Index Numbers

Chapter 15

Learning Objectives

- Compute and interpret a simple index.
- Descried the difference between a weighted and an unweighted index.
- Compute and interrupt a Laspeyres price index.
- Compute and interrupt a Paasche price index.
- Compute and interrupt a value index.

What is an index number?

- An index or index number measures the change in a particular item (typically a product or service) between two time periods.
- An index number is a number that expresses the relative change in price, quantity, or value compared to a base period.
- An index can also compare one item with another.

Example 1

According to the Bureau of Labor Statistics, in 2000 the average hourly earnings of production workers was $14.02. In 2009, it was $18.62. What is the index of hourly earnings of production workers for 2009 based on 2000 data?

$$P = \frac{\text{average hourly earnings } in\ 2009}{\text{average hourly earnings } in\ 2000}(100) = \frac{\$18.62}{\$14.02}(100) = 132.81$$

Thus, the hourly earnings in 2009 compared to 2000 were 132.81 percent. This means there was a 32.81 percent increases in hourly earnings during the period.

Example 2

The population of the Saudi administrative area of Northern Borders in 2004 was 279,971, and for Al-Riyadh it was 5,458,273. what is the population index of Northern Borders compared to Al-Riyadh?

The index of population for Northern Border is 5.1, found by:

$$P = \frac{\textit{Population of}\ \text{Northern Borders}}{\textit{Population of}\ \text{Al-Riyadh}}(100) = \frac{279{,}971}{5{,}458{,}273}(100) = 5.1$$

This indicates that the population of Northern Border is 5.1 percent of the population of Al-Riyadh, or the population of Northern Border is 94.9 percent less than the population of Al-Riyadh $(100 - 5.1 = 94.9)$.

Example 3

The following table shows the number of passengers (in millions) for the five busiest airport in the United States in 2009. What is the index for Atlanta (ATL), Chicago(ORD), Los Angeles (LAX), and Dallas/Ft (DFW). worth compared to Denver (DEN).

Airport	Number of Passengers (millions)
ATL	90.0
ORD	69.3
LAX	59.5
DFW	57.0
DEN	51.4

Notes

- The index of the average hourly earnings of production workers (132.81) is a percentage, but the percent symbol is usually omitted.
- Each index has a **based period**. In the example regarding the average hourly earnings of production workers, we used 2000 as the base period.
- Most business and economic indexes are computed to the nearest whole number, such as 214 or 94, or the nearest tenth of a percent such as 83.4 or 118.7.

Why Convert Data to Indexes

- Indexes allow us to express a change in price quantity, or value as percent.
- An index is a convenient way to express a change in a diverse group of items.
- Converting data to indexes also makes it easier to assess the trend in a series composed of exceptionally large numbers.

Constriction of Index numbers

- We already discussed the construction of simple price index.
- The Simple price index formula using 100 as the base value for any given period:

$$P = \frac{p_t}{p_0}(100)$$

P: is the simple price index.

p_0: the base period price.

p_t: selected period price.

Example 4

Suppose the price of a fall weekend package (including loading and all meals) at Tryon Mountain Lodge in western North Carolina in 2000 was \$450. The price rose to \$795 in 2010. What is the price index for 2010 using 2000 as the base period and 100 as the base value?

$$P = \frac{p_t}{p_0}(100) = \frac{\$795}{\$450}(100) = 176.7$$

Interpreting this result, the price of the fall weekend package increased 76.7 percent from 2000 to 2010.

Example 5

Listed below are the top steel-producing regions for the year 2009. Express the amount produced by China, the European Union, Japan, and Russia as an index, using the United States as a base. What percent more steel does China produce than the United States?

Region	Amount (millions of tons)
People's Republic of China	500.5
European Union	198.0
Japan	118.7
United States	91.4
Russia	68.5

Example 6

The average hourly earnings of production workers for January of selected years are given below.

Year	Average Hourly Earnings
1995	\$11.65
2000	14.02
2005	16.13
2010 (May)	19.01

(a) Using 1995 as the base period and 100 as the base value, determine the indexes for the other years. Interpret the index.

Unweighted Indexes

- In many situation, we wish to combine several items and develop an index to compare the cost of this aggregation of items in two different time periods. For Example, we might be interested in an index for items that relate to expense of operating and maintaining an automobile. The items in the index might include tires, oil changes, and gasoline prices. There are several way we can combine the items to determine the index.

Example 7

- The following table show the prices for several food items for the years 1999 and 2009. We would like to develop an index for this group of food items for 2009 using 1999 as the base. This is written in the abbreviation code 1999= 100.

TABLE 15–2 Computation of Index for Food Price 2009, 1999 = 100

Item	1999 Price	2009 Price
Bread, white, cost per pound	$ 0.87	$ 1.28
Eggs, dozen	1.05	2.17
Milk, gallon, white	2.94	3.87
Apples, Red Delicious, 1 pound	0.86	1.16
Orange Juice, 12 oz concentrate	1.75	2.54
Coffee, 100% ground roast, 1 pound	3.43	3.68
Total	$10.90	$14.70

Simple Average of the Price Indexes

- Simple average of the price relatives:

$$P = \frac{\sum P_i}{n}$$

P_i: the simple index for each item.

n: the number of items.

Cont. Example 7

- Applying the simple average of price relatives

$$P = \frac{\sum P_i}{n} = \frac{147.1 + \cdots + 107.3}{6} = 145.5$$

This indicates that the mean of the group of indexes increased 45.5 percent from 1999 to 2009.

Simple Average of the Price Indexes

- A positive feature of simple average of price indexes is that we would obtain the same value for the index regardless of the unites of measure.
- A negative feature of this index is that it fails to consider the relative importance of the items included in the index.

Simple Aggregate Index

- A second possibility is to sum the prices (rather than the indexes) for the two periods and then determine the index based on the totals. The formula is:

$$P = \frac{\sum p_t}{\sum p_0}(100)$$

P: simple aggregate index

p_t: the price for the selected period

p_0: the price for the base period

Cont. Example 7

- Applying the simple aggregate index:

$$P = \frac{1.28 + \cdots + 3.68}{0.87 + \cdots + 3.43}(100) = \frac{14.70}{10.90}(100) = 134.9$$

This means that the aggregate group of prices had increased 34.9 percent in the 10-year period.

Simple Aggregate Index

- Because the value of a simple aggregate index can be influenced by the units of measurement, it is not used frequently.
- Also it fails to consider the relative importance.

Weighted Indexes

- Tow methods of computing a weighted price index are the **Laspeyres** method and the **Passche** method. They differ only in the period used for weighting. The Laspeyres uses base-period weights. The Passche uses current-year weights.

Laspeyres Price index

$$P = \frac{\sum p_t q_0}{\sum p_0 q_0}(100)$$

P is the price index

p_t is the current price

p_0 is the price in the base period

q_0 is the quantity used in the base period

Example 8

The prices for the six food items from Table 15–2 are repeated below in Table 15–3. Also included is the number of units of each consumed by a typical family in 1999 and 2009.

TABLE 15–3 Price and Quantity of Food Items in 1999 and 2009

Item	1999 Price	1999 Quantity	2009 Price	2009 Quantity
Bread, white, cost per pound	$0.87	50	$1.28	55
Eggs, dozen	1.05	26	2.17	20
Milk, gallon, white	2.94	102	3.87	130
Apples, Red Delicious, 1 pound	0.86	30	1.16	40
Orange Juice, 12 oz concentrate	1.75	40	2.54	41
Coffee, 100% ground roast, 1 pound	3.43	12	3.68	12

Determine a weighted price index using the Laspeyres method. Interpret the result.

Example 8

The weighted price index for 2009 is 137.0, found by

$$P = \frac{\Sigma p_t q_0}{\Sigma p_0 q_0}(100) = \frac{\$695.72}{\$507.64}(100) = 137.0$$

Based on this analysis, we conclude that the price of this group of items has increased 37.0 percent in the ten-year period. The advantage of this method over the simple aggregate index is that the weight of each of the items is considered. In the simple aggregate index, coffee had about 40 percent of the weight in determining the index. In the Laspeyres index, the item with the most weight is milk, because the product of the price and the units sold is the largest.

Paasche Price Index

$$P = \frac{\sum p_t q_t}{\sum p_0 q_t}(100)$$

P is the price index

p_t is the current price

p_0 is the price in the base period

q_t is the quantity used in the current period

Example 9

Use the information from example 8 to determine the Passche index and interpret the result.

The Paasche index is 135.6, found by

$$P = \frac{\Sigma p_t q_t}{\Sigma p_0 q_t}(100) = \frac{\$811.60}{\$598.36}(100) = 135.6$$

This result indicates that there has been an increase of 35.6 percent in the price of this "market basket" of goods between 1999 and 2009. That is, it costs 35.6 percent more to purchase these items in 2009 than it did in 1999. The Laspeyres index is more widely used because there are fewer pieces of data to update each period. The Consumer Price Index, the most widely reported index, is an example of a Laspeyres index.

Laspeyres versus Paasche Index

- **Laspeyres**
  - Advantages Requires quantity data from only the base period. This allows a more meaningful comparison over time. The changes in the index can be attributed to changes in the price.
  - Disadvantages Does not reflect changes in buying patterns over time. Also, it may overweight goods whose prices increase.
- **Paasche**
  - Advantages Because it uses quantities from the current period, it reflects current buying habits.
  - Disadvantages It requires quantity data for the current year. Because different quantities are used each year, it is impossible to attribute changes in the index to changes in price alone. It tends to overweight the goods whose prices have declined. It requires the prices to be recomputed each year.

Fisher's Ideal Index

- Laspeyres' index tends to overweight goods whose prices have increased. Paasche's index, on the other hand, tends to overweight goods whose prices have gone down.
- **Fisher's ideal index** was developed in an attempt to offset these shortcomings.
- It is the geometric mean of the Laspeyres and Paasche indexes.

$$\text{Fisher's ideal index} = \sqrt{(\text{Laspeyres' index})(\text{Paasche's index})}$$

Example 10

Determine Fisher's ideal index from the data in example 8.

Fisher's ideal index is 136.3.

$$\text{Fisher's ideal index} = \sqrt{(\text{Laspeyres' index})(\text{Paasche's index})}$$
$$= \sqrt{(137.0)(135.6)} = 136.3$$

Value Index

- A **value index** measures changes in both the price and quantities involved.
- A value index, such as the index of department store sales, needs the original base-year prices, the original base-year quantities, the present-year prices, and the present year quantities for its construction.
- Its formula is:

$$V = \frac{\sum p_t q_t}{\sum p_0 q_0}(100)$$

Example 11

The number of items produced by Houghton Products for 1996 and 2009 and the wholesale prices for the two periods are:

	Price		Number Produced	
Item Produced	1996	2009	1996	2009
Shear pins (box)	$ 3	$4	10,000	9,000
Cutting compound (pound)	1	5	600	200
Tie rods (each)	10	8	3,000	5,000

(a) Find the value index of production for 2009 using 1996 as the base period.
(b) Interpret the index.

Example 11

a. $$P = \frac{\$4(9{,}000) + \$5(200) + \$8(5{,}000)}{\$3(10{,}000) + \$1(600) + \$10(3{,}000)}(100)$$
$$= \frac{\$77{,}000}{60{,}600}(100) = 127.1$$

b. The value of sales has gone up 27.1 percent from 1996 to 2009.