Natural Exponential Function
Math 106
Lecture 8

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$$\ln : (0, \infty) \to \mathbb{R}, \ln x = \int_1^x \frac{1}{t} dt$$

where $f(t) = \frac{1}{t} dt$ is continuous function on any interval not contains 0. So, it is integrable on interval from 1 to x.

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$$y = \exp x \Leftrightarrow \ln y = x$$

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- The function exp is incrasing.
- Since ln and exp are inverse function. So,

$$\ln e^x = x, \quad \forall x \in \mathbb{R},$$

$$e^{\ln x} = x, \quad \forall x \in (0, \infty).$$

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- $\lim_{x\to-\infty} \exp(x) = 0$.

ln x, exp x

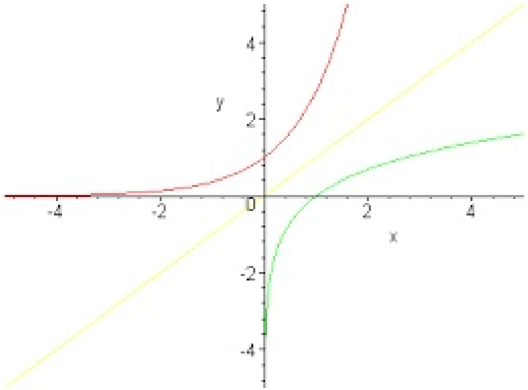

Figure: $\ln(x), \exp(x)$.

Derivative of Natural Exponential Function :

$$\frac{d}{dx}(e^x) = e^x \quad \forall x \in \mathbb{R},$$

$$\frac{d}{dx}(e^{f(x)}) = e^{f(x)} f'(x) \quad \forall x \in D(f').$$

EX: Find the derivative:

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$$(3)\sin x\exp\left(\cos x\right)\exp\left(x\right).$$

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- $\int e^{ax} dx = \frac{1}{a} e^{ax} + c, \quad \forall a \neq 0$

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$$(5) \int e^{3x} \sin(1 + e^{3x}) dx.$$

Thm:

$$\lim_{n\to\infty}(1+\frac{x}{n})^n = e^x, \quad \forall x \in \mathbb{R}.$$

Thanks for listening.