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Improper Integral Residue Theorem

Asked 1 year, 3 months ago Active 1 year, 3 months ago Viewed 192 times



I'm stuck on a question involving evaluating improper integrals using the residue theorem. Here's what I'm trying to evaluate:

$$0 \qquad \int_{-\infty}^{\infty} \frac{1}{(1+x^2)^3} dx$$



To begin with, we can define a contour integral over the the semi-circle disc above the Real axis called C which can be broken up in two parts involving the Real number line, and the curve of the semi-circular disc

 $\int_{C} rac{1}{(1+z^{2})^{3}} dz = \int_{-R}^{R} rac{1}{(1+x^{2})^{3}} dx + \int_{Cr} rac{1}{(1+z^{2})^{3}} dz$

We can see that the left-hand side of the equation has poles at i and -i with multiplicity 3 respectively. Using the Residue Theorem

$$\int_C rac{1}{(1+z^2)^3} dz = 2\pi i (3rac{1}{((i)+i)^3})$$

Because there are 3 instances of the i being the pole inside the region defined. Also:

$$\int_{Cr} rac{1}{\left(1+z^2
ight)^3} dz = 0$$

Because parameterizing $z=Re^{it}$ and setting the limit as R goes to infinity will show the integrand to be zero.

That leaves us with the final answer of

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complex analysis - Improper Integral Residue Theorem - Mathematics Stack Exchange The correct answer is $\frac{1}{8}$ and Tm not quite sure now that is formed.

Any help would be appreciated. Thanks.

complex-analysis

indefinite-integrals

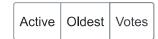
residue-calculus

asked Dec 13 '18 at 17:01



HoopsMcCann

2 Answers





The residue of $\frac{1}{(1+z^2)^3}$ at i is $-\frac{3i}{16}$ and therefore your integral is equal to

1



$$2\pi i \times \frac{-3i}{16} = \frac{3\pi}{8}$$



In order to compute that residue, you can use the fact that

$$\frac{1}{(1+z^2)^3} = \frac{\frac{1}{(z+i)^3}}{(z-i)^3}$$

and that therefore the residue that we are interested in is $\frac{\varphi''(i)}{2}$, where $\varphi(z)=\frac{1}{(z+i)^3}$.

And, as I wrote, $\frac{\varphi''(i)}{2} = -\frac{3i}{16}$.

edited Dec 13 '18 at 17:13

answered Dec 13 '18 at 17:06



José Carlos Santos

Why can't we find the residue by multiplying the original function by $(z-1)^3$, isn't that how you find a residue usually? - HoopsMcCann Dec 13 '18 at 17:39

Why $(z-1)^3$? The number 1 has nothing to do with this problem. It should be $(z-i)^3$, and that is exactly what I did. - José Carlos Santos Dec 13 '18 at 17:51

Typo on my end. So the idea is, if we have a higher order pole, we differentiate it until the order of it is 1, and divide by how many times we differentiated factorial? - HoopsMcCann Dec 13 '18 at 17:56

I would not put it like that, but that's the right idea. – José Carlos Santos Dec 13 '18 at 18:08



0

By the Cauchy integral formula

if a is inside the contour, and f(z) is holomorphic inside the contour.

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anu

$$\int_{\gamma} rac{f(z)}{(z-a)^n} dz = 2\pi i rac{1}{(n-1)!} f^{(n-1)}(a)$$

in this case let a=i

and
$$f(z) = \frac{1}{(z+i)^3}$$

answered Dec 13 '18 at 17:18

