prove that a group of order 9 is abelian.
>
>Thax

Here is an elementary proof, assuming no more than Lagrange's Thm (every element has order that divides 9).

If an element c has order 9, then 1, c, c^2 ... c^8 is the whole group, and is obviously Abelian.

If not, then every element except 1 has order 3, x^3 = 1 and x^2 =/= 1. Say 'a' is such an element, and 'b' is another, different from 1, a, a^2. So you also have b and b^2.

Now ab is distinct from the above 5 elements (e.g. ab = b^2 implies a = b). Same for ab^2, (a^2)b, and (a^2)b^2. Likewise, these elements are distinct from each other; so that's the whole group.
[eg b = ab^2 --> 1 = b(b^(-1)) = (ab^2)(b^(-1) = ab, but then b must be the (unique!) inverse of a, which is a^2, as a^3 = 1; and so on.]

So ba must be one of the last 4 elements. If ba = ab^2 then bab = ab^3 = a. But abab ab = 1; so aaab = 1 or b = 1, which cannot be. Likewise you show ba = (a^2)b leads to contradiction, and so does ba = (a^2)b^2. Hence ba = ab, and easily any two elements commute.
[eg (a^2)b = aab = aba = baa = b(a^2) and so on]