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**OPER 441: Modeling and Simulation**  
**Exercises Sheet #6**

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**Question 1:**

Consider the triangular distribution:

$$F(x) = \begin{cases} 0 & x < a \\ \frac{(x-a)^2}{(b-a)(c-a)} & a \leq x \leq c \\ 1 - \frac{(b-x)^2}{(b-a)(b-c)} & c < x \leq b \\ 1 & b < x \end{cases}$$

- a) Use a spreadsheet to generate 1000 observations of the triangular distribution with  $a = 2$ ,  $c = 5$ ,  $b = 10$ .
- b) Use your favorite statistical software to make a histogram of 1000 observations from your implementation of the triangular distribution with  $a = 2$ ,  $c = 5$ ,  $b = 10$ .

**Question 2:**

The times to failure for an automated production process have been found to be randomly distributed according to a Rayleigh distribution:

$$f(x) = \begin{cases} 2\beta^{-2}xe^{-(x/\beta)^2} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

Setup a spreadsheet to generate 5 random numbers from your algorithm with  $\beta = 2$ .

**Question 3:**

Setup a spreadsheet to generate 5 random numbers from the negative binomial distribution with parameters ( $r = 4$ ,  $p = 0.4$ ) using:

- a) The convolution method  
b) The number of Bernoulli trials to get 4 successes.

**Question 4:**

Setup a spreadsheet that will generate 30 observations from the following probability density function using the Acceptance-Rejection algorithm for generating random variates.

$$f(x) = \begin{cases} \frac{3x^2}{2} & -1 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

### Question 5:

Parts arrive to a machine center with three drill presses according to a Poisson distribution with mean  $\lambda$ . The arriving customers are assigned to one of the three drill presses randomly according to the respective probabilities  $p_1$ ,  $p_2$ , and  $p_3$  where  $p_1 + p_2 + p_3 = 1$  and  $p_i > 0$  for  $i = 1, 2, 3$ . What is the distribution of the inter-arrival times to each drill press? Specify the parameters of the distribution.

a) Suppose that  $p_1$ ,  $p_2$ , and  $p_3$  equal to 0.25, 0.45, and 0.3 respectively and that  $\lambda$  is equal to 12 per minute. Setup a spreadsheet to generate the first 3 arrival times.

This question demonstrates the splitting property of a Poisson distribution. Each machine experience a Poisson process with mean  $\lambda \times p_i$ . Thus, the distribution of the inter-arrival times to each drill press will be exponential with mean  $1/(\lambda \times p_i)$

Because of the splitting rule for Poisson processes, the drill presses each see arrivals according to the following three Poisson processes:

$$\lambda_1 = \lambda p_1 = 12 * 0.25 = 3$$

$$\lambda_2 = \lambda p_2 = 12 * 0.45 = 5.4$$

$$\lambda_3 = \lambda p_3 = 12 * 0.3 = 3.6$$