}$) & log $pc$ versus log $Sw$ for sample SJ13.

Figure 12: Slope of the first procedure versus slope of the second procedure.

Table 1: Petrophysical model showing the three Shajara Reservoir Units with their corresponding values of Frequency spectral radiation fractal dimension and capillary pressure fractal dimension.

<table>
<thead>
<tr>
<th>Formation</th>
<th>Reservoir</th>
<th>Sample</th>
<th>Porosity %</th>
<th>k(md)</th>
<th>Positive slope of the first procedure Slope=3-Df</th>
<th>Negative slope of the second procedure Slope=Df-3</th>
<th>Frequency spectral radiation fractal dimension</th>
<th>Capillary pressure fractal dimension</th>
</tr>
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<tbody>
<tr>
<td>Permo-Carboniferous Shajara Formation</td>
<td>Upper Shajara Reservoir</td>
<td>SJ13</td>
<td>25</td>
<td>973</td>
<td>0.2128</td>
<td>-0.2128</td>
<td>2.7872</td>
<td>2.7872</td>
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<tr>
<td></td>
<td></td>
<td>SJ12</td>
<td>28</td>
<td>1440</td>
<td>0.2141</td>
<td>-0.2141</td>
<td>2.7859</td>
<td>2.7859</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SJ11</td>
<td>36</td>
<td>1197</td>
<td>0.2414</td>
<td>-0.2414</td>
<td>2.7586</td>
<td>2.7586</td>
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<tr>
<td>Middle Shajara Reservoir</td>
<td></td>
<td>SJ9</td>
<td>31</td>
<td>1394</td>
<td>0.2214</td>
<td>-0.2214</td>
<td>2.7786</td>
<td>2.7786</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SJ8</td>
<td>32</td>
<td>1344</td>
<td>0.2248</td>
<td>-0.2248</td>
<td>2.7752</td>
<td>2.7752</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SJ7</td>
<td>35</td>
<td>1472</td>
<td>0.2317</td>
<td>-0.2317</td>
<td>2.7683</td>
<td>2.7683</td>
</tr>
<tr>
<td>Lower Shajara Reservoir</td>
<td></td>
<td>SJ4</td>
<td>30</td>
<td>176</td>
<td>0.3157</td>
<td>-0.3157</td>
<td>2.6843</td>
<td>2.6843</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SJ3</td>
<td>34</td>
<td>56</td>
<td>0.5621</td>
<td>-0.5621</td>
<td>2.4379</td>
<td>2.4379</td>
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<tr>
<td></td>
<td></td>
<td>SJ2</td>
<td>35</td>
<td>1955</td>
<td>0.2525</td>
<td>-0.2252</td>
<td>2.7748</td>
<td>2.7748</td>
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<td></td>
<td></td>
<td>SJ1</td>
<td>29</td>
<td>1680</td>
<td>0.2141</td>
<td>-0.2141</td>
<td>2.7859</td>
<td>2.7859</td>
</tr>
</tbody>
</table>
On the other hand, the Upper Shajara reservoir was separated from the Middle Shajara reservoir by yellow green mudstone as shown in Figure 1. It is defined by three samples so called SJ11, SJ12, SJ13 as explained in Table 1. Their positive slopes of the first procedure and negative slopes of the second procedure are displayed in Figure 9-11 and Table 1. Moreover, their frequency spectral radiation fractal dimension and capillary pressure fractal dimension are also higher than those of sample SJ3 and SJ4 from the Lower Shajara Reservoir due to an increase in their permeability as simplified in Table 1.

![Figure 13: Frequency spectral radiation fractal dimension versus capillary pressure fractal dimension.](image)

Overall a plot of positive slope of the first procedure versus negative slope of the second procedure as described in Figure 12 reveals three permeable zones of varying Petrophysical properties. These reservoir zones were also confirmed by plotting frequency spectral radiation fractal dimension versus capillary pressure fractal dimension as described in Figure 13. Such variation in fractal dimension can account for heterogeneity which is a key parameter in reservoir quality assessment.

**Conclusion**

The sandstones of the Shajara Reservoirs of the Permian Carboniferous Shajara Formation were divided here into three units based on frequency spectral radiation fractal dimension. The Units from base to top are: Lower Shajara Frequency Spectral Radiation Fractal Dimension Unit, Middle Shajara Frequency Spectral Radiation Fractal Dimension Unit, and Upper Shajara Frequency Spectral Radiation Fractal Dimension Unit. These units were also proved by capillary pressure fractal dimension. The fractal dimension was found to increase with increasing grain size and permeability owing to possibility of having interconnected channels.

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**References**


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