Chapter(10) One-Sample Tests of Hypothesis Examples

What is a Hypothesis?
Hypothesis is a statement about a population parameter developed for the purpose of testing.

What is Hypothesis Testing?
Hypothesis testing is a procedure, based on sample evidence and probability theory, used to determine whether the hypothesis is a reasonable statement and should not be rejected, or is unreasonable and should be rejected.

Why we conduct the Hypothesis Testing? And How?

- We conduct the hypothesis test to find out is the difference between sample statistic (which is used to estimate the population parameter) and the value of population parameter (hypothesized value) is difference due to :
  - **Sampling error** (estimation error) which is occur when we draw a random sample from population.
  - Or is it **significant difference?** (Means actually there is difference which is has meaning in the study).

By using the sample information (mean, size, and variance)
In simple word Hypothesis Test is Statistic Test (Z test, t test, F test and chi square)
Example:
If we need to know the value of IQ score of students in the KSU we draw random sample from this population (KSU) and compute the mean of IQ from sample data which is equal =120 , but there is A researcher hypotheses that the IQ of the whole population is equal to 125 (μ = 125) ,here we need to know if the difference between the sample mean 120 and the hypothesized mean 125 is it Significant difference Or it is difference due to sampling error (by chance)? To answer this question we can conduct test statistic which is called Hypothesis Test, and we can construct Confidence interval which is already known.

Hypothesis Testing Steps

Step (1): State the Null (H_0) and alternate (H_1) hypothesis

- Null hypothesis (H_0):

Is the hypothesis being tested , here we write the hypothesized value which is called reference value (the value which we assume that equal to population parameter) and in null hypothesis only those operators are allowed ("=", "≥", "≤")because in this hypothesis always we assume that there is no difference at all (because we notice that the equal operator).

- Alternate hypothesis (H_1):

A statement that is accepted if the sample data provide sufficient evidence that the null hypothesis is false. (e.g. "≠" "<" and ">")

The alternative hypothesis which is called the researcher hypothesis it contains the opposite mathematical operation that are found in means always, we say that there is significant difference (means the population parameter will be <,>, ≠) and in this hypothesis we write what the researcher hypothesis about population parameter.

When we reject the Null Hypothesis that means we accept the alternative hypothesis and Vice Versa (this will be our decision at the end of test)

Important Things to Remember about H_0 and H_1

- H_0: H subzero or H not.
- H_0 and H_1 are mutually exclusive and collectively exhaustive
- H_0 is always presumed to be true
- H_1 has the burden of proof
- A random sample (n) is used to "*reject* H_0"
- If we conclude 'do not reject H_0', this does not necessarily mean that the null hypothesis is true, it only suggests that there is not sufficient evidence to reject H_0; rejecting the null hypothesis then, suggests that the alternative hypothesis may be true.
- Equality is always part of H0 (e.g. "=" , "≥" , "≤").
- "≠" "<" and ">" always part of H_1

Keywords	Inequality Symbol	Part of:
Larger (or more) than	>	H_1
Smaller (or less)	<	H_1
No more than	≤	H_0
At least	≥	H_0
Has increased	>	H_1
Is there difference?	≠	H_1
Has not changed	=	H_0
Has "improved", "is better than". "is more effective"	See left text	H_1

Types of Tests

According the alternative hypothesis the hypothesis tests can be one or two tailed.

Case1: One- tailed test (Right-tailed)

For Example:

$$H_0 : \pi \le \pi_0 \qquad\qquad H_0 : \mu \le \mu_0$$
$$H_1 : \pi > \pi_0 \quad \textbf{Or} \quad H_1 : \mu > \mu_0$$

Case2: One- tailed test (left-tailed)

For Example

$$H_0 : \pi \ge \pi_0 \qquad\qquad H_0 : \mu \ge \mu_0$$
$$H_1 : \pi < \pi_0 \quad \textbf{Or} \quad H_1 : \mu < \mu_0$$

Case3: Two-tailed test

For Example:

$$H_0 : \pi = \pi_0 \qquad\qquad H_0 : \mu = \mu_0$$
$$H_1 : \pi \ne \pi_0 \quad \textbf{Or} \quad H_1 : \mu \ne \mu_0$$

Step (2): Select a level of significance:

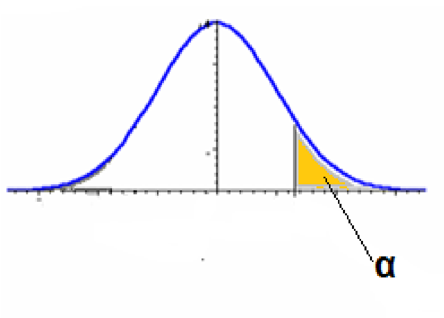
Level of significance (α) : The probability of rejecting the null hypothesis when it is true.

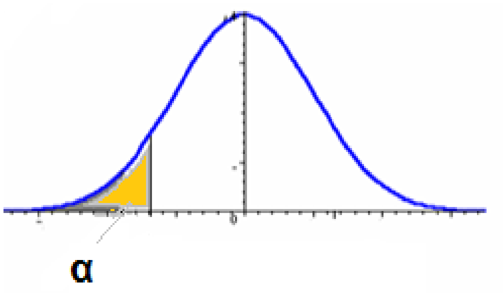
e.g. We often take about testing at a 1% significance level (probability 0.01 of rejecting true H_0)

Relationship between the alternative hypothesis and the level of significance (α):

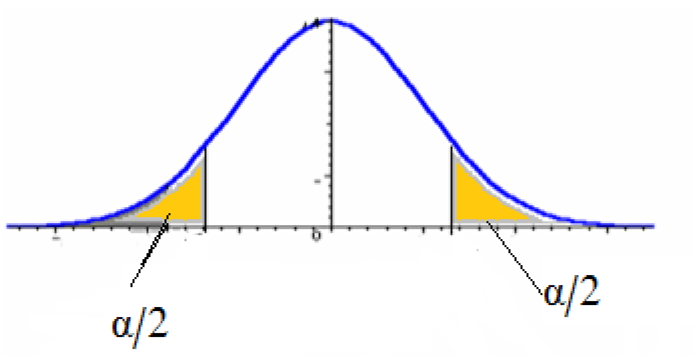
Case1& 2: If the alternative hypothesis is one- tailed test (Right or left-tailed); (α) appear in the one side of the curve.

Right tailed   Left tailed





Case3: If the alternative hypothesis is two- tailed test (α) appear in the left & right sides of the curve , we divide (α) by 2 (α/2)



Step3: Select the Test Statistic (computed value)

The test Statistic: A value, determined from sample information, used to determine whether to reject the null hypothesis.

There are many different statistical tests. The choice of which test to use depends on several factors:

- The type of data. – The distribution of the data.
- The type of study design.

Example of test statistic used to test hypothesis: Z ,t, F

Step (4): Selected the Critical value

Critical value: The dividing point between the region where null Hypothesis is rejected and the region where it is not rejected.

The critical value is read from the statistical tables and determined by:

- α. - Type of test.
- Sometimes by the degrees of freedom.

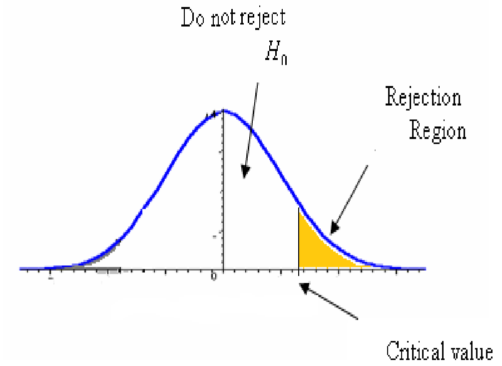
Step (5): Formulate the Decision Rule and Make a Decision

Decision rule: We will compare the computed value with critical value (table value) and make our decision:

Reject H_0 when observed test-statistic equals or exceeds Critical value.
(By other mean; when the test statistic falls in the rejection region).
Otherwise, Fail to Reject (Retain) H_0

Case1:

Reject H_0 if T.S > (C.V)

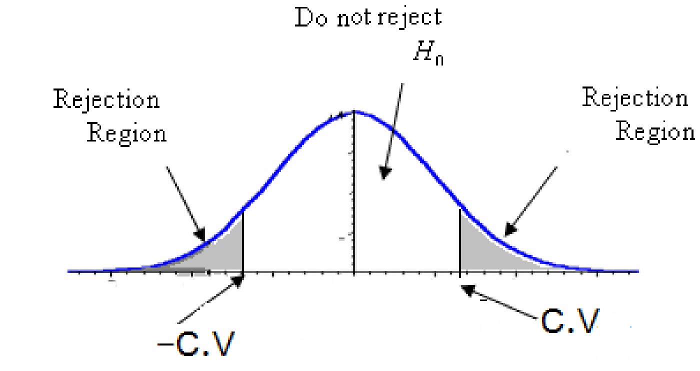


Case2:

Reject H_0 if T.S <-(C.V) or |T. S|> (C.V)

Case 3:

Reject H_0 if |T. S|> (C.V) which means T.S> (C.V) or T.S<-(C.V)



- Results of a Hypothesis test :

Either: Accept the null hypothesis (H_0) as reasonable possibility	It is A week conclusion ; not significant result means the difference only cause of sampling error
Or: Reject the null hypothesis and Accept the researcher hypothesis (H_1)	It is A strong conclusion ;a significant result (means there is difference)

Testing for apopulation mean

Step (1): State the null (H_0) and alternate (H_1) hypothesis

$H_0: \mu \le \mu_0$ **or** $H_0: \mu \ge \mu_0$ **or** $H_0: \mu = \mu_0$

$H_1: \mu > \mu_0$ $\quad$ $H_1: \mu < \mu_0$ $\quad$ $H_1: \mu \ne \mu_0$

Step (2): Select a level of significance.

Step (3): Select the Test Statistic (computed value)

- **Known population Standard Deviation**

$$Z_c = \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

- **unknown population Standard Deviation(** $n \ge 30$ **)**

$$Z_c = \frac{\bar{X} - \mu_0}{\frac{S}{\sqrt{n}}}$$

- **unknown population Standard Deviation(** $n < 30$ **)**

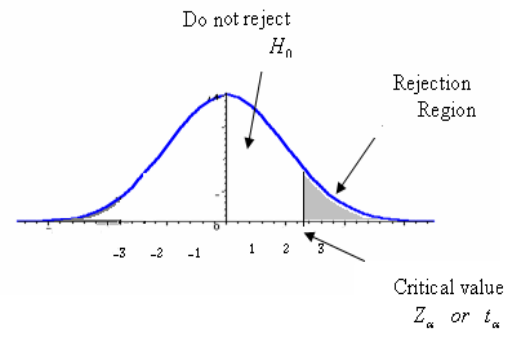
$$t_c = \frac{\bar{X} - \mu_0}{\frac{S}{\sqrt{n}}}$$

Step (4): Selected the Critical value

Types of test	Z	t
The one – tailed test (Right)	Z_{α}	$t_{(\alpha, n-1)}$
The one – tailed test (left)	$-Z_{\alpha}$	$-t_{(\alpha, n-1)}$
The two – tailed test	$\pm Z_{\alpha/2}$	$\pm t_{\left(\frac{\alpha}{2}, n-1\right)}$

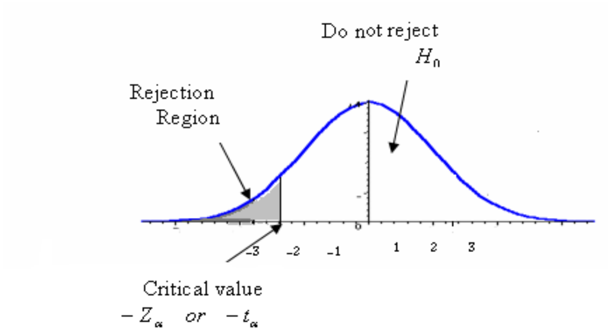
Step (5): Formulate the Decision Rule and Make a Decision

Case1: Reject H_0 if $Z_c > Z_\alpha$ $\quad$, $t_c > t_{v,\alpha}$



Case2: Reject H_0 **if** $|Z_c| > Z_\alpha$ $\quad$, $\quad$ $|t_c| > t_{v,\alpha}$

This means $\quad$ $Z_c < -Z_\alpha$ $\quad$, $t_c < -t_{v,\alpha}$

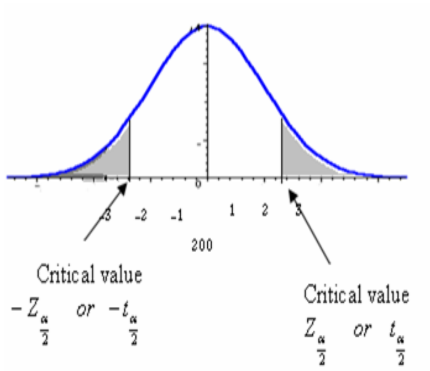


Case 3: Reject H$_0$ if $|Z_c| > Z_{\frac{\alpha}{2}}$, $|t_c| > t_{v,\frac{\alpha}{2}}$ that is:

$$Z_c > Z_{\frac{\alpha}{2}} \quad , t_c > t_{v,\frac{\alpha}{2}}$$

or

$$Z_c < -Z_{\frac{\alpha}{2}} \quad , \; t_c < -t_{v,\frac{\alpha}{2}}$$



Known population Standard Deviation
Example (1)
Jamestown Steel Company manufactures and assembles desks and other office equipment at several plants in western New York State. The weekly production of the Model A325 desk at the Fredonia Plant follows the normal probability distribution with a mean of 200 and a standard deviation of 16. Recently, because of market expansion, new production methods have been introduced and new employees hired. The vice president of manufacturing would like to investigate whether there has been a change in the weekly production of the Model A325 desk. Is the mean number of desks produced at Fredonia Plants different from 200 at the 0.01 significance level? IF the vice president takes sample of 50 weeks and he finds the mean is 203.5, use the statistical hypothesis testing procedure to investigate whether the production rate has changed from 200 per week**.**
Solution:
Step (1):State the null hypothesis and the alternate hypothesis.

H_0: $\mu = 200$
H_1: $\mu \neq 200$

This is Two-tailed test
(Note: keyword in the problem “has changed”)
Step (2): Select the level of significance.

α = 0.01 as stated in the problem

Step (3): Select the test statistic

Use Z-distribution since **σ** is known

$$\mu_0 = 200 \quad , \quad \overline{X} = 203.5 \quad , \quad \sigma = 16 \quad , \quad n = 50$$

$$Z = \frac{\overline{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{203.5 - 200}{\frac{16}{\sqrt{50}}} = \frac{3.5}{2.2627} = 1.55$$

Step (4): Formulate the decision rule (Critical value)

$Z_{\frac{\alpha}{2}} = Z_{\frac{0.01}{2}} = Z_{0.005}$ 0.5 – 0.005 =0.4950

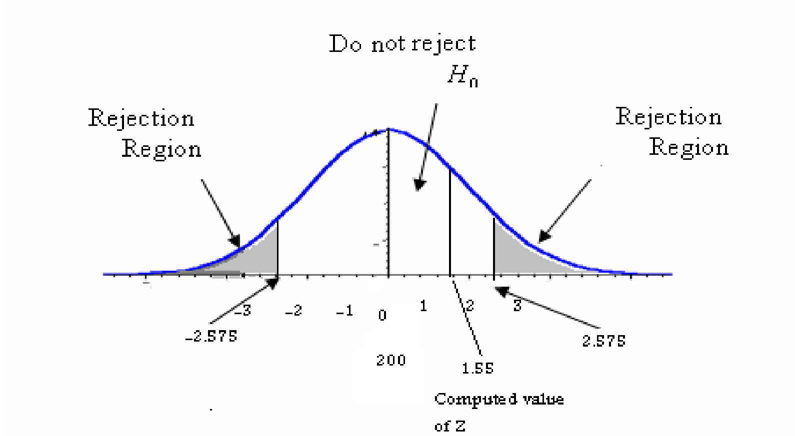
$Z_{0.005} = \pm 2.58$

Reject H_0 if $Z_c > 2.58$ or $Z_c < -2.58$

Step (5): Make a decision and interpret the result.

Reject H_0 if $Z_c < -Z_{\frac{\alpha}{2}}$ or $Z_c > Z_{\frac{\alpha}{2}}$

$Z_c = 1.55 < 2.58$



Because 1.55 does not fall in the rejection region, H0 is not rejected at significance level 0.01. We conclude that the population mean is not different from 200. So we would report to the vice president of manufacturing that the sample evidence does not show that the production rate at the Fredonia Plant has changed from 200 per week. The difference of 3.5 units between the historical weekly production rate and that last year can reasonably be attributed to sampling error.

Example (2)

Suppose you are buyer of large supplies of light bulbs. You want to test at the 5% significant level, the manufacturer's claim that his bulbs last more than 800 hours, you test 36 bulbs and find that the sample mean is 816 hours , with standard deviations 70 hours .should you accept the claim?

Solution:

Step 1: State the null hypothesis and the alternate hypothesis.

$H_0 : \mu \leq 800$

$H_1 : \mu > 800$

This is one-tailed test (right),

(Note: keyword in the problem "more than")

Step 2: Select the level of significance.

$\alpha = 0.05$ as stated in the problem

Step 3: Select the test statistic.

Use Z-distribution since σ is known

$\mu_0 = 800$, $\bar{X} = 816$, $S = 70$, $n = 36$

$$Z = \frac{\bar{X} - \mu_0}{\frac{S}{\sqrt{n}}} = \frac{816 - 800}{\frac{70}{\sqrt{36}}} = \frac{16}{11.6667} = 1.37$$

Step 4: Formulate the decision rule.

Reject H_0 if $Z_c > Z_\alpha$

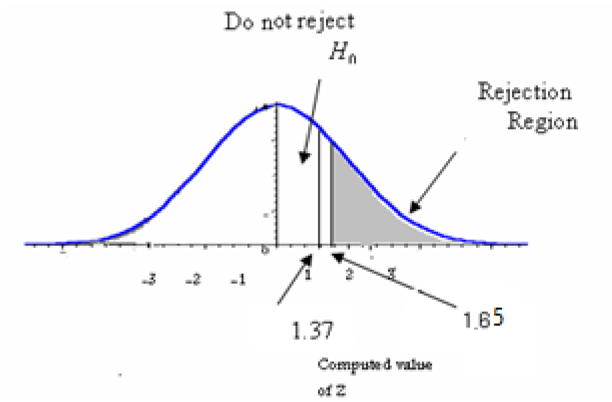
$Z_\alpha = Z_{0.05}$ $= 0.5 – 0.05 = 0.4500$

$Z_{0.05} = 1.65$

Step 5: Make a decision and interpret the result.

Reject H_0 if $Z_c > 1.65$

$Z_c = 1.37 < 1.65$



Because 1.37 does not fall in the rejection region, H_0 is not rejected. We conclude that the population mean is less than or equal 800. The decision that we don't reject the null hypothesis at significant level 0.05, but we reject the alternative hypothesis the manufacturer's claim that $\mu > 800$.

The difference of 16 hours between sample and production can reasonably be attributed to sampling error.

Example (3)

The McFarland Insurance Company Claims Department reports the mean cost to process a claim is \$60. An industry comparison showed this amount to be larger than most other insurance companies, so the company instituted cost-cutting measures. To evaluate the effect of the cost-cutting measures, the Supervisor of the Claims Department selected a random sample of 26 claims processed last month. The sample information is reported below.

$n = 26$, $\overline{X} = 56.42$, $S = 10.04$

At the .01 significance level is it reasonable a claim is now less than \$60?

Solution:

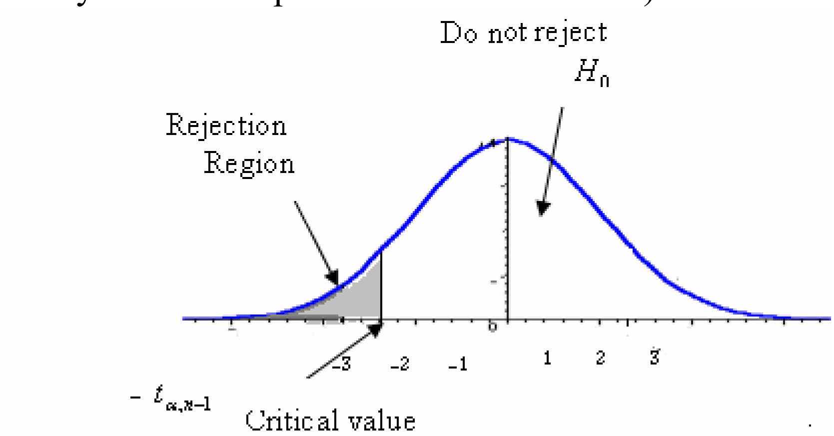
Step 1: State the null hypothesis and the alternate hypothesis.

$H_0 : \mu \geq \$60$

$H_1 : \mu < \$60$

This is one-tailed test (Left)

(Note: keyword in the problem "now less than")



Step 2: Select the level of significance.

α = 0.01 as stated in the problem

Step 3: Select the test statistic.

Use t-distribution since σ is unknown

$\mu_0 = 60$, $\overline{X} = 56.42$, $S = 10.04$, $n = 26$

$$t_c = \frac{\overline{X} - \mu_0}{\frac{S}{\sqrt{n}}} = \frac{56.42 - 60}{\frac{10.04}{\sqrt{26}}} = \frac{-3.58}{1.9690} = -1.82$$

Step 4: Formulate the decision rule.

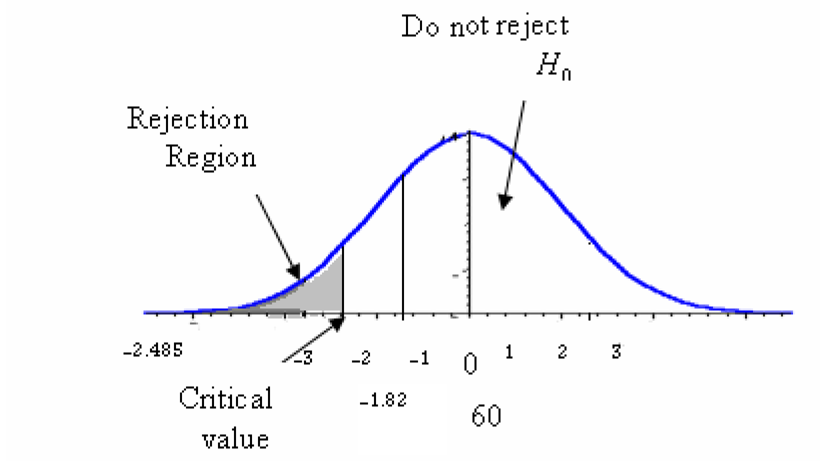
Reject H_0 if $t_c < -t_{\alpha,n-1}$

$-t_{\alpha,n-1} = -t_{0.01,25} = -2.485$

Step 5: Make a decision and interpret the result.

Reject H_0 if $t_c < -2.485$

$t_c = -1.82 > -2.485$



Because -1.82 does not fall in the rejection region, H_0 is not rejected at the .01 significance level. We have not demonstrated that the cost-cutting measures reduced the mean cost per claim to less than \$60. The difference of \$3.58 (\$56.42 - \$60) between the sample mean and the population mean could be due to sampling error.

P-VALUE

The p-value or observed of significance level of a statistical test is the smallest value of α for which H_0 can be rejected. It is the actual risk of committing a type I error, if H_0 is rejected based on the observed value of the test statistic. The p-value measures the strength of the evidence against H_0.

The probability of observing samples data by chance under the Null hypothesis (i.e. null hypothesis is true).

In testing a hypothesis, we can also compare the *p*-value to with the significance level (α).

- **If the *p*-value < significance level(α) , H_0 is rejected (means significant result)**
- **If the *p*-value $\geq$ significance level (α), H_0 is not rejected. (means not significant result).**

Compute the P- value (Only we used the Z distribution)

Case 1 : If the one – tailed test (Right)

$$\boxed{p-value = P(Z > z_c) = 0.5 - \Phi(z_c)}$$

Case 2: If the one – tailed test (left)

$$\boxed{p-value = P(Z < -z_c) = 0.5 - \Phi(z_c)}$$

Case3: If the two – tailed test

$$p-value = P(Z > z_c) + P(Z < -z_c) = [0.5 - \Phi(z_c)] + [0.5 - \Phi(z_c)] = 1 - 2\,\Phi(z_c)$$

Or

$$\boxed{p-value = 2P(Z > |z_c|) = 2[0.5 - \Phi(z_c)]}$$

Example (4) : Refer to example (1)

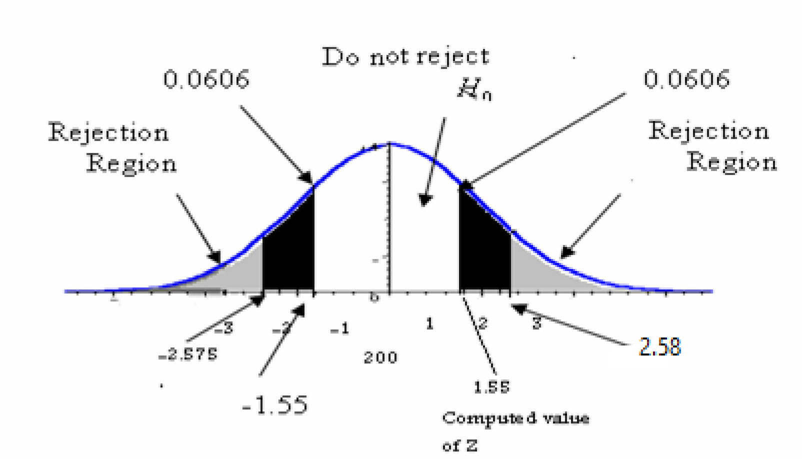
$H_0 : \mu = 200 \quad H_1 : \mu \neq 200$

$\mu_0 = 200$, $\overline{X} = 203.5$, $\sigma = 16$, $n = 50$, $\alpha = 0.01$

$$p - value = 2P(Z > |z_c|) = 2[0.5 - \Phi(z_c)]$$
$$= 2P(Z > 1.55) = 2[0.5 - \Phi(1.55)] = 2[0.5 - 0.4394] = 2 \times 0.0606 = 0.1212$$

$P - value = 0.1212 > \alpha = 0.01$

$\therefore Do \ \ not \ \ reject \ H_0$



Test of hypothesis concerning a population proportion

Step (1) : State the Null(H_0) and alternate(H_1) hypothesis

$H_0 : \pi \le \pi_0$ or $H_0 : \pi \ge \pi_0$ or $H_0 : \pi = \pi_0$

$H_1 : \pi > \pi_0$ $\quad$ $H_1 : \pi < \pi_0$ $\quad$ $H_1 : \pi \ne \pi_0$

Step (2): Select a level of significance:

Step (3): Select the Test Statistic (computed value)

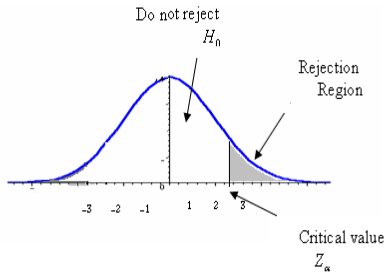
$$Z_c = \frac{p - \pi_0}{\sqrt{\dfrac{\pi_0(1-\pi_0)}{n}}}$$

Step (4): Selected the Critical value

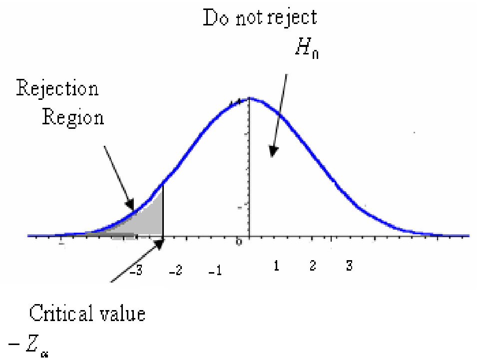
The one – tailed test (Right)	Z_α
The one – tailed test (left)	$-Z_\alpha$
The two – tailed test	$\pm Z_{\alpha/2}$

Step (5): Formulate the Decision Rule and Make a Decision

Case1: Reject H_0 if $Z_c > Z_\alpha$

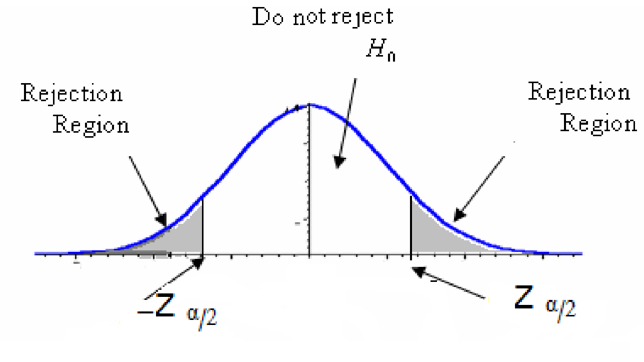


Case2: Reject H_0 if $|Z_c| > Z_\alpha$ or $Z_c < -Z_\alpha$



Case3: Reject H_0 if

$Z_c > Z_{\frac{\alpha}{2}}$ or $Z_c < -Z_{\frac{\alpha}{2}}$



Example (5)

Suppose prior elections in a certain state indicated it is necessary for a candidate for governor to receive at least 80 percent of the vote in the northern section of the state to be elected. The incumbent governor is interested in assessing his chances of returning to office. A sample survey of 2,000 registered voters in the northern section of the state revealed that 1540 planned to vote for the incumbent governor. Using the hypothesis-testing procedure, assess the governor's chances of reelection (the level of significance is 0.05).

Solution:

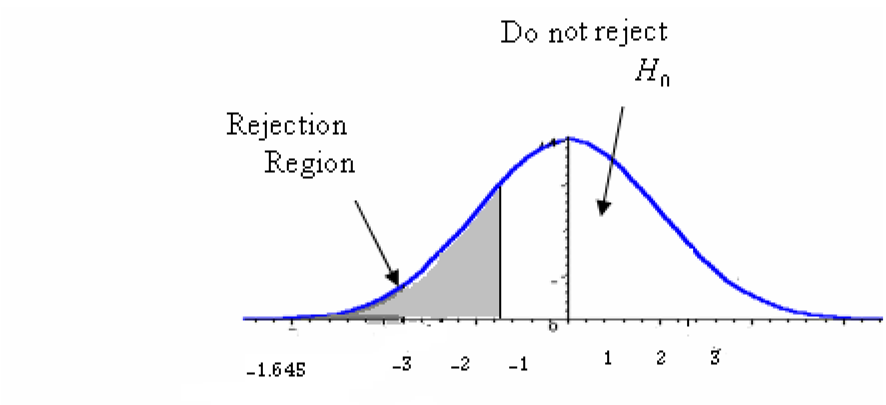
Step 1: State the null hypothesis and the alternate hypothesis.

$H_0 : \pi \geq 0.80$

$H_1 : \pi < 0.80$

This is one-tailed test (Left)

(Note: keyword in the problem "at least")



Step 2: Select the level of significance.

α = 0.05 as stated in the problem

Step 3: Select the test statistic.

Use Z-distribution since the assumptions are met and $n\pi$ and $n(1-\pi) \geq 5$

$\pi_0 = 0.80$, $p = \frac{1540}{2000} = 0.77$, $n = 2000$

$$Z_c = \frac{p - \pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}} = \frac{0.78 - 0.80}{\sqrt{\frac{0.8 \times 0.20}{2000}}} = \frac{-0.03}{0.0089} = -3.37$$

Step 4: Formulate the decision rule.

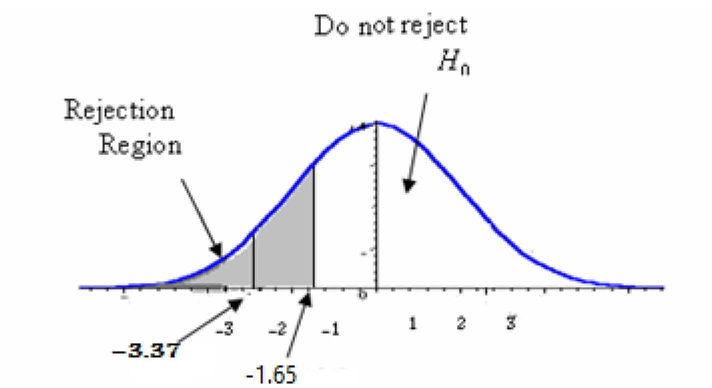
Reject H_0 if $Z_c < -Z_\alpha$

$Z_{0.05} = -1.65$

Step 5: Make a decision and interpret the result.

Reject H_0 if $Z_c < -1.65$

$Z_c = -3.37 < -1.65$

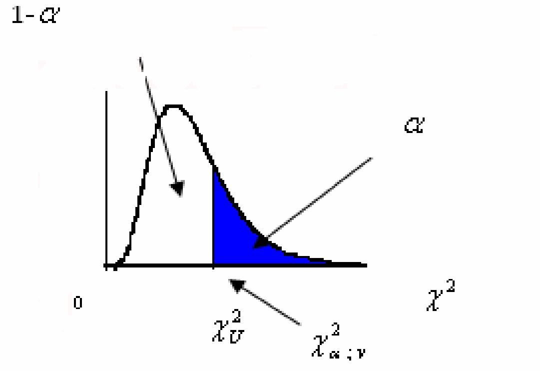


Reject H_0

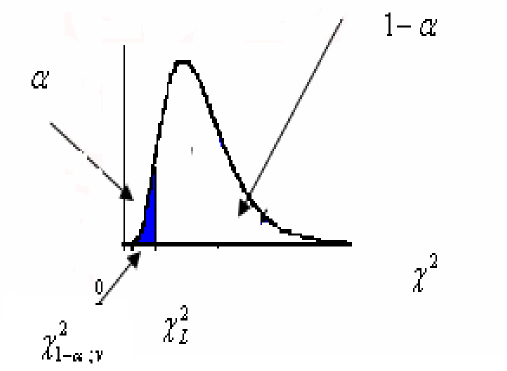
The computed value of (-3.37) is in the rejection region, so the null hypothesis is rejected at the .05 level. The difference of 2.5 percentage points between the sample percent (77 percent) and the hypothesized population percent (80) is statistically significant. The evidence at this point does not support the claim that the incumbent governor will return to the governor's mansion for another four years.

Test of hypothesis concerning apopulation variance

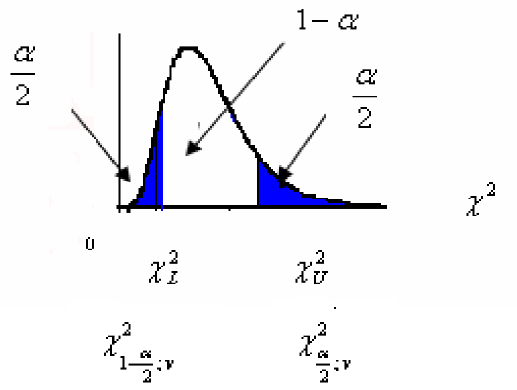
Step (1) : State the Null (H_0) and alternate (H_1) hypothesis



Case 1: $H_0: \sigma^2 \le \sigma_0^2$
$H_1: \sigma^2 > \sigma_0^2$



Case 2: $H_0: \sigma^2 \ge \sigma_0^2$
$H_1: \sigma^2 < \sigma_0^2$



Case 3: $H_0: \sigma^2 = \sigma_0^2$
$H_1: \sigma^2 \ne \sigma_0^2$

Step (2): Select a level of significance
Step (3): Select the Test Statistic (computed value)

$$\chi^2 = \frac{(n-1)S^2}{\sigma^2}$$

Step (4): Selected the Critical value

The one – tailed test (Right)	$\chi^2_{(\alpha,n-1)}$
The one – tailed test (left)	$\chi^2_{(1-\alpha,n-1)}$
The two – tailed test	$\chi^2_{\left(\frac{\alpha}{2},n-1\right)}$ **&** $\chi^2_{\left(1-\frac{\alpha}{2},n-1\right)}$

Step (5): Formulate the Decision Rule and Make a Decision

Case1: The one – tailed test (Right)

Reject H_0 if $\chi^2_c > \chi^2_{(\alpha,n-1)}$

Case2: The one – tailed test (left)

Reject H_0 if $\chi^2_c < \chi^2_{(1-\alpha,n-1)}$

Case3: The two – tailed test; rejectH_0 if

$$\chi^2_c > \chi^2_{\left(\frac{\alpha}{2},n-1\right)}$$

Or

$$\chi^2_c < \chi^2_{\left(1-\frac{\alpha}{2},n-1\right)}$$

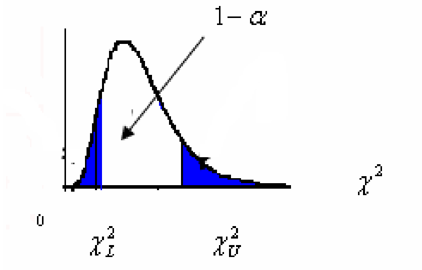
Example (6)

A sample of size 10 produced a variance of 14.Is this sufficient to reject the null hypothesis that σ^2 is equal to 6
When tested using a 0.05 level of significance?

Step 1: State the null hypothesis and the alternate hypothesis.

$H_0 : \sigma^2 = 6$

$H_1 : \sigma^2 \neq 6$



This is two-tailed test

(Note: keyword in the problem "that σ^2 is equal to 6")

Step 2: Select the level of significance.

α = 0.05 as stated in the problem

$$\frac{\alpha}{2} = \frac{0.05}{2} = 0.025$$

$$1 - \frac{\alpha}{2} = 1 - \frac{0.05}{2} = 0.975$$

Step 3: Select the test statistic.

$$\chi_c^2 = \frac{(n-1)S^2}{\sigma^2} = \frac{(10-1)14}{6} = \frac{126}{6} = 21$$

Step 4: Formulate the decision rule (Critical value)

$\chi^2_{0.025\ ;,9} = 19.022$, $\chi_{0.975;\ 9} = 2.7003$

Right tail areas for the Chi-square Distribution					
V df\area	Q				
	0.250	0.100	0.050	0.025	0.010
1	1.3233	2.7055	3.8415	5.0239	6.6349
2	2.7726	4.6052	5.9915	7.3778	9.2104
3	4.1083	6.2514	7.8147	9.3484	11.3449
4	5.3853	7.7794	9.4877	11.1433	13.2767
5	6.6257	9.2363	11.0705	12.8325	15.0863
6	7.8408	10.6446	12.5916	14.4494	16.8119
7	9.0371	12.0170	14.0671	16.0128	18.4753
8	10.2189	13.3616	15.5073	17.5345	20.0902
9	11.3887	14.6837	16.9190	19.0228	21.6660
10	12.5489	15.9872	18.3070	20.4832	23.2093

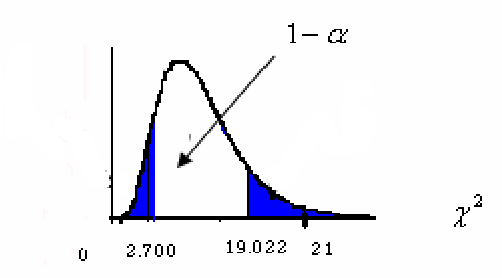
Right tail areas for the Chi-square Distribution					
V df\area	Q				
	0.750	0.900	0.950	0.975	0.990
1	0.101531	0.015791	0.003932	0.000982	0.000157
2	0.575364	0.210721	0.102586	0.050636	0.020100
3	1.212532	0.584375	0.351846	0.215795	0.114832
4	1.922568	1.063624	0.710724	0.484419	0.297107
5	2.674604	1.610309	1.145477	0.831209	0.554297
6	3.454598	2.204130	1.635380	1.237342	0.872083
7	4.254852	2.833105	2.167349	1.689864	1.239032
8	5.070642	3.489537	2.732633	2.179725	1.646506
9	5.898823	4.168156	3.325115	2.700389	2.087889
10	6.737199	4.865178	3.940295	3.246963	2.558199
11	7.584145	5.577788	4.574809	3.815742	3.053496

Step 5: Make a decision and interpret the result.

Reject H_0 if $\chi_c^2 > \chi^2_{\frac{\alpha}{2},v} = \chi^2_{0.025\,;.9} = 19.022$,

Or $\chi_c^2 < \chi^2_{1-\frac{\alpha}{2},v} = \chi_{0.975\,;\,9} = 2.7003$

The decision is to reject the null hypothesis, because the computed χ^2 Value (21) is larger than the critical value (19.022).
We conclude that there is a difference



The Types of errors

Null Hypothesis	Does Not RejectH_0	Rejects H_0
H_0 **is true**	**Do not rejecting The null hypothesis,** H_0 **,when It is true(**$1-\alpha$**)** **{Correct decision}**	**rejecting The null hypothesis,** H_0 **,when It is true(**α**)** **{Type I error}**
H_0 **is false**	**Do not rejecting The null hypothesis,** H_0 **,when It is false(**β**)** **{Type II error}**	**rejecting The null hypothesis,** H_0 **,when It is false (Power) (**$1-\beta$**)** **{Correct decision}**

Note:

The quantity ($1-\beta$) is called the power of the test because it measures the probability of taking the action that we wish to have occur-that is ,rejecting the H0 when it is false and H1 is true.

($1-\beta$**) = P(rejecting the H0 when it is false)**

Ideally, you would like (α) to be small and the power ($1-\beta$) to be large.

Example (7)

A manufacturer purchases steel bars to make cotter pins. Past experience indicates that the mean tensile strength of all incoming shipments is greater than 10,000 psi and that the standard deviation, σ, is 400 psi. In order to make a decision about incoming shipments of steel bars, the manufacturer set up this rule for the quality-control inspector to follow: "Take a sample of 100 steel bars, at the .05 significance level.

Suppose the unknown population mean of an incoming lot, designated μ_1 is really 10120 psi. Find

a .The type I error (Rejecting the null hypothesis, H_0, when It is true (α).

b. The correct decision (Do not rejecting the null hypothesis, H_0, when It is true ($1-\alpha$).

c .The type II error (Do not rejecting the null hypothesis, H_0, when It is false (β).

d .The correct decision (Rejecting The null hypothesis, H_0, when It is false ($1-\beta$).

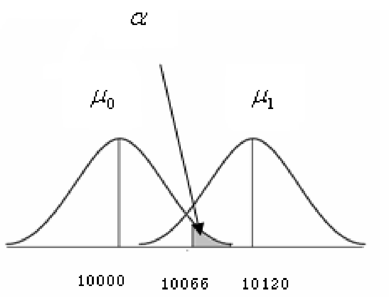
Solution:

$$H_0 : \mu \leq 10000 \quad Z = 1.645$$
$$H_1 : \mu > 10000$$

$$\mu_0 + Z_\alpha \frac{\sigma}{\sqrt{n}} = 10000 + 1.645 \frac{400}{10} = 10000 + 65.8 = 10066$$

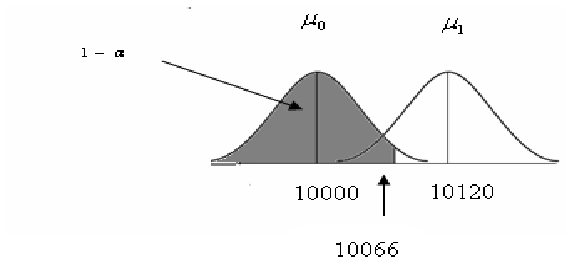
a .The type I error (where rejecting the null hypothesis, H_0, when it is true (α)).

$\alpha = 0.05$



b. The correct decision (Do not rejecting The null hypothesis, H_0, when It is true ($1-\alpha$).

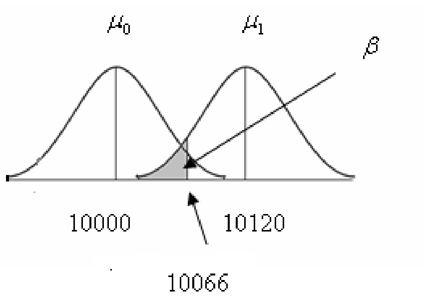
$1-\alpha = 1-.05 = 0.95$



c .The type II error (where accepting the null hypothesis, H_0, when it is false (β)).

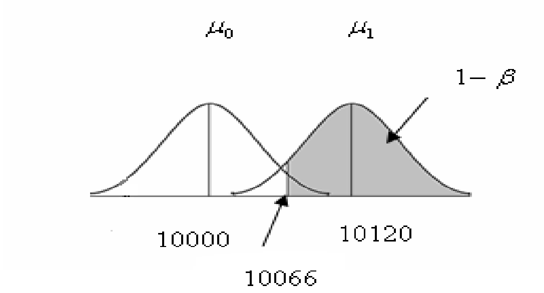
$$P\left(Z < \frac{\overline{X}_c - \mu_1}{\frac{\sigma}{\sqrt{n}}}\right) = P\left(Z < \frac{10066-10120}{\frac{400}{\sqrt{100}}}\right) = P\left(Z < \frac{-54}{40}\right) = 0.5 - P(-1.35 < Z < 0)$$

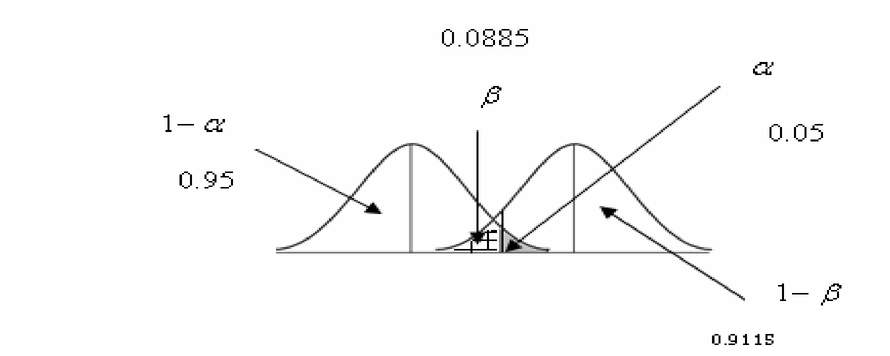
$0.5 - \Phi(1.35) = 0.5 - 0.4115 = 0.0885 \therefore \beta = 0.0885$



d .The correct decision (where H_0 is false and reject it ($1-\beta$)).

$1-\beta = 1-0.0885 = 0.9115$





Example (8)

A manufacturer purchases steel bars to make cotter pins. Past experience indicates that the mean tensile strength of all incoming shipments is less than 10,000 psi and that the standard deviation, σ, is 400 psi. In order to make a decision about incoming shipments of steel bars, the manufacturer set up this rule for the quality-control inspector to follow: "Take a sample of 100 steel bars, at the .05 significance level.

Suppose the unknown population mean of an incoming lot, designated μ_1 is really 9900psi.find:

a .The type I error (Rejecting the null hypothesis, H_0, when It is true (α)).

b. The correct decision (Do not rejecting the null hypothesis, H_0, when It is true ($1-\alpha$)).

c .The type II error (Do not rejecting the null hypothesis, H_0, when It is false (β)).

d .The correct decision (Rejecting the null hypothesis, H_0, when It is false ($1-\beta$)).

Solution:

$H_0: \mu \geq 10000$

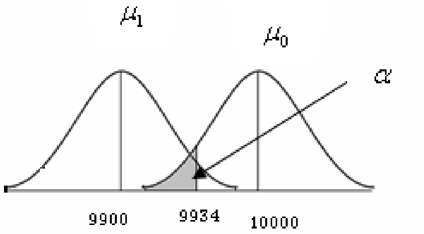
$H_1: \mu. < 10000$

$Z = 1.645$

$$\mu_0 - Z_\alpha \frac{\sigma}{\sqrt{n}} = 10000 - 1.645\frac{400}{10} = 10000 - 65.8 = 9934$$

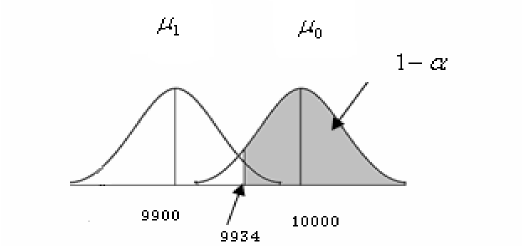
a .The type I error (where rejecting the null hypothesis, H_0, when it is true (α)).

$\alpha = 0.05$



b. The correct decision (Do not rejecting the null hypothesis, H_0, when It is true ($1-\alpha$)).

$1-\alpha = 1-0.05 = 0.95$



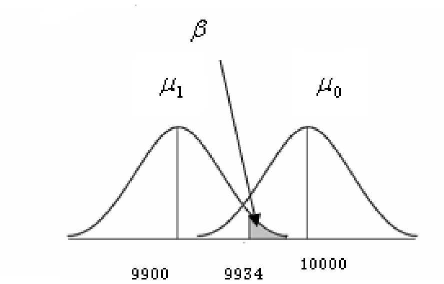
c .The type II error (where accepting the null hypothesis, H_0, when it is false (β)).

$$P\left(Z > \frac{\overline{X}_c - \mu_1}{\frac{\sigma}{\sqrt{n}}}\right)$$

$$= P\left(Z > \frac{9934-9900}{\frac{400}{\sqrt{100}}}\right) = P\left(Z > \frac{34}{40}\right) = 0.5 - P(0 < Z < 0.85)$$

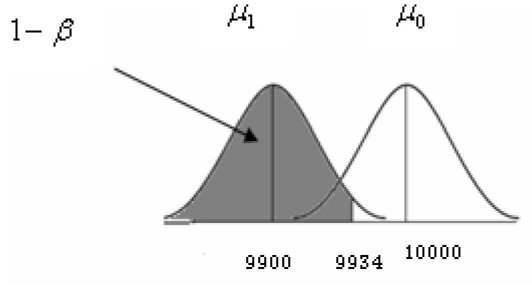
$0.5 - \Phi(0.85) = 0.5 - 0.3023 = 0.1977$

$\therefore \beta = 0.1977$



d .The correct decision (where H_0 is false and reject it ($1-\beta$)).

$1-\beta = 1-0.1977 = 0.8023$

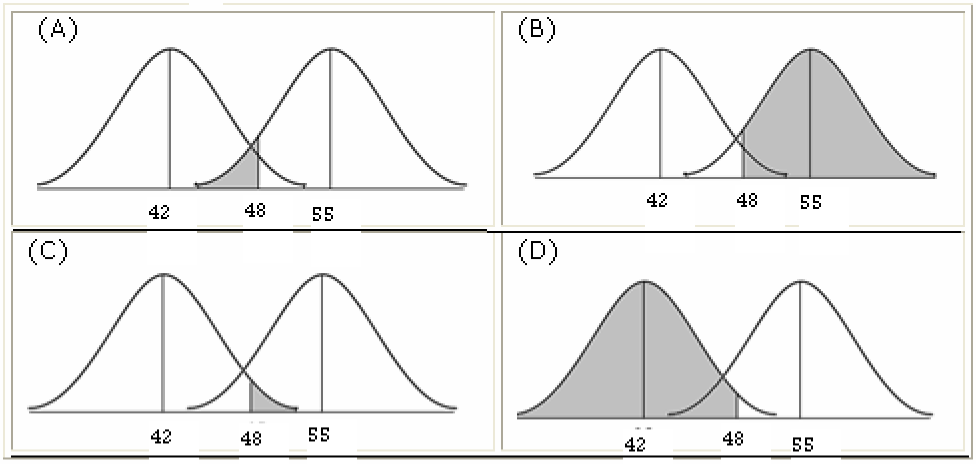


Example (9)

If

$\mu_1 = 55$, $H_0 : \mu \leq 42$

$H_1 : \mu > 42$



Complete the following statements:

a. The probability of not rejecting the null hypothesis when it is true is the shaded area in diagram..........

b. The probability of Type I error is the shaded area in diagram…….

c. The probability of Type II error is the shaded area in diagram…….

d .The probability of rejecting the null hypothesis when it is false is the shaded area in. diagram..........

Solution:

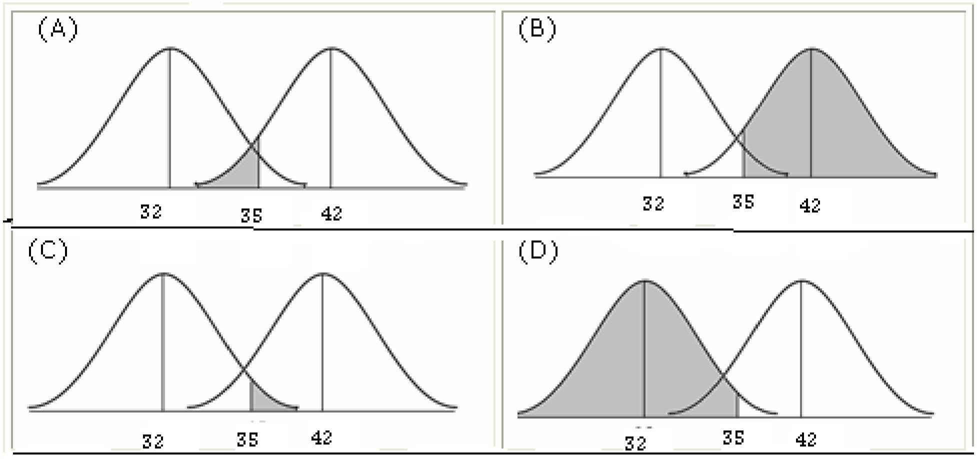
a. Diagram D
b. Diagram C
c. Diagram A
d. Diagram B

Example (10)

If

$\mu_1 = 32$, $H_0 : \mu \geq 42$

$H_1 : \mu < 42$



Complete the following statements:

a. The probability of not rejecting the null hypothesis when it is true is the shaded area in diagram..........

b. The probability of Type I error is the shaded area in diagram…….

c. The probability of Type II error is the shaded area in diagram…….

d .The probability of rejecting the null hypothesis when it is false is the shaded area in diagram.........

Solution:

a. Diagram B
b. Diagram A
c. Diagram C
d. Diagram D

Example (11)

Null Hypothesis	Does Not Reject H_0	Rejects H_0
H_0 is true	Do not rejecting The null hypothesis, H_0 ,when It is true($1-\alpha$) Example √ $H_0 : \mu \geq 60$ x $H_1 : \mu < 60$ If the decision is Do not reject H_0 Let $\mu_1 = 80$ $\therefore$ The decision is correct Do not rejecting The null hypothesis, H_0 ,when It is true($1-\alpha$)	rejecting The null hypothesis, H_0 ,when It is true(α) Example x $H_0 : \mu \geq 60$ √ $H_1 : \mu < 60$ If the decision is reject H_0 Let $\mu_1 = 80$ $\therefore$ The decision is incorrect rejecting The null hypothesis, H_0 ,when It is true(α)
H_0 is false	Do not rejecting The null hypothesis, H_0 ,when It is false(β) Example √ $H_0 : \mu \geq 60$ x $H_1 : \mu < 60$ If the decision is Do not reject H_0 Let $\mu_1 = 50$ $\therefore$ The decision is incorrect Do not rejecting The null hypothesis, H_0 ,when It is false(β)	rejecting The null hypothesis, H_0 ,when It is false (Power)($1-\beta$) Example x $H_0 : \mu \geq 60$ √ $H_1 : \mu < 60$ If the decision is reject H_0 Let $\mu_1 = 50$ $\therefore$ The decision is correct rejecting The null hypothesis, H_0 ,when It is false (Power)($1-\beta$)

Note about P_c :

$H_0 : \pi \le \pi_0$ $H_1 : \pi > \pi_0$,	$\pi_0 + Z_\alpha \sqrt{\frac{\pi_0(1-\pi_0)}{n}}$,	$\beta = P\left(Z < \frac{P_c - \pi_1}{\sqrt{\frac{\pi_1(1-\pi_1)}{n}}} \right)$

$H_0 : \pi \ge \pi_0$ $H_1 : \pi < \pi_0$,	$\pi_0 - Z_\alpha \sqrt{\frac{\pi_0(1-\pi_0)}{n}}$,	$\beta = P\left(Z > \frac{P_c - \pi_1}{\sqrt{\frac{\pi_1(1-\pi_1)}{n}}} \right)$

Test of hypothesis about single population mean

Example 1(ch10)

In the following example, the population distribution is normal with known standard deviation and the hypothesis test is right-tailed.

A hypothesis test is to be performed to determine whether the mean waiting time during peak hours for customers in a supermarket has increased from the previous mean waiting time of 8.2 minutes. Previous experience indicates that the waiting time follows a normal distribution with standard deviation equal 3.8 minutes. To test the hypothesis, a random sample of 25 customers will be selected yields mean $\bar{x} = 9.75$.. Answer the questions 1 to 10.

Question 1

The null and alternative hypotheses are...

(A) $H_0 : \mu \geq 8.2$ & $H_1 : \mu < 8.2$	(B) $H_0 : \mu = 8.2$ & $H_1 : \mu \neq 8.2$
(C) $H_0 : \mu \leq 8.2$ & $H_1 : \mu > 8.2$	(D) $H_0 : \mu \neq 8.2$ & $H_1 : \mu = 8.2$

Question 2:

This hypothesis test is classifies as...

(A) Right-tailed	(B) Two-tailed
(C) Multi-tailed	(D) left-tailed

Question 3

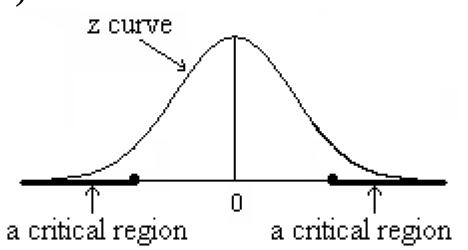
The appropriate test statistic and its distribution under the null hypothesis is...

(A) $Z = \dfrac{\overline{X} - \mu_0}{\sigma / \sqrt{n}} \tilde{\sim} N(0,1)$	(B) $Z = \dfrac{\overline{X} - \mu_0}{\sigma / \sqrt{n}} \sim N(0,1)$
(C) $T = \dfrac{\overline{X} - \mu_0}{S / \sqrt{n}} \sim t_{24}$	(D) $Z = \dfrac{\overline{X} - \mu}{S / \sqrt{n}} \sim N(0,1)$

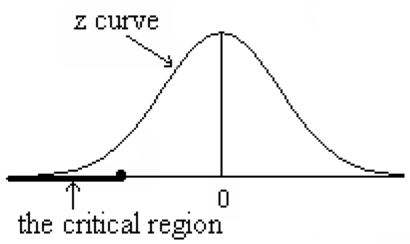
Question 4

The critical region is best described by figure....

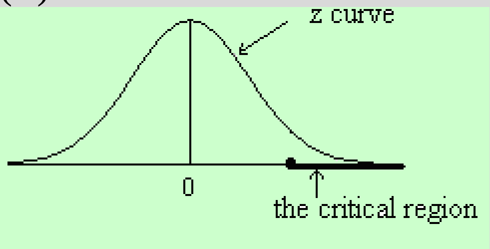
(A)



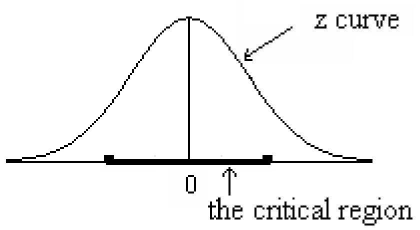
(B)



(C)



(D)



Question 5

With significance level equal 0.05, the decision criterion for the hypothesis test in terms of the computed value of the test statistic is....

(A) Reject H_0 if $z_c < -1.645$	(B) Reject H_0 if $z_c > 1.96$
(C) Reject H_0 if $Z_c > 1.645$ or $Z_c < -1.645$	(D) Reject H_0 if $z_c > 1.645$

Question 6

With significance level equal 0.05, the decision criterion for the hypothesis test in terms of the computed value of the estimator of the pupation parameter($\bar{x}_c$)is

(A)) 9.45	(B) 3.98	(C) 8.90	(D) 6.95

Solution:

$$\bar{X}_c = \mu_0 + Z_\alpha\left(\frac{\sigma}{\sqrt{n}}\right) = 8.2 + 1.645\left(\frac{3.8}{\sqrt{25}}\right) = 9.45$$

Question 7:

The computed value of our test statistic is....

(A) -2.04	(B) 3.98	(C) 2.04	(D) 0.54

Solution: $z_c = \dfrac{9.75-8.2}{3.8/\sqrt{25}} = 2.04$

Question 8

The decision would be to....

(A) Cannot be determined
(B) Do not reject the null hypothesis.
(C) Reject the null hypothesis.
(D) Reject the alternative hypothesis.

Question 9

Suppose that in fact the waiting time is increased to 9 minutes ($\mu_1 = 9.9$), then the decision has been made is...

(A) Committing Type I error	(B) Committing Type II error
(C) Correct decision$(1-\alpha)$	(D) Correct decision$(1-\beta)$

Question 10

The power of our test at $\mu_1 = 9.9$ minutes is....

(A) 0.2224	(B) 0.7224	(C) 0.7776	(D) 0.2776

Solution:

Power equals the probability of rejecting the null hypothesis while the alternative hypothesis is false. We compute it as follows:

At $\mu_1 = 9.9$

$$\beta = P(Z < Z_c) = P\left(Z < \frac{\overline{X}_c - \mu_1}{\sigma/\sqrt{n}}\right) = P\left(Z < \frac{9.45-9.9}{3.8/\sqrt{25}}\right) = P(Z < -0.59)$$

= 0.5-0.2224=0.2776

Power=1-β=1-0.2776=7224

Amina Ali Saleh
Professor of Statistics

Question 11

A 95% confidence interval is $8.3 < \mu < 11.2$. The null hypothesis is $\begin{array}{l} H_0 : \mu = 8.2 \\ H_1 : \mu \neq 8.2 \end{array}$

What is the decision?

(A)	Reject the null hypothesis.
(B)	Do not Reject the null hypothesis.
(C)	Can not be determined
(D)	Reject the alternative hypothesis.

End of example 1

Example 2(ch10)

In the following example, the population distribution is approximately normal with known standard deviation and the hypothesis test is left-tailed

The students at the university claim that the average student must travel for at least 25 minutes in order to reach the university. The university admissions office obtained a random sample of 36 one-way travel times from students. The sample had a mean of 19.4 minutes. Assume that the population standard deviation is 9.6 minutes. Does the admissions office have sufficient evidence to reject the student's claim?
Answer the following questions

Question 1
The null and alternative hypotheses are...

(A) $H_0 : \mu < 25$ & $H_1 : \mu \geq 25$	(B) $H_0 : \mu = 25$ & $H_1 : \mu > 25$
(C) $H_0 : \mu \neq 25$ & $H_1 : \mu < 25$	(D) $H_0 : \mu \geq 25$ & $H_1 : \mu < 25$

Question 2:
This hypothesis test is classifies as...

(A) Right-tailed	(B) Two-tailed
(C) Multi-tailed	(D) left-tailed

Question 3:
The appropriate test statistic and its distribution under the null hypothesis is...

(A) $Z = \dfrac{P - \pi_0}{\sqrt{\dfrac{\pi_0(1-\pi_0)}{n}}} \sim N(0,1)$	(B) $T = \dfrac{\overline{X} - \mu_0}{\sigma / \sqrt{n}} \overset{\sim}{\sim} N(0,1)$
(C) $Z = \dfrac{\overline{X} - \mu_0}{\sigma / \sqrt{n}} \overset{\sim}{\sim} N(0,1)$	(D) $Z = \dfrac{\overline{X} - \mu_0}{\sigma / \sqrt{n}} \sim N(0,1)$

Question 4

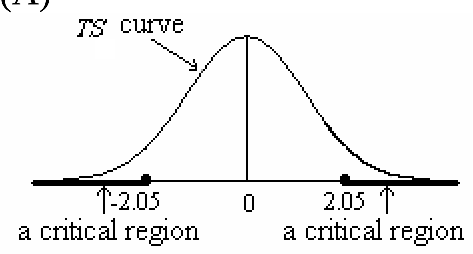
With significance level equal 0.02, the decision criterion for the hypothesis test in terms of the computed value of the test statistic (Z_c) is....

(A) Reject H_0 if Z_c < -2.05
(B) Reject H_0 if Z_c > 1.96
(C) Reject H_0 if Z_c > 2.05 or $Z_c < -2.05$
(D) Reject H_0 if Z_c <-1.645

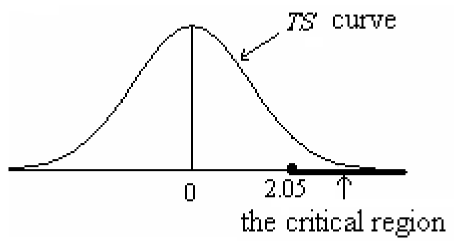
Question 5

With level of significance 2%, the critical region is best described by figure....

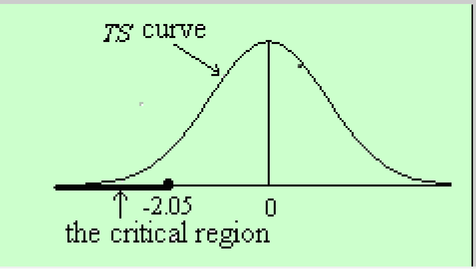
(A)



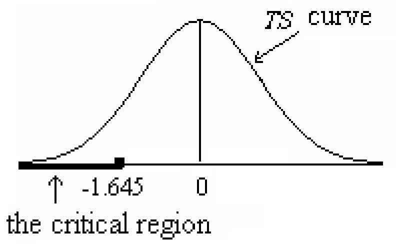
(B)



(C)



(D)



Question 6:

With significance level equal 0.02, the decision criterion for the hypothesis test in terms of the computed value of $\bar{X}$, $\bar{x}_c$ is....

(A) 4.67	(B) 3.98	(C) 22.37	(D) 21.72

$$\bar{X}_c = \mu_0 + Z_\alpha\left(\frac{\sigma}{\sqrt{n}}\right) = 25 - 2.05\left(\frac{9.6}{\sqrt{36}}\right) = 21.72$$

Question 7:
The computed value of our test statistic is....

(A) -3.5	(B) 1.24	(C) 3.5	(D) -2.36

Question 8:
The decision would be to....

(A) Cannot be determined
(B) Do not Reject the null hypothesis.
(C) Reject the null hypothesis.
(D) Reject the alternative hypothesis.

Question 9:
Suppose that in fact the traveling time is 20 minutes ($\mu_1 = 20$), then the decision has been made is...

(A) Committing Type I error	(B) Committing Type II error
(C) Correct decision$(1-\alpha)$	(D) Correct decision$(1-\beta)$

Question 10:
The p-value of this test statistic is....

(A) 0.0002	(B) 2.05	(C) 0.025	(D) 0.05

Solution:

P-value= $P(Z < z_c | H_0)$

$= P(Z < -3.5 | \mu = 25)$

$= P(Z < -3.5) = 0.5 - P(0 < Z < 3.5)$

$= 0.5 - .4998 = 0.0002$

p-value=0.0002< α =0.02

$\therefore$ Reject H_0

Question 11:

The power of our test at $\mu_1 = 20$ minutes is....

(A) 0.2224	(B) 0.8599	(C) 0.7776	(D) 0.7224

Solution:

$$\beta = P(Z > Z_c) = P\left(Z > \frac{\bar{X}_c - \mu_1}{\sigma/\sqrt{n}} \right) = P\left(Z > \frac{21.72 - 20}{9.6/6} \right) = P\left(Z > \frac{1.72}{1.6} \right)$$

$$= P(Z > 1.08) = 0.5 - \phi(1.08) = 0.5 - 0.3599 = 0.1401$$

$$power = 1 - \beta = 1 - 0.1401 = 0.8599$$

End of example 2

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Test of Hypothesis about Single Population Proportion (large Samples)

Example 3(ch10)

In the following example, the sample is large such that $n\pi > 5$ ***&*** $n(1-\pi) > 5$ ***and the hypothesis test is Right-tailed.***

It assumed from last experience that 75% of sports viewers are male. A famous sport newspaper reports that this proportion is greater than 0.75. A random sample of 400 season ticket holders reveals that 350 are male. We wish to test the above hypothesis.

Answer the following questions

Question 1:

The null and alternative hypotheses are...

(A) $H_0: \pi \geq 0.75$ & $H_1: \pi < 0.75$	(B) $H_0: \pi < 0.75$ & $H_1: \pi \geq 0.75$
(C) $H_0: \pi \leq 0.75$ & $H_1: \pi > 0.75$	(D) $H_0: \pi \neq 0.75$ & $H_1: \pi = 0.75$

Question2:

This hypothesis test is classifies as...

(A) Two-tailed	(B) Right-tailed
(C) Opposite-tailed	(D) left-tailed

Question 3:

The appropriate test statistic and its distribution under the null hypothesis is...

(A) $Z = \dfrac{P-\pi_0}{\sqrt{\pi_0(1-\pi_0)/n}} \sim N(0,1)$	(B) $T = \dfrac{\bar{X}-\mu_0}{S/\sqrt{n}} \sim t_8$
(C) $Z = \dfrac{\bar{X}-\mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$	(D) $\chi^2 = \dfrac{P-\pi_0}{\sqrt{\pi_0(1-\pi_0)/n}} \sim \chi_1^2$

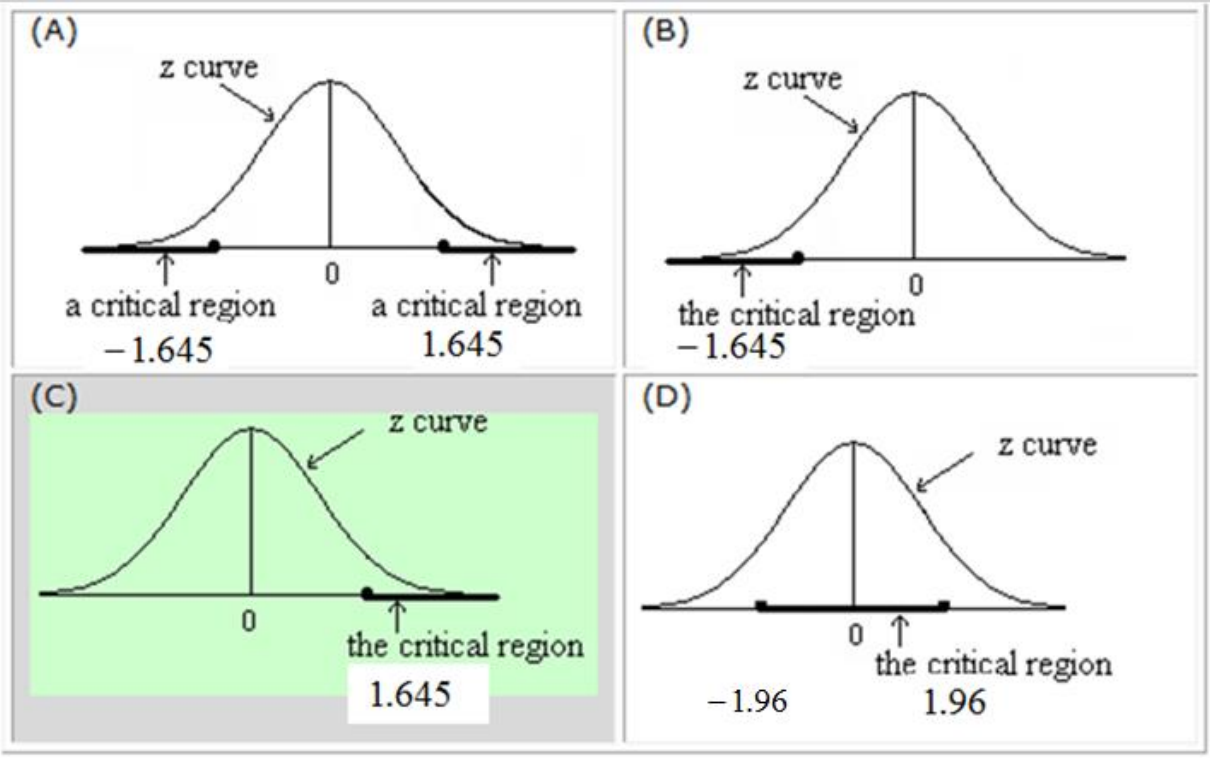
Question 4:

With significance level equal 0.05, the decision criterion for the hypothesis test in terms of the computed value of the test statistic (Z_c) is....

(A) Reject H_0 if Z_c<-1.96	(B) Reject H_0 if Z_c> 1.96
(C) Reject H_0 if Z_c> 1.96 or $Z_c < -1.96$	(D) Reject H_0 if Z_c>1.645

Question 5:

With level of significance 5%, the critical region is best described by figure....



Question 6:

With significance level equal 0.05, the decision criterion for the hypothesis test in terms of the computed value of P, p_c is....

(A) 0.708	(B) 0.978	(C) 0.786	(D) 0.6734

Solution:

We reject the null hypothesis if either z_c>1.645

$$z_c > 1.645 \Rightarrow P_c = \pi_0 + Z_\alpha \sqrt{\frac{\pi_0(1-\pi_0)}{n}} \Rightarrow P_c = 0.75 + 1.645\sqrt{\frac{0.75(0.25)}{400}} \Rightarrow P_c = 0.786$$

Question 7:

The computed value of our test statistic is....

(A) 0.01	(B) 5.99	(C) 0.23	(D) -0.01

Solution:

$$z_c = \frac{350/400 - 0.75}{\sqrt{(.75)(.25)/400}} = \frac{0.875 - 0.75}{0.0217} = \frac{0.88 - 0.75}{0.0217} = 5.99$$

Question 8:
The decision would be to....

(A) Do not Reject the null hypothesis
(B) Cannot be determined.
(C) Reject the null hypothesis.
(D Reject the alternative hypothesis.

Question 9:
Suppose that in fact the true proportion is 0.85, then the decision has been made is... α

(A) Rejecting the true hypothesis(α) type1 error	(B)Do not Rejecting the false hypothesis (β) type11 error.
(C) Do not rejecting the true hypothesis($1-\alpha$)Correct decision	(D) Rejecting the false hypothesis($1-\beta$)Correct decision

Question10:
Suppose that in fact the true proportion is 0.74, then the decision has been made is...

(A) Rejecting the true hypothesis(α) type1 error	(B)Do not Rejecting the false hypothesis (β) type11 error.
(C) Do not rejecting the true hypothesis($1-\alpha$)Correct decision	(D) Rejecting the false hypothesis($1-\beta$)Correct decision

Question11:
The power of our test at $\pi_1 = 0.85$ is....

(A) 0.0087	(B) 0.037	(C) 0.352	(D) 0.9998

Solution:

$P_c = 0.786$

$$\beta = P\left(< \frac{P_c - \pi_1}{\sqrt{\frac{\pi_1(1-\pi_1)}{n}}} \right)$$

$$\beta = P\left(< \frac{0.786 - 0.85}{\sqrt{\frac{0.85(0.15)}{400}}} \right) = P(Z < -3.58)$$

$\beta = 0.5 - \phi(3.58) = 0.5 - 0.4998 = 0.0002$

Power= $1-\beta = 1 - 0.0002 = 0.9998$

Question12: A 95% confidence interval is $0.15 \leq \pi \leq 0.87$. The null hypothesis is $H_0 : \pi = 0.96$
What is the decision?

(A) Reject the null hypothesis.
(B) Do not Reject the null hypothesis.
(C) Cannot be determined
(D) Reject the alternative hypothesis.

Question13: A 95% confidence interval is $0.15 \leq \pi \leq 0.87$. The null hypothesis is $H_0 : \pi = 0.77$
What is the decision?

(A) Reject the null hypothesis.
(B) Do not Reject the null hypothesis.
(C) Cannot be determined
(D) Do not Reject Reject the alternative hypothesis.

End of example 3

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Test of hypothesis about Tow population means
(Independent Samples)
Known variances

Example 4(ch11)
Exercise 30 page 393

A coffee manufacturer is interested in whether the **mean** daily consumption of regular-coffee drinkers is less than that of decaffeinated-coffee drinkers. Assume **the population standard deviation** for those drinking regular coffee is 1.20 cups per day and 1.36 cups per day for those drinking decaffeinated coffee. A random sample of 50 regular-coffee drinkers showed a mean of 4.35 cups per day. Another random sample (independent of the first) of 40 decaffeinated-coffee drinkers showed a mean of 5.84 cups per day.

Answer questions 1 to 15.

Remark: It is preferable to summarize the information as follows:

	Regular-coffee drinkers (1)	decaffeinated-coffee drinkers (2)
σ_i	$\sigma_1 = 1.20$	$\sigma_2 = 1.36$
n_i	$n_1 = 50$	$n_2 = 40$
$\bar{x}_i$	$\bar{x}_1 = 4.35$	$\bar{x}_2 = 5.84$

Question 1

The parameter(s) under testing is (are)...

(A) One population mean	(B) Two population means
(C) One population variance	(D) Two population proportions.

Question 2

The null hypothesis is...

(A) $\mu_1 \le \mu_2$	(B) $\mu_1 = \mu_2$
(C) $\mu_1 \ge \mu_2$	(D) $\pi_1 \ge \pi_2$

Question 3:

This hypothesis test is classifies as...

(A) Right-tailed	(B) Two-tailed	(C) Multi-tailed	(D) left-tailed

Question 4:
The appropriate test statistic and its distribution under the null hypothesis is...

(A) $Z = \dfrac{P_1 - P_2}{\sqrt{\dfrac{P_1(1-P_1)}{n_1} + \dfrac{P_2(1-P_2)}{n_2}}}$ ~ $N(0,1)$	(B) $Z = \dfrac{\bar{X}_1 - \bar{X}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}} \sim N(0,1)$
(C) $T = \dfrac{\bar{X}_1 - \bar{X}_2}{S_P\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}} \sim N(0,1)$	(D) $Z = \dfrac{\bar{X}_1 - \bar{X}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}} \sim N(0,1)$

Question 5:
With significance level equal 0.05, the decision criterion for the hypothesis test in terms of the computed value of the test statistic is....

(A) Reject H_0 if Z_c < -1.645	(B) Reject H_0 if Z_c > 1.96
(C) Reject H_0 if Z_c > 1.645 or $Z_c < -1.645$	(D) Reject H_0 if Z_c > 1.645

Question 6:
A point estimation of the difference $\mu_1 - \mu_2$ is...

(A) -0.16	(B) -10	(C) -1.49	(D) 1.49

Question 7:
The standard deviation of the statistic $\bar{X}_1 - \bar{X}_2$ is given by the formula....

(A) $\sqrt{\dfrac{P_1(1-P_1)}{n_1} + \dfrac{P_2(1-P_2)}{n_2}}$	(B) $\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}$
(C) $S_P\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}$	(D) $\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}$

Question 8:
The value of the standard deviation of the statistic $\bar{X}_1 - \bar{X}_2$ equals...

(A) 0.274	(B) 0.0628	(C) 0.058	(D) 0.241

Question 9:
The computed value of our test statistic is....

(A) -23.726	(B) 23.726	(C) -5.438	(D) 5.936

Solution:

$$z_c = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{-1.49}{0.274} = -5.438$$

Question 10:

The decision would be to....

(A) Can not be determined
(B) Do not reject the null hypothesis.
(C) Reject the null hypothesis.
(D) Reject the alternative hypothesis.

Question 11:

Suppose that in fact the mean daily consumption of regular-coffee drinkers is less than that of decaffeinated-coffee drinkers by 2.5 cups, then the decision has been made is...

(A) Committing Type I error	(B) Committing Type II error
(C) Correct decision($1-\alpha$)	(D) Correct decision($1-\beta$)

Question 12:

Suppose that in fact the mean daily consumption of regular-coffee drinkers is greater than that of decaffeinated-coffee drinkers by 2.5 cups, then the decision has been made is...

(A) Committing Type I error	(B) Committing Type II error
(C)) Correct decision($1-\alpha$)	(D) Correct decision($1-\beta$)

End of example 4