Problem 1

A cylindrical metal rod having a diameter of 14.5 mm and a gauge length of 40 mm is to be subjected to a tensile load along the rod axis. The following load-elongation data was measured in the lab under a tensile test:

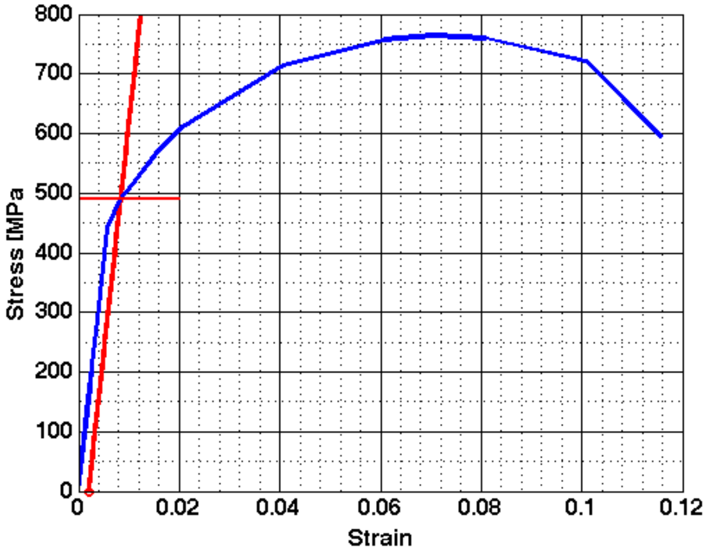
Length (mm)	Load (N)
40.00	0.00
40.07	22251.00
40.09	28609.00
40.11	34966.00
40.15	47681.00
40.23	72956.00
40.33	80669.00
40.43	84840.00
40.63	93969.00
40.83	100970.00
41.63	117820.00
42.43	125130.00
42.83	126240.00
43.23	125530.00
44.03	119230.00
44.63	98140.00

Compute the following:

a. The modulus of elasticity?
b. The yield strength using 0.2% offset method
c. The ductility using percent elongation
d. Calculate the strain hardening exponent n and the stress coefficient K?
e. The elastic and plastic (permanent) strains at applied loads of 28609 Newton and 100970 Newton?

<u>Solution</u>:

The stress-strain curve:



Length (mm)	Load (N)	Strain	Stress (Mpa)
40.00	0.00	0	0
40.07	22251.0	0.00175	134.75
40.09	28609.0	0.00225	173.25
40.11	34966.0	0.00275	211.75
40.15	47681.0	0.00375	288.75
40.23	72956.0	0.00575	441.81
40.33	80669.0	0.00825	488.52
40.43	84840.0	0.01075	513.78
40.63	93969.0	0.01575	569.06
40.83	100970.	0.02075	611.46
41.63	117820.	0.04075	713.5
42.43	125130.	0.06075	757.77
42.83	126240.	0.07075	764.49
43.23	125530.	0.08075	760.19
44.03	119230.	0.10075	722.04
44.63	98140.	0.11575	594.32

a. The modulus of elasticity (10 points):
 Pick any two points in the elastic region and calculate the slope ➔ E = 77 GPa

b. The yield strength using 0.2% offset method (5 points):
Yield strength = 490 MPa (see the red lines in the above figure)

c. The ductility using percent elongation (5 points):
El%= (fracture strain –elastic strain) *100 = (0.11575-0.005)*100= 11.0750 %

d. The strain hardening exponent n and the stress coefficient K (10 points):
Pick any two points in the uniform plastic region (between yield strength and ultimate tensile strength):

Eng. strain	Eng. stress (Mpa)	True strain	True Stress (Mpa)
0.0158	569.06	0.0156	578.0227
0.0208	611.46	0.0205	624.1478

n = (log(624.1478)-log(578.0227)) / (log(0.0205)-log(0.0156)) = 0.28
You may get different strain hardening exponents depending on the selected points (e.g. 0.2823, 0.2810, 0.2612, 0.2033, and 0.1234)
K= 578.0227 / $0.0156^{0.281}$ = 624.1478 / $0.0205^{0.281}$ = 1861 MPa

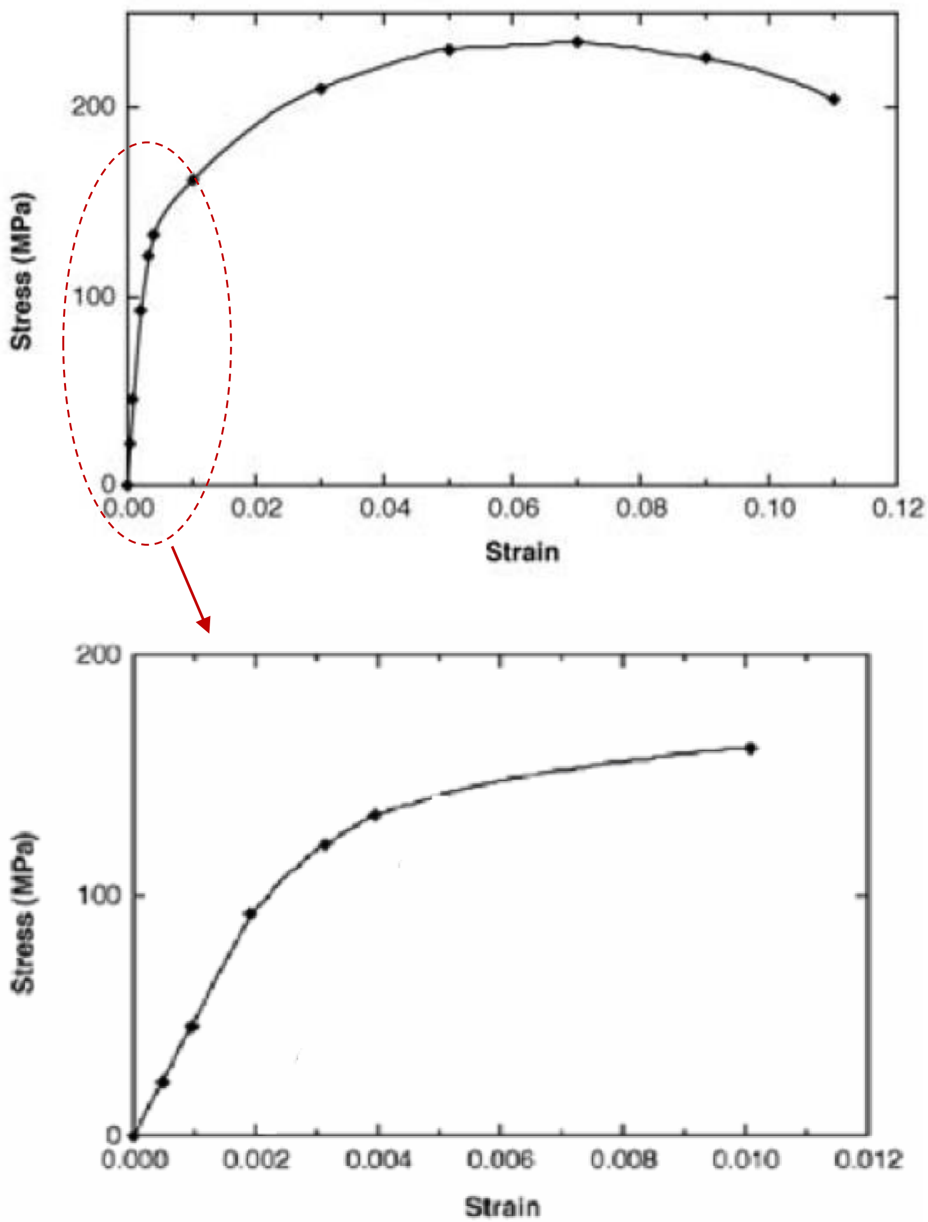
e. The elastic and plastic (permanent) strains at applied loads of 28609 Newton and 100970 Newton (10 points):

- For 28609 N: Since this point is located in the elastic region,
  Total strain = 0.00225
  Elastic strain =Total strain = 0.00225
  Plastic strain = 0

- For 100970 N: this point is located in the elastic-plastic region,
  Total strain = 0.02075
  Elastic strain = 611.46 / 77 x 10^3 = 0.007941 (can also be obtained from the figure)
  Plastic strain =Total strain – Elastic strain = 0.02075 - 0.007941 = 0.012809

Problem 2

The engineering stress – strain curve is shown for a cylindrical tensile specimen of an aluminum alloy. The initial part is also shown on a separate graph. Initial cross section area is 40 mm^2 and initial length is 200 mm. Poisson ratio is given as 0.3.



Use the above stress – strain response to calculate the following:

a) Modulus of elasticity.
b) Offset yield strength.
c) Permanent strain upon unloading from 150 MPa.
d) Modulus of resilience.
e) Change in diameter at a stress of 200 MPa.
f) Modulus of toughness.
g) Strength coefficient (k)
h) Strain hardening exponent (n)
i) What is the instantaneous cross section area at an engineering strain of 0.02.
j) What is the instantaneous length at an engineering strain of 0.02.
k) Does the alloy undergo plastic deformation at a load of 7 KN.

Solution:

a) Modulus of elasticity:
It can be estimated from the slope of the initial loading segment:
E~ 50 GPa ($\pm 10\ GPa$)

b) Offset yield strength: σ_y ~ 150 MPa ($\pm 10\ MPa$)

c) Permanent strain upon unloading from 150 MPa:

$\varepsilon_{plastic} = \varepsilon_{total} - \varepsilon_{elastic}$= 0.0047-0.002=0.0027

d) Modulus of resilience: U=$\frac{1}{2}\frac{\sigma_y^2}{E} = 2.25\ X\ 10^5\ \text{J/m}^3$ (for σ_y = 150 MPa and E=50 GPa)

e) Change in diameter at a stress of 200 MPa:
Since this point is located in the uniform plastic region (region after yield and before necking), we can assume that the volume is constant and find the change in diameter from the relation:
$A_i l_i = A_o l_o$
l_i can be found from the strain at 200 MPa as follow:

$$\varepsilon_{at\ \sigma=200\ MPa} \sim 0.025 = \frac{l_i - l_o}{l_o}$$

$$\longrightarrow l_i = l_o(1 + 0.025) = 200 * (1 + 0.025) = 205\ mm$$

$$A_i l_i = A_o l_o \longrightarrow A_i = \frac{A_o l_o}{l_i} = \frac{40 * 200}{205} = 39.024\ mm^2$$

$$A_i = \frac{\pi}{4} D_i^2 \longrightarrow D_i = 7.0489\ mm$$

$$\Delta D = D_i - D_0 = 7.1365 - 7.0489 = 0.087596\ mm$$

f) Toughness
Calculate the area under the engineering stress-strain curve.

g) Strength coefficient, K
The strength coefficient k can be found from the following relation

$$\sigma_t = k\ \varepsilon_t^n$$

From the given engineering stress-strain curve, pick any two points in the uniform deformation region:

$$\sigma_1 = 179\ MPa \quad and \quad \sigma_2 = 195\ MPa$$
$$\varepsilon_1 = 0.015748 \quad and \quad \varepsilon_2 = 0.021654$$

The corresponding true stress and true strain values become

$$\sigma_{t1} = \sigma_1(1+\varepsilon_1) = 182\ MPa \quad and \quad \sigma_{t2} = \sigma_2(1+\varepsilon_2) = 199\ MPa$$
$$\varepsilon_{t1} = \ln(1+\varepsilon_1) = 0.015625 \quad and \quad \varepsilon_{t2} = \ln(1+\varepsilon_2) = 0.021423$$

Substitute the above values in the relation: $\sigma_t = k\ \mathcal{E}_t^n$

$$182 = k\ (0.015625)^n$$
$$199 = k\ (0.021423)^n$$

Solving the above two equations:

$$\log(182) = \log(k) + n * log\ (0.015625)$$
$$\log(199) = \log(k) + n * log\ (0.021423)$$

$$\rightarrow n = \frac{\log(182) - \log(199)}{log\ (0.015625) - og\ (0.021423)} = 0.283$$

Hence,

$$\log(182) = \log(k) + n * log\ (0.015625)$$
$$\log(k) = \log(182) - 0.283 * log\ (0.015625) = 6.38$$
$$k = \exp(6.38) = 590.5\ MPa$$

h) Strain hardening exponent (n)
From part (g) above, n=0.283

i) What is the instantaneous cross section area at an engineering strain of 0.02?
The engineering stress at 0.02 is 190 MPa
The corresponding true stress is $\sigma_t = 190(1+0.02) = 194\ MPa$
Since $\sigma = \frac{F}{A_o} \quad \rightarrow \quad F = \sigma\ A_o = 190 * 40 = 7600\ N$
Since $\sigma_t = \frac{F}{A_i} \quad \rightarrow \quad A_i = \frac{F}{\sigma_t} = \frac{7600}{194} = 39.175\ mm^2$

j) What is the instantaneous length at an engineering strain of 0.02?

$$0.02 = \frac{l_i - l_o}{l_o}$$
$$\longrightarrow l_i = l_o(1+0.02) = 200 * (1+0.02) = 204\ mm$$

k) Does the alloy undergo plastic deformation at a load of 7 KN?
The engineering stress: $\sigma = \frac{F}{A_o} = \frac{7000}{40} = 175\ MPa > \sigma_y$
➔ The alloy undergoes plastic deformation at F=7 KN