Natural Logarithmic Function
Math 106
Lecture 7

Dr. Nasser Bin Turki

King Saud University
Department of Mathematics

2015

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$$\int t^r \, dt = \frac{t^{r+1}}{r+1} + c,$$

where $r - 1$
However, this is not true in the case $r = -1$ becuse we will get 0 in the denominator. So, in this chapter we will look at this problm.

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$$\ln : (0, \infty) \to \mathbb{R}, \ln x = \int_1^x \frac{1}{t} dt$$

where $f(t) = \frac{1}{t} dt$ is continuous function on any interval not contains 0. So, it is integrable on interval from 1 to x.

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- The function ln is incrasing.

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- $\lim_{x\to 0^+} \ln x = -\infty$.

$\ln x$

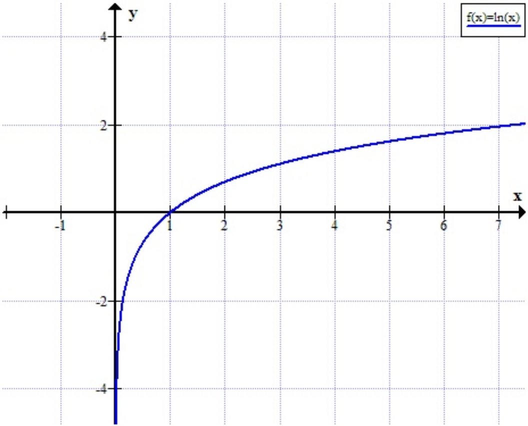

Figure: $\ln x$.

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$$\frac{d}{dx}\ln g(x) = \frac{g'(x)}{g(x)}.$$

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$$y = \frac{\sqrt[5]{x^2}\sin^4 x}{x^3\sqrt{2x}}.$$

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- $\int \csc x dx = \ln |\csc x - \cot x| + c$.

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$$(2) \int \frac{1}{x - x\ln x}dx.$$

Thanks for listening.