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A Rock Mechanical Model for Overbalanced, Managed Pressure, and Underbalanced Drilling Applications

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Abstract. In this work rock mechanics principles are used to elaborate a mathematical model to predict wellbore pressure required for safe overbalanced (OBD), managed pressure (MPD), and underbalanced (UBD) drilling based on laboratory evaluation of representative core samples from the formation to be drilled. The elaborated model combines the linear-poroelastic solution of stresses around circular boreholes and Mohr-Coulomb failure criterion. The model compares the induced stresses caused by the application of drilling (wellbore pressure) with the allowable induced stresses based on laboratory measurements of the formation mechanical properties (unconfined compressive strength and poison's ratio), failure criteria (apparent cohesion and angle of internal friction), in-situ principle stresses, and wellbore trajectory (vertical, directional, or horizontal). Three hypothetical formations, very weak (ISRM grade R1), weak (ISRM grade R2), and medium strong (ISRM grade R3) rocks were used to illustrate the application of the model. It was found that the safe windows for OBD and UBD drilling in vertical wells are much wider than those of the horizontal wells for all studied rocks. Also it was found that the safe drilling window width increases as the rock strength increases for both vertical and horizontal wells. Furthermore, it was found that it is extremely difficult to use UBD to drill horizontal wells parallel to the maximum principle horizontal in-situ stress in the very weak rock (R1) under the studied conditions. Therefore, the model in this paper provides a reliable tool for the prediction of optimum drilling window required for borehole stability and drilling safety.

Keywords: *underbalanced drilling; overbalanced drilling; managed pressure drilling; linear poroelastic; Mohr-Coulomb failure criterion, critical wellbore pressure.*

§1. Introduction

The most common way of drilling a well today is by overbalance pressure (OBD). This method is the way it has been used since the beginning of the developing petroleum industry. By drilling overbalanced, the bottom hole (wellbore) pressure (P_w) is kept higher than the formation pore fluid pressure (P_p) at all times while drilling the well. To keep the well overbalanced at all times requires adjustments of the mud weight during the whole drilling operation. The designed mud weight must be lower than the formation fracture pressure, but higher than the formation pore fluid pressure. The main advantages of the conventional drilling (OBD) are: it is well-known drilling technique, its safety issues are very well known, requires fewer personnel to operate, more economical, requires less rig space, provides good borehole stability, and there is no need for handling of hydrocarbons during OBD. On the other hand, the main disadvantages of the conventional drilling (OBD) are: potential damage of the formation, potential mud loss to the formation, provides low rate of penetration through harder formations, potential for differential sticking, and potential for getting a kick in case of a section with unknown pore pressure. Recently, underbalanced drilling (UBD), and managed pressure drilling (MPD) techniques are developed (see Figure 1). The size of the operational margin for UBD or OBD mainly depends on the formation strength.

Managed pressure drilling (MPD) is an adaptive drilling process to precisely control the annular pressure profile throughout the wellbore (*Philip*, 2006).

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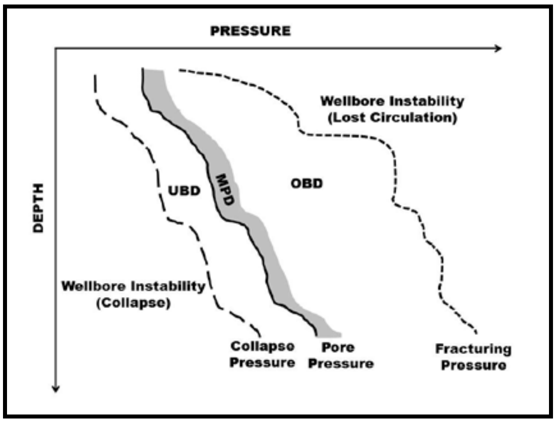

Fig. 1. Drilling windows for various types of drilling operations

MPD normally avoids the flow of the formation fluids into the wellbore and control, reduce drilling cost, and increase safety. Both UBD and MPD techniques employ a closed pressure-controlled system, making them ideal for pressure control. Managed Pressure Drilling is a drilling tool that is increasingly being recognized by operators and regulators as it is able to enhance safety, efficiency and lower cost. MPD aims to resolve long-lasting drilling problems that contribute to non-productive time such as well instability, stuck pipe, lost circulation, and well control incidents.

MPD is drilling with a controlled annulus and controlled returns to surface using an equivalent mud weight that is maintained at, or marginally above, formation pressure by manipulation of a dedicated choke device or other methods. The key point is that reservoir fluid is not intended to reach the surface. The main advantages of managed pressure drilling are: it improves rate of penetration, extends bit life, minimizes drillstring differential sticking, minimizes lost circulation, reduces the number of casing strings required to access the target, requires a simpler equipment package to satisfy safety considerations for the well, and reducing the day rate compared with UBD. In the other hand, the main disadvantages of the managed pressure drilling are: may not be capable of solving the problems encountered, such as when fracture pressure is too close to pore pressure and when variations occur in pore and fracture pressures in different intervals within the same open hole. The third possible drilling operation is the underbalanced drilling (UBD). UBD is a procedure to drill oil and gas wells where the pressure in the wellbore is kept lower than the pressure of the fluid in the formation (reservoir) being drilled. In UBD, formation fluid flows into the wellbore and up to the surface.

The major advantages of UBD are reducing formation damage in the reservoir, caused by mud solids and liquids invasion and shale swelling, maximizing hydrocarbon production, minimizing lost circulation, increasing drilling rates with certain rock types, extending bit life, reducing rock chip hold own, and minimizing the need for well stimulation. The disadvantages of the underbalanced drilling technique are as follows: well instability, well control issues, and detection of kick need to be considered when choosing to utilize underbalanced drilling, drillstring vibrations are often more pronounced, Higher drag and torque will be experienced, surface cleaning equipment must be made available and may have to accommodate a complex mixture of fluids and cuttings, aeration of drilling fluids can create a complicated hydraulic profile, compressors will need to be rented, considerably increasing the daily drilling cost, an explosion potential exists due to the formation of hydrocarbon/oxygen mixtures and the frictional sources of ignition downhole, and air in contact with steel drillstring is a corrosion risk.

§2. Objective of the Study

The objective of this work is to elaborate a mathematical model able to predict the wellbore pressure drop, balance pressure, and overbalance pressure (ΔP_w) required for safe UBD, MPD, or OBD respectively based on rock mechanics principles and the laboratory characterization of representative core samples.

§3. The Mathematical Model

Rock mechanics principles are used to solve many problems facing the oil industry such as sand production, wellbore instability, hydraulic fracturing, etc. (*Musaed*, 1998).

In this work a mathematical model based on rock mechanics principals is elaborated to predict wellbore pressure required for safe underbalanced or overbalanced drilling processes. This model combines in-situ principal stresses, well inclination, well orientation, formation strength criteria and formation physical properties. Formation rock failure criteria are evaluated using Mohr-Coulomb failure criterion which is one of the most famous and applied rock failure criteria (*Fjaer et al.*, 1992). This criterion is shown in Equation 3.1:

$$\tau_f = \tau_o + \sigma_f \, Tan\phi \tag{3.1}$$

Three principal in-situ stresses acting deep in the earth are the vertical principal in-situ stress (σ_v), the maximum horizontal principal in-situ stress (σ_H) and the minimum horizontal principal in-situ stress (σ_h).

Wellbore instability can be predicted when these principal in-situ stresses are transformed parallel to the wellbore axis (for inclined or horizontal wells as shown in Figure 2) using the matrices shown in Equations 3.2 and 3.3 (*Fjaer et al.*, 1992; *Musaed*, 1997):

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{zz} \end{bmatrix} = A \begin{bmatrix} \sigma_H \\ \sigma_h \\ \sigma_v \end{bmatrix}$$

$$A = \begin{bmatrix} Cos^2\beta \, Cos^2\alpha & Sin^2\beta \, Cos^2\alpha & Sin^2\alpha \\ Sin^2\beta & Cos^2\beta & 0 \\ Cos^2\beta \, Sin^2\alpha & Sin^2\beta \, Sin^2\alpha & Cos^2\alpha \end{bmatrix} \tag{3.2}$$

$$\begin{bmatrix} \tau_{yz} \\ \tau_{zx} \\ \tau_{yx} \end{bmatrix} = \frac{1}{2} B \begin{bmatrix} \sigma_H \\ \sigma_h \\ \sigma_v \end{bmatrix}$$

$$B = \begin{bmatrix} Sin2\beta \, Sin\alpha & -Sin2\beta \, Sin\alpha & 0 \\ Sin2\alpha \, Cos\beta & Sin^2\beta \, Sin2\alpha & -Sin2\alpha \\ Cos^2\beta \, Sin^2\alpha & -Sin2\beta \, Cos\alpha & 0 \end{bmatrix} \tag{3.3}$$

The drilling process generates an induced stresses acting on the wall of a borehole (*Jaeger*, 1979). These stresses are, the vertical induced stress (σ_z), the radial induced stress (σ_r) and the tangential induced stress (σ_θ) which can be computed using Equations set 3.4:

$$\begin{aligned} \bar{\sigma}_r &= P_w - P_p = P_{wc} \\ \bar{\sigma}_\theta &= \left(\sigma_x + \sigma_y - P_p - P_w\right) - \\ &\quad 2\left(\sigma_x - \sigma_y\right) Cos2\theta - 4\tau_{xy} \, Sin2\theta \\ \bar{\sigma}_z &= \sigma_{zz} - Pp - 2v\left(\sigma_x - \sigma_y\right) Cos\,2\theta - \\ &\quad 4v\tau_{xy} \, Sin2\theta \\ \tau_{\theta z} &= 2\left[-\tau_{zx} Sin\theta + \tau_{yz} Cos\theta\right] \end{aligned} \tag{3.4}$$

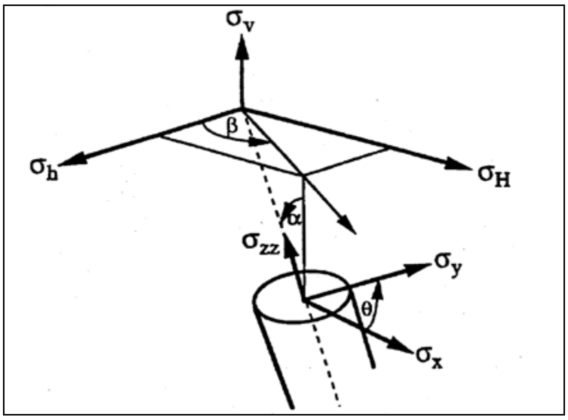

Fig. 2. Stresses transformation for a directional well.

By knowing the magnitude of the wellbore (mud) pressure, the induced principal stresses acting on the wall of a borehole can be computed as shown in Equations set 3.5 as follows:

$$\begin{aligned} \bar{\sigma}_1 &= \sigma_r = P_w - P_p = P_{wc} \\ \bar{\sigma}_2 &= \frac{1}{2}(\sigma_\theta + \sigma_z) - \frac{1}{2}\sqrt{(\sigma_\theta - \sigma_z)^2 + 4\tau_{\theta z}} \\ \bar{\sigma}_3 &= \frac{1}{2}(\sigma_\theta + \sigma_z) + \frac{1}{2}\sqrt{(\sigma_\theta - \sigma_z)^2 + 4\tau_{\theta z}} \end{aligned} \tag{3.5}$$

Finally, borehole instability using underbalanced drilling can be predicted by comparing the computed drilling induced and the experimentally measured shear stresses (i.e. failure criterion) as shown in Figure 3 and Equations 3.6 and 3.7 as follows:

$$\left(\tau_f\right)_{Criterion} = \tau_o + \left[\left(\frac{\bar{\sigma}_1 + \bar{\sigma}_3}{2}\right) + \left(\frac{\bar{\sigma}_1 - \bar{\sigma}_3}{2}\right) Cos\,2\theta\right] tan\phi \tag{3.6}$$

$$\left(\tau_f\right)_{Drilling} = \left[\frac{\bar{\sigma}_1 - \bar{\sigma}_3}{2}\right] Sin\,2\theta \tag{3.7}$$

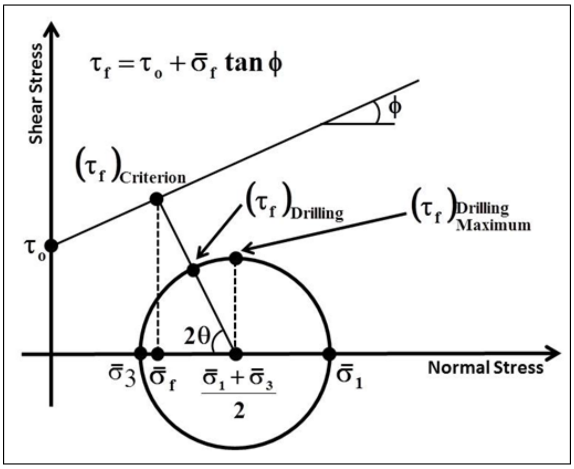

Fig. 3. Representation of drilling induced failure possibility

Wellbore fracturing pressure limit can be estimated using the following relationship (*Brady et al.*, 1985):

$$(P_W)_f = \left[\frac{2\,\tau_O\, Cos\,\phi}{1+Sin\,\phi}\right] \qquad (3.8)$$

Therefore, borehole instability (collapse) will take place if the model predicted (drilling) shear stress is equal or greater than the laboratory measured (failure criterion) shear stress. In the other hand, lost circulation (fracturing) occurs if the wellbore pressure exceeds the fracturing pressure of the formation. Table 1 lists the hypothetical data used to validate the mathematical model and to predict the wellbore pressure required for safe UBD, MPD, and OBD operations from wellbore instability prospects. These data are a modification of a real case vertical well of an oil field in china (*Qiang*, 2015). Three hypothetical formations were used, very weak (R1), weak (R2), and medium strong (R3) classified according to International Society for Rock Mechanics (*ISRM*, 1978). Full data is shown in Table 1.

Tab. 1. Hypothetical data used for model verification.

Rock Type	ISRM Grade	UCS, MPa	Poisson's Ratio	Friction Angle, (Degree)	Depth, m	Pore Pressure, MPa
Very Weak	R1	6.90	0.20	21.0	2730	51
Weak	R2	20.6	0.23	26.0	2730	51
Medium Strong	R3	35.6	0.25	31.4	2730	51
Principal In-Situ Stresses Gradients, psi/ft (MPa/m)				σ_V	1.10 (0.025)	
				σ_H	1.00 (0.023)	
				σ_h	0.93 (0.021)	
Angular Position around the Wellbore, degree				θ	Zero	
Wellbore Inclination from Vertical, degree				α	Zero or 90	
Wellbore Orientation Angle from, degree				β	Zero or 90	

§4. Results and Discussion

The developed model was used to predict the safe wellbore pressure required for safe underbalanced drilling, managed pressure drilling, and overbalanced drilling in vertical and horizontal wells in three types of formations (rock grades) (*ISRM*, 1978) namely, very weak (R1), weak (R2), and medium strong (R3) rocks as shown in Table 1.

Figures 4 - 6 are the predictions for safe wellbore pressure windows for hypothetical vertical wells.

For the very weak formation (R1), it was found that there are three safe possibilities to utilize OBD, MPD, and UBD with minimum wellbore pressure (collapse) and maximum wellbore pressure (lost circulation) limits of -19.4 MPa and +3.2 MPa respectively as shown in Figure 4.

For the weak formation (R2), stable vertical wells can be drilled using UBD, MPD, or OBD. The minimum wellbore pressure (collapse) and maximum wellbore pressure (lost circulation) limits for this case are -29.2 MPa and +8.1 MPa respectively as shown in Figure 5.

For the medium strong formation (R3), stable vertical wells can be drilled using UBD, MPD, or OBD. The minimum wellbore pressure (collapse) and maximum wellbore pressure (lost circulation) limits for this case are -39.4 MPa and +11.2 MPa respectively as shown in Figure 6.

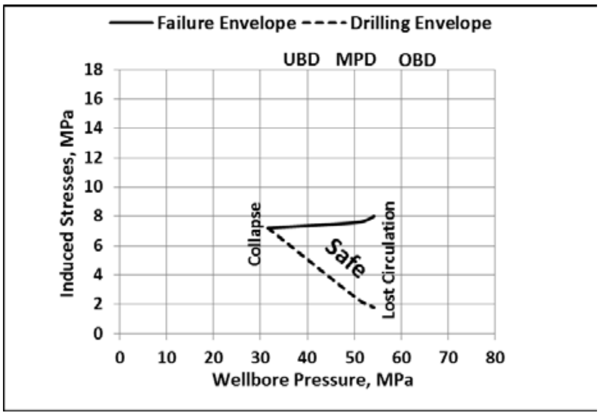

Fig. 4. Vertical well in very weak formation (R1).

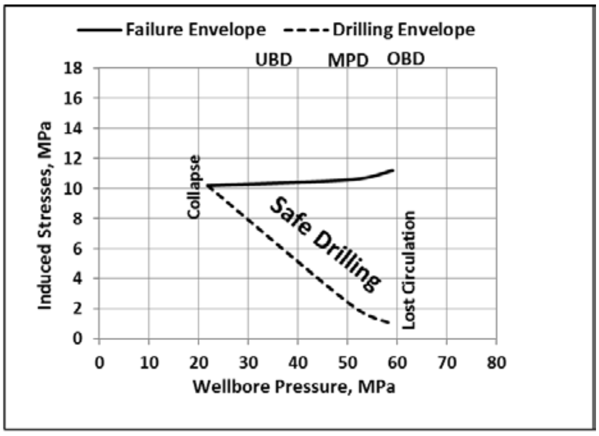

Fig. 5. Vertical well in weak formation (R2).

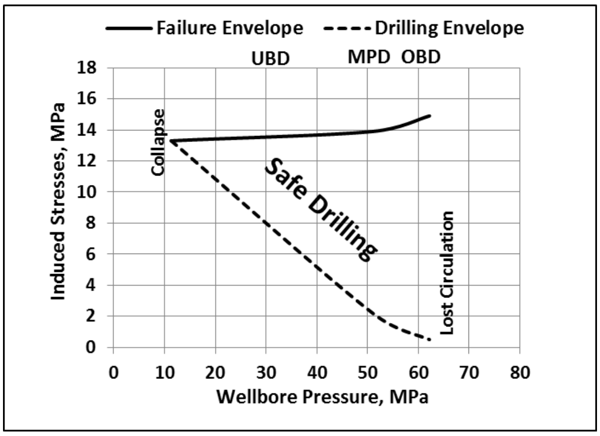

Fig. 6. Vertical well in medium strong formation (R3).

It is clear that the safe drilling window was getting wider as the formation to be drilled is getting stronger as shown in Figure 7. Table 2 summarizes all the studied cases.

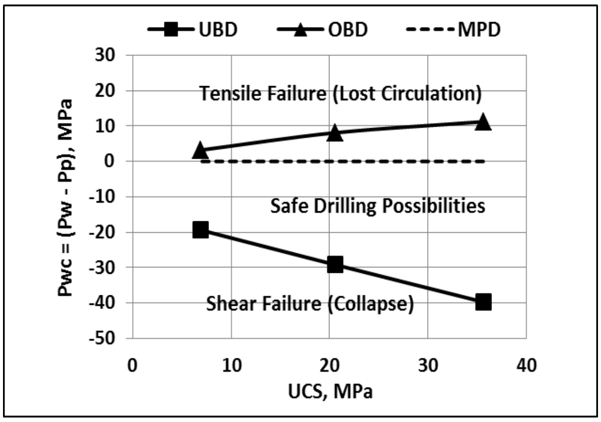

Fig. 7. Summary of the studied vertical drilling cases.

For horizontal well drilling possibilities, it was found that it is unsafe to utilize UBD or MPD in the very weak (R1) formation and safe for both the weak (R1) and medium strong (R3) formations. The safe wellbore pressure for horizontal wells drilled parallel to the minimum horizontal principal in-situ stress in the medium strong (R3) formation is ranging between -38.2 MPa and +11.2 as shown in Figure 8 and Table 3. For horizontal wells drilled parallel to the maximum horizontal principal in-situ stress in the medium strong formation (R3), the safe wellbore pressure range was -33.3 MPa to +11.2 MPa as shown in Figure 9 and Table 3.

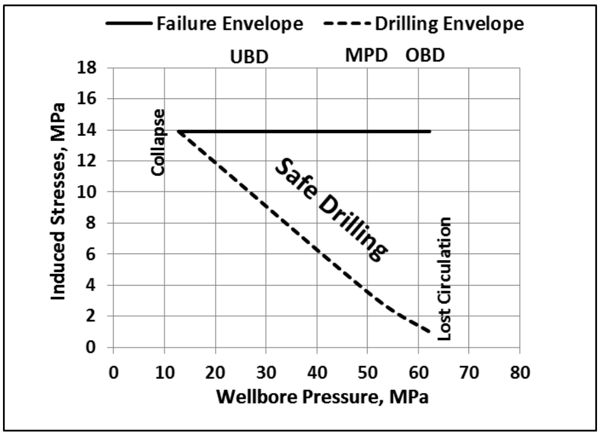

Fig. 8. Horizontal well (//σ_H) in medium strong formation (R3).

Data tabulated in Tables 2 - 5 are plotted in Figure 10. It can be noticed that the safe drilling window decreases as the formation strength decreases.

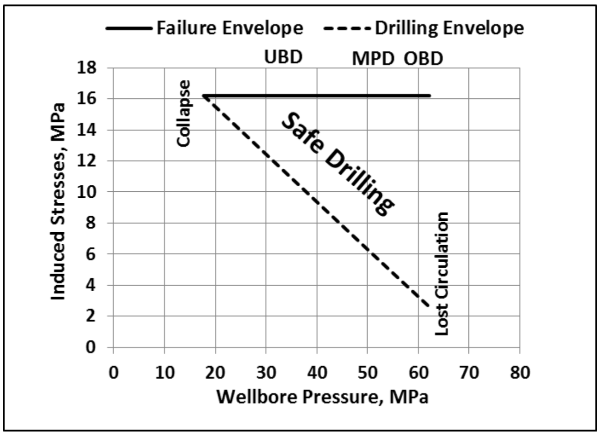

Fig. 9. Horizontal well (//σ_h) in medium strong formation (R3).

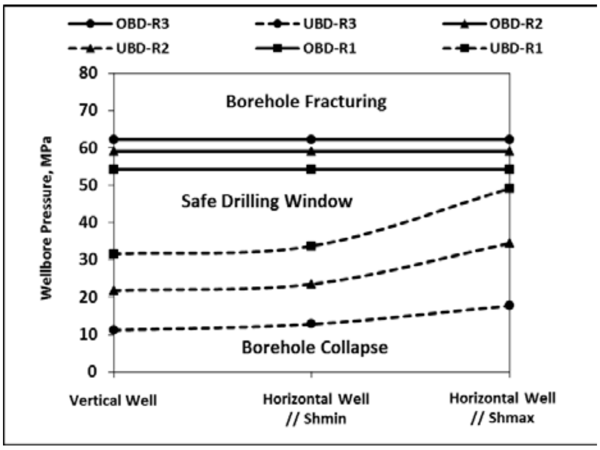

Fig. 10. Safe drilling in three different horizontal well orientations and formations strengths.

Furthermore, the most decease in drilling window was for the horizontal well drilled parallel to the maximum principal horizontal in-situ stress. This difference is attributed to the difference in shear stresses at each orientation which is equal to 0.10 psi/ft (0.02 MPa/m) for horizontal wells drilled parallel to the maximum horizontal principal in-situ stress and 0.17 psi/ft (0.04 MPa/m) for horizontal wells drilled parallel to the minimum horizontal principal in-situ stress.

§5. Conclusions

Based on the analysis of the output data obtained using the mathematical model and the hypothetical formation data presented in this study, the following conclusions are attained:

1. The decision of utilizing UBD, MPD, or OBD process is highly dependent on the formation strength, formation pore fluid pressure, and the principal in-situ stresses acting on the area under consideration.
2. In underbalanced drilling, the window of safe wellbore pressure for vertical wells is much wider than the window of the horizontal wells under the same conditions.
3. In overbalanced drilling, the same window was applicable in both vertical and horizontal wells under the same conditions.
4. In underbalanced drilling, the order of stability decrease (based on well configurations) are vertical wells, horizontal wells drilled parallel to the minimum horizontal principal in-situ stress, and horizontal wells drilled parallel to the maximum horizontal principal in-situ stress accordingly.
5. This study provides a new simple and reliable model for the prediction of optimum drilling mud window required for safe OBD, MPD, and UBD processes to be performed in various types of formations (rocks).
6. The model presented in this study can be improved if the drilling mud cooling effect (thermal stresses) and mud physico-chemical effect (swelling stresses) are incorporated.

Tab. 2. Summary of the results of drilling vertical hypothetical wells in R1, R2 and R3 formations.

Formation Type	Well Type	UCS, MPa	Poisson's Ratio	Friction Angle, Degree	Safe Drilling Window, MPa					
					OBD		MPD		UBD	
					P_{wc}= (P_w-P_p)	P_w	P_{wc}= (P_w-P_p)	P_w	P_{wc}= (P_w-P_p)	P_w
Very Weak (R1)	Vertical	6.9	0.20	21	+3.2	54.2	0	51	-19.4	31.6
Weak (R2)	Vertical	20.6	0.23	26	+8.1	59.1	0	51	-29.2	21.8
Med. Strong (R3)	Vertical	35.6	0.25	31.4	+11.2	62.2	0	51	-39.9	11.1

Tab. 3. Summary of the results of drilling vertical and horizontal hypothetical wells in R1 formation.

Formation Type	**Well Type**	**UCS, MPa**	**Poisson's Ratio**	**Friction Angle, Degree**	**Safe Drilling Window, MPa**					
					OBD		**MPD**		**UBD**	
					P_{wc}= (P_w-P_p)	**P_w**	**P_{wc}= (P_w-P_p)**	**P_w**	**P_{wc}= (P_w-P_p)**	**P_w**
Very Weak (R1)	Vertical	6.9	0.20	21	+3.2	54.2	0	51	-19.4	31.6
	Horizontal// σ_h	6.9	0.20	21	+3.2	54.2	0	51	-17.3	33.7
	Horizontal// σ_H	6.9	0.20	21	+3.2	54.2	0	51	-1.9	49.1

Tab. 4. Summary of the results of drilling vertical and horizontal hypothetical wells in R2 formation.

Formation Type	**Well Type**	**UCS, MPa**	**Poisson's Ratio**	**Friction Angle, Degree**	**Safe Drilling Window, MPa**					
					OBD		**MPD**		**UBD**	
					P_{wc} = (P_w-P_p)	**P_w**	**P_{wc} = (P_w-P_p)**	**P_w**	**P_{wc} = (P_w-P_p)**	**P_w**
Weak (R2)	Vertical	20.6	0.23	26	+8.1	59.1	0	51	-29.2	21.8
	Horizontal// σ_h	20.6	0.23	26	+8.1	59.1	0	51	-27.5	23.5
	Horizontal// σ_H	20.6	0.23	26	+8.1	59.1	0	51	-16.6	34.4

Tab. 5. Summary of the results of drilling vertical and horizontal hypothetical wells in R3 formation.

Formation Type	**Well Type**	**UCS, MPa**	**Poisson's Ratio**	**Friction Angle, Degree**	**Safe Drilling Window, MPa**					
					OBD		**MPD**		**UBD**	
					P_{wc} = (P_w-P_p)	**P_w**	**P_{wc} = (P_w-P_p)**	**P_w**	**P_{wc} = (P_w-P_p)**	**P_w**
Medium Strong (R3)	Vertical	35.6	0.25	31.4	+11.2	62.2	0	51	-39.9	11.2
	Horizontal// σ_h	35.6	0.25	31.4	+11.2	62.2	0	51	-38.2	12.8
	Horizontal// σ_H	35.6	0.25	31.4	+11.2	62.2	0	51	-33.3	17.7

List of symbols

A	Normal stresses transformation matrix
B	Shear stresses transformation matrix
MPD	Managed pressure drilling
OBD	Overbalanced drilling
P_p	Pore fluid pressure, in MPa
P_w	Wellbore pressure, in MPa
P_{wc}	Critical wellbore pressure, in MPa
$(P_w)_f$	Formation fracturing pressure, in MPa
Shmax	Min. in-situ horizontal principal stress, in MPa
SHmax	Max. in-situ horizontal principal stress, in MPa
R1	Very weak rock
R2	Weak rock
R3	Medium strength rock
UBD	Underbalanced drilling
UCS	Unconfined compressive strength, in MPa
α	Vertical inclination angle, in degree
β	Horizontal orientation angle, in Degree
θ	Angular position around the well, in Degree
ΔP_w	Wellbore pressure drop, MPa
ϕ	Friction angle, degree
σ_f	Normal stress at failure, in MPa
σ_h	Min. in-situ horizontal principal stress, in MPa
σ_H	Max. in-situ horizontal principal stress, in MPa
σ_v	Vertical in-situ principal stress, in MPa
σ_o	Unconfined compressive strength, MPa
σ_r	Drilling induced radial stress, MPa
σ_x	Transformed horizontal stress, in MPa
σ_y	Transformed horizontal stress, in MPa
σ_z	Transformed vertical stress, in MPa
σ_{zz}	Drilling induced vertical stress, in MPa
σ_θ	Drilling induced tangential stress, in MPa
$\bar{\sigma}_1$	Effective maximum drilling induced stress, in MPa
$\bar{\sigma}_3$	Effective minimum drilling induced stress, in MPa
τ_f	Shear stress at failure, in MPa
τ_o	Apparent cohesion, in MPA

τ_{yz}	Shear stress at yz plan, in MPa
τ_{zx}	Shear stress at zx plan, in MPa
$\tau_{\theta z}$	Drilling induced tangential stress, in MPa
$(\tau_f)_{Criterion}$	Mohr-Coulomb shear failure, in MPa
$(\tau_f)_{Drilling}$	Drilling induced shear failure, in MPa

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