Chapter 2

Motion in One Dimension

Lecture Content

- General problem solving strategy
- Position, velocity, and speed (Average and instantaneous)
- Acceleration
- Motion diagrams
- Constant acceleration motion

Problem solving strategy

Conceptualise

Categorise

Analyse

Finalise

Problem solving strategy (1)
Conceptualise

- Think about and understand the situation (Diagrams, figures, tables)
- Construct a movie in your mind of what is happening
- Make quick sketch to the problem
- Focus on numerical and algebraic information
- Look for key phrases (stops, start from rest, free falling)
- Focus on the expected result of solving the problem (What is exactly the question is asking for?)
- Don't forget to incorporate information from your own experience and common sense (can the speed of an ordinary car reach 1000 km/hr?)

Problem solving strategy (2) Catagorise

- After you understand the problem, simplify the problem. By removing non important details to the solution.
- Categorise the problem, is it simple plug-in problem? Or you need to think and analyse more deeply.
- Have you seen this problem before? Do u solve similar problem before?

Problem solving strategy (3)
Analyse

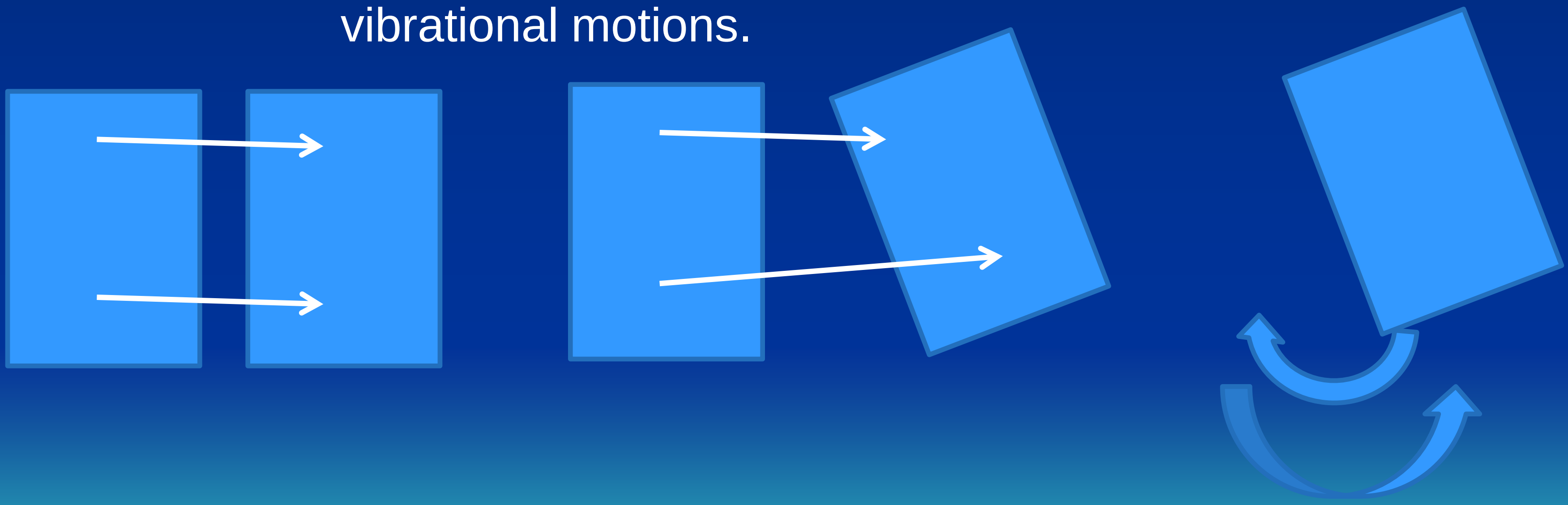
- Select relevant equations to solve the problem
- Use algebra and Calculus to solve for the unknown parameters
- Calculate the result and round it to the appropriate significant figures.

Problem solving strategy (4) Finalise

- This is the most important part.
- Examine the numerical result, Does it have the correct unit?
- Does it meet you expectations of you conceptualisation in stage 1
- Does it make sense?
- Think about how this problem compares with the other you already solved before.
- Is it new problem you didn't solve before? Make it as a model for next problems

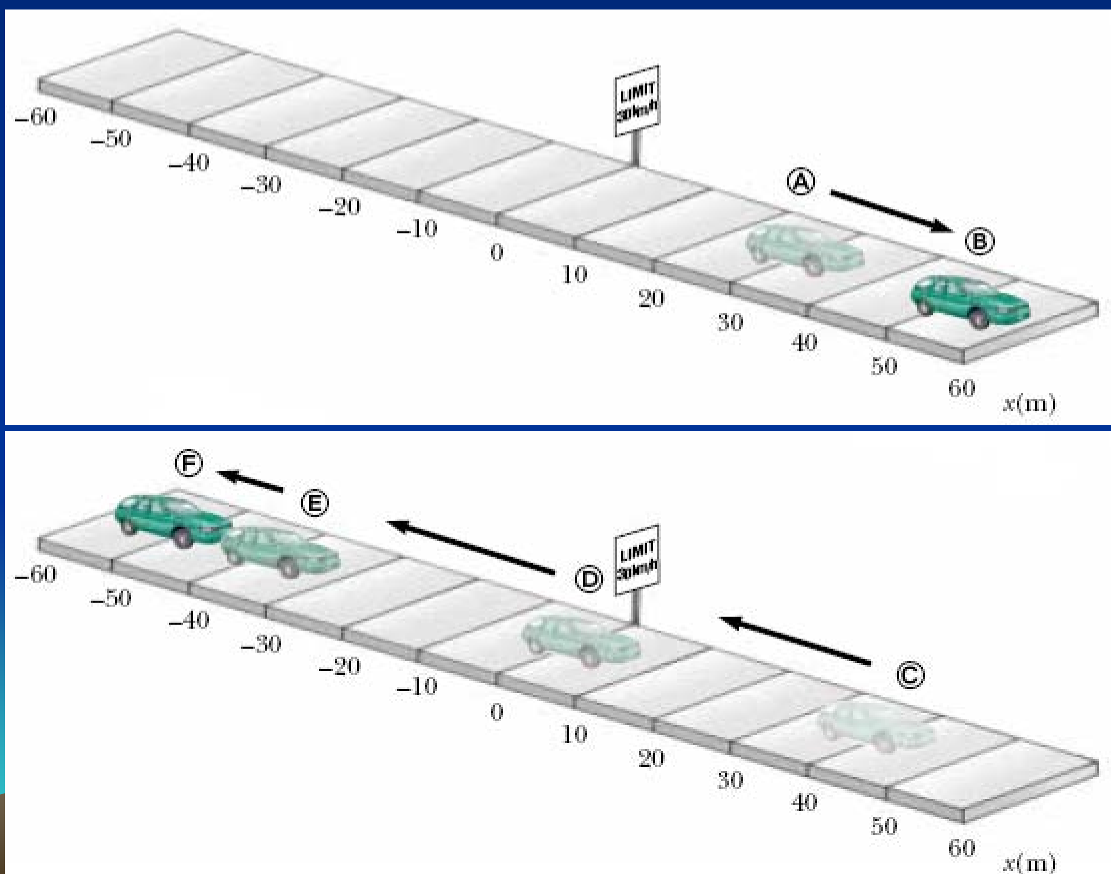
Motion

- Study the kinematics (motion in space and time) of particles (having mass and negligible size) without studying the cause of motion.
- Motion types: translational, rotational, and vibrational motions.



Position

- If particle position in space is know at all time, the motion can be evaluated.
- Position is the location of particle in space at certain time and usually with respect to a chosen reference.



Position of the Car at Various Times

Position	t(s)	x(m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53

Position – time graph

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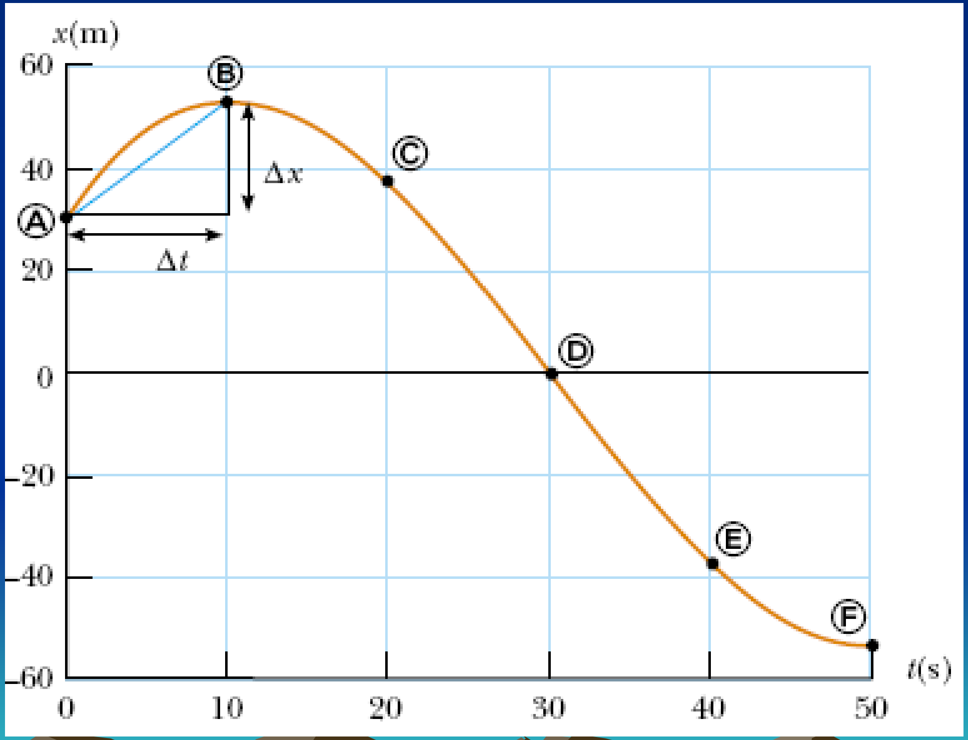

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Displacement

- Displacement is defined as the change (Δ) in particle position during time interval.
- Initial position is x_i and final position is x_f , displacement = $\Delta x = x_f - x_i$ (can be positive or negative)
- Distance is the length of the path travelled (always positive).
- Distance is not the same as the displacement.
- Displacement is a vector, while distance is scalar.
- Examples

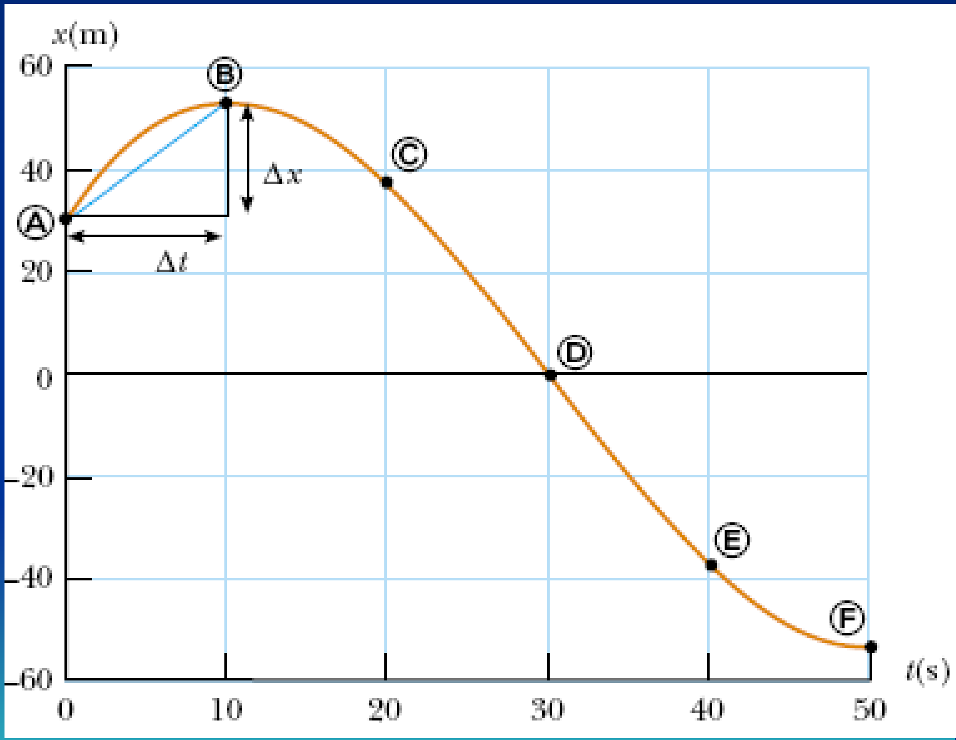
Velocity

- Average velocity is defined as ratio of the displacement (*Δx*) of a particle in a time interval (*Δt*).

$$\bar{v}_x \equiv \frac{\Delta x}{\Delta t}$$

- The average velocity can be positive or negative.
- Average speed is defined as the ratio of the distance of a particle to the time interval.

$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}}$$



Instantaneous Velocity

- When the time interval becomes small, approaches zero, the limiting velocity is the instantaneous velocity

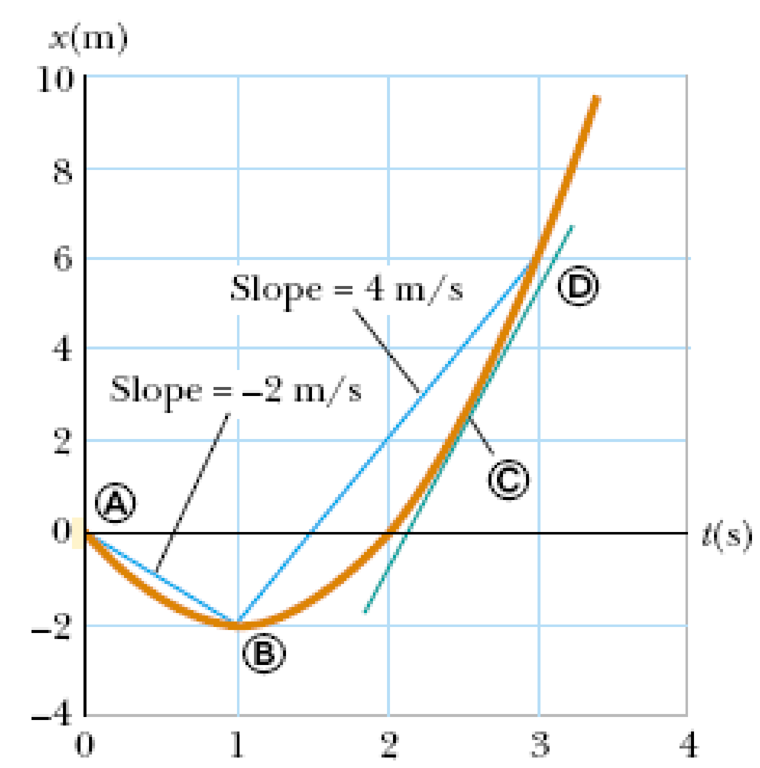
$$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

$$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

- We will use the term velocity to denote for instantaneous velocity
- Instantaneous speed is the absolute value of the velocity.

Example

A particle moves along the x axis. Its position varies with time according to the expression $x = -4t + 2t^2$ where x is in meters and t is in seconds.[3] The position–time graph for this motion is shown in Figure 2.4. Note that the particle moves in the negative x direction for the first second of motion, is momentarily at rest at the moment $t = 1$ s, and moves in the positive x direction at times $t > 1$ s.



(A) Determine the displacement of the particle in the time intervals $t = 0$ to $t = 1$ s and $t = 1$ s to $t = 3$ s.

(B) Calculate the average velocity during these two time intervals.

(C) Find the instantaneous velocity of the particle at $t = 2.5$ s.

$$\Delta x_{A\to B} = x_f - x_i = x_B - x_A = [-4(1) + 2(1)^2] - [-4(0) + 2(0)^2] = -2\text{ m}$$

$$\Delta x_{B\to D} = x_f - x_i = x_D - x_B = [-4(3) + 2(3)^2] - [-4(1) + 2(1)^2] = +8\text{ m}$$

$$\bar{v}_{x(A\to B)} = \frac{\Delta x_{A\to B}}{\Delta t} = \frac{-2\text{ m}}{1\text{ s}} = -2\text{ m/s}$$

$$\bar{v}_{x(B\to D)} = \frac{\Delta x_{B\to D}}{\Delta t} = \frac{8\text{ m}}{2\text{ s}} = +4\text{ m/s}$$

$$v_x \equiv \lim_{\Delta t\to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

$$v_x = +6\text{ m/s}$$

Acceleration

- Average accelerationis defined as ratio of the change in velocity (*Δv*) of a particle in a time interval (*Δt*).
- Average acceleration can be positive or negative.

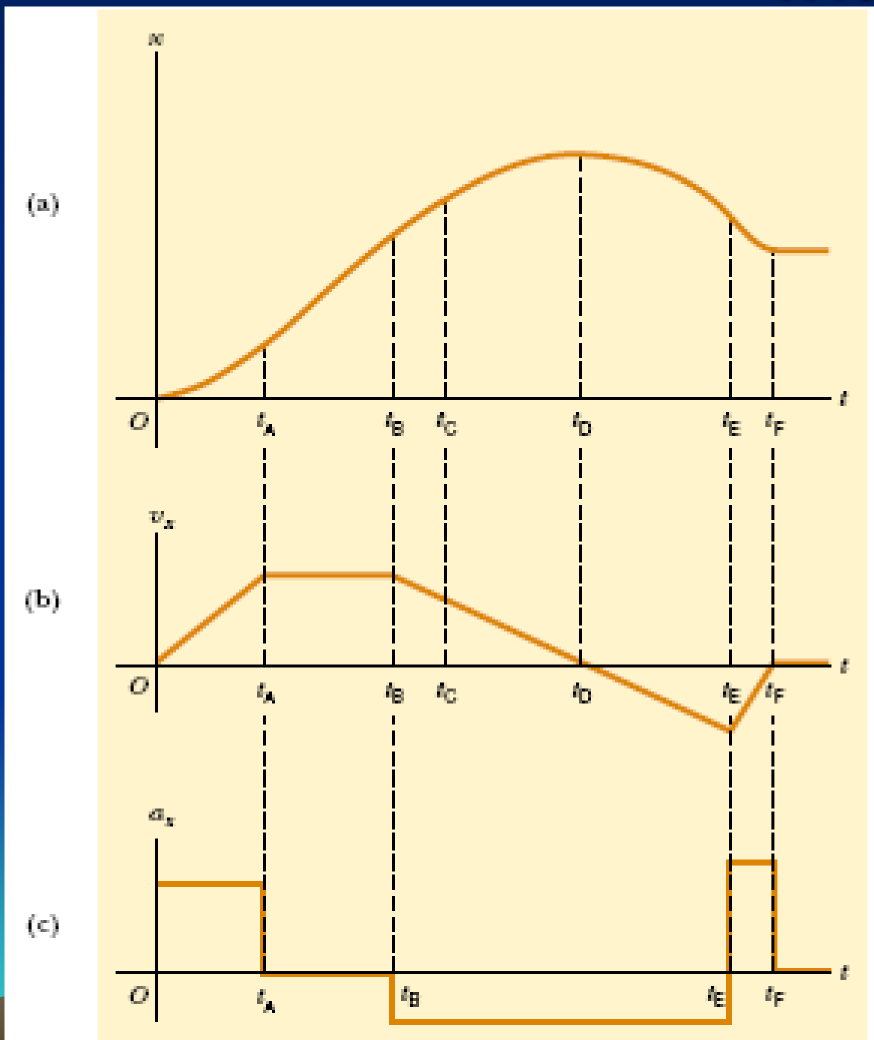
$$\bar{a}_x \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$$

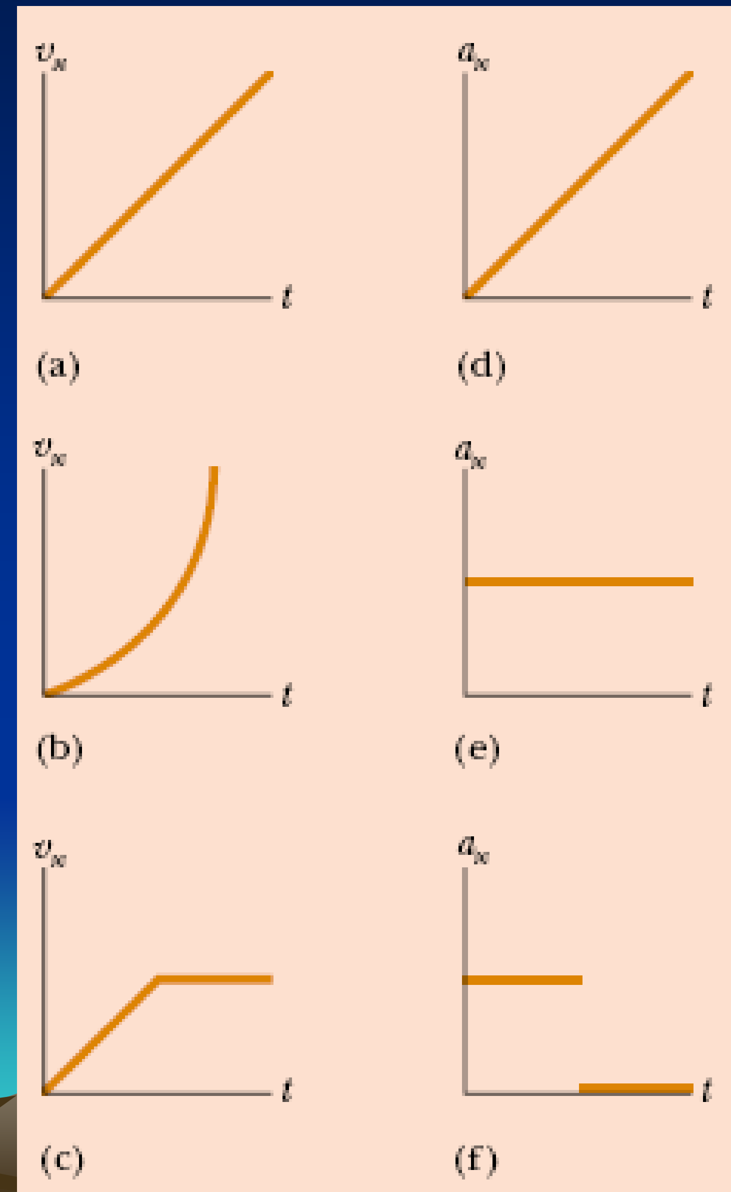
- The instantaneous acceleration is given by

$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$$

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

Graphical representation of position, velocity and acceleration with time





Example

The velocity of a particle moving along the x axis varies in time according to the expression $v_x = (40 - 5t^2)$ m/s, where t is in seconds.

(A) Find the average acceleration in the time interval $t = 0$ to $t = 2.0$ s.

(B) Determine the acceleration at $t = 2.0$ s.

Constant acceleration motion (in one dimension)

$$a_x = \frac{v_{xf} - v_{xi}}{t - 0}$$

$$v_{xf} = v_{xi} + a_x t \qquad \text{(for constant } a_x\text{)}$$

$$\bar{v}_x = \frac{v_{xi} + v_{xf}}{2} \qquad \text{(for constant } a_x\text{)}$$

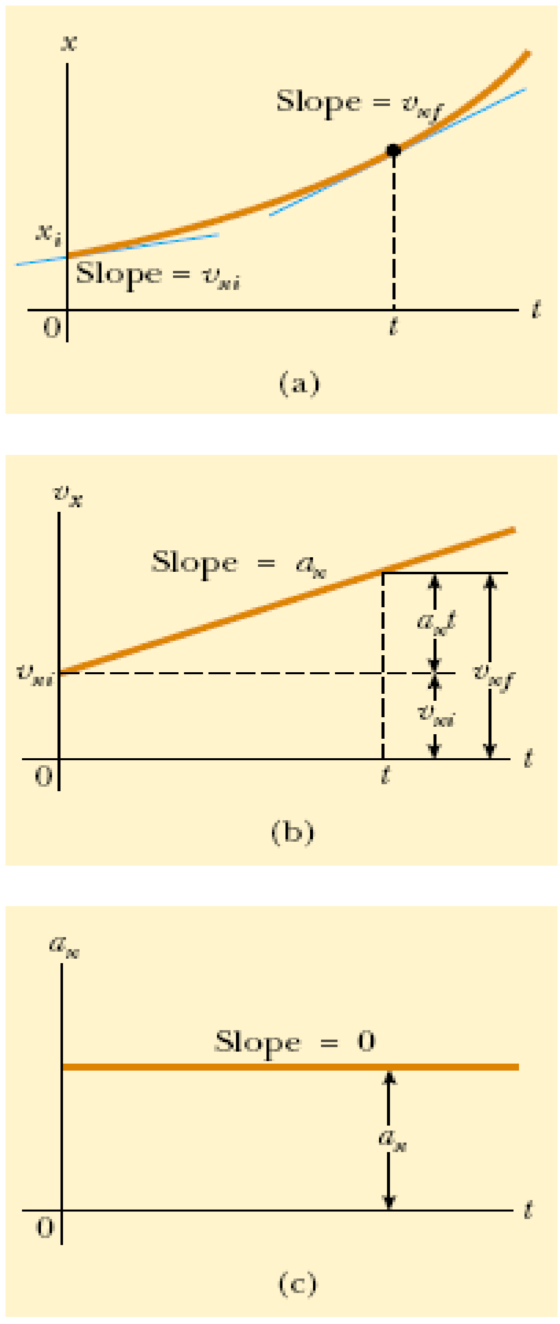
$$x_f - x_i = \bar{v}t = \tfrac{1}{2}(v_{xi} + v_{xf})t$$

$$x_f = x_i + \tfrac{1}{2}(v_{xi} + v_{xf})t \qquad \text{(for constant } a_x\text{)}$$

$$x_f = x_i + \tfrac{1}{2}[v_{xi} + (v_{xi} + a_x t)]t$$

$$x_f = x_i + v_{xi}t + \tfrac{1}{2}a_x t^2 \qquad \text{(for constant } a_x\text{)}$$

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i) \qquad \text{(for constant } a_x\text{)}$$



Example

A jet lands on an aircraft carrier at 140 mi/h (≈ 63 m/s).

(A) What is its acceleration (assumed constant) if it stops in 2.0 s due to an arresting cable that snags the airplane and brings it to a stop?

(B) If the plane touches down at position $x_i = 0$, what is the final position of the plane?

A

$$a_x = \frac{v_{xf} - v_{xi}}{t} \approx \frac{0 - 63\ \mathrm{m/s}}{2.0\ \mathrm{s}}$$

$$= -31\ \mathrm{m/s^2}$$

B

$$x_f = x_i + \tfrac{1}{2}(v_{xi} + v_{xf})t =$$

$$= 0 + \tfrac{1}{2}(63\ \mathrm{m/s} + 0)(2.0\ \mathrm{s})$$

$$= 63\ \mathrm{m}$$

Example

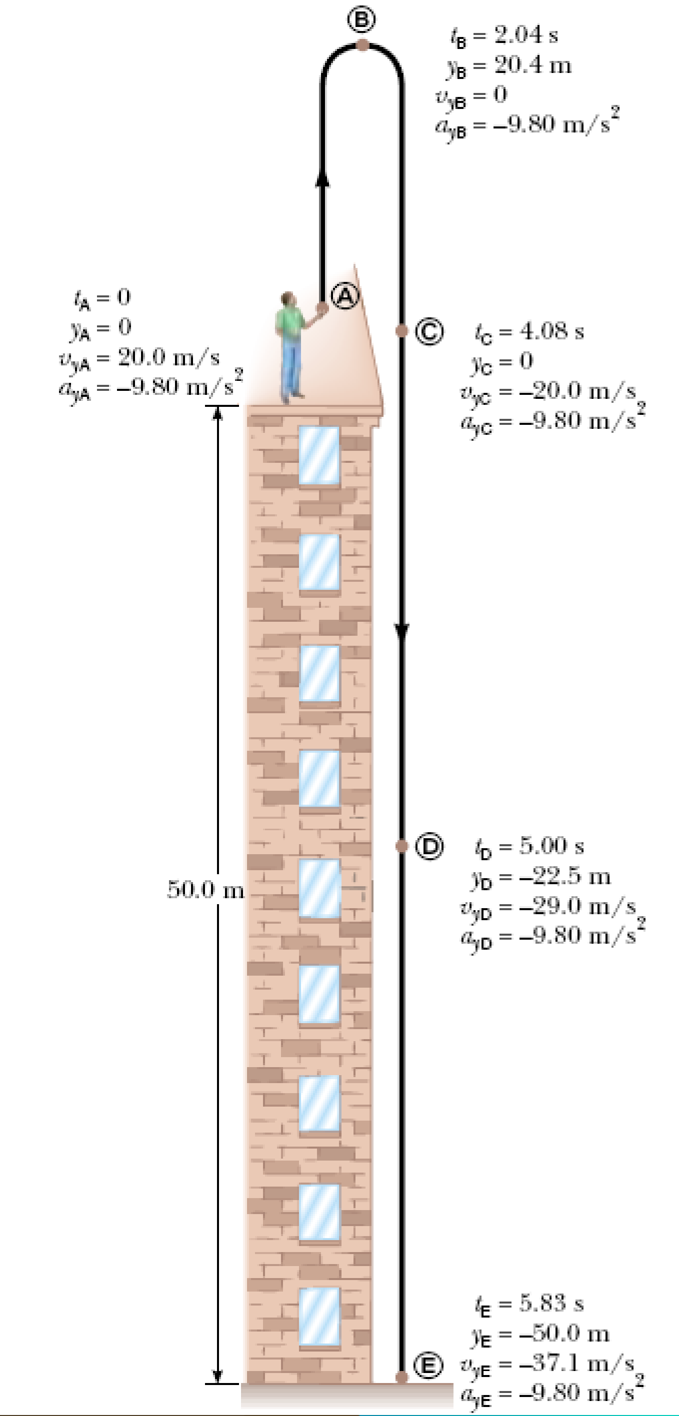
A stone thrown from the top of a building is given an initial velocity of 20.0 m/s straight upward. The building is 50.0 m high, and the stone just misses the edge of the roof on its way down, as shown in Figure 2.14. Using $t_A = 0$ as the time the stone leaves the thrower's hand at position Ⓐ, determine **(A)** the time at which the stone reaches its maximum height, **(B)** the maximum height, **(C)** the time at which the stone returns to the height from which it was thrown, **(D)** the velocity of the stone at this instant, and **(E)** the velocity and position of the stone at $t = 5.00$ s.

(A) At maximum height $v_{yB} = 0$, g = – 9.80 m/s2

$$v_{yB} = v_{yA} + a_y t$$

$$0 = 20.0\ \text{m/s} + (-9.80\ \text{m/s}^2)t$$

$$t = t_B = \frac{20.0\ \text{m/s}}{9.80\ \text{m/s}^2} = 2.04\ \text{s}$$



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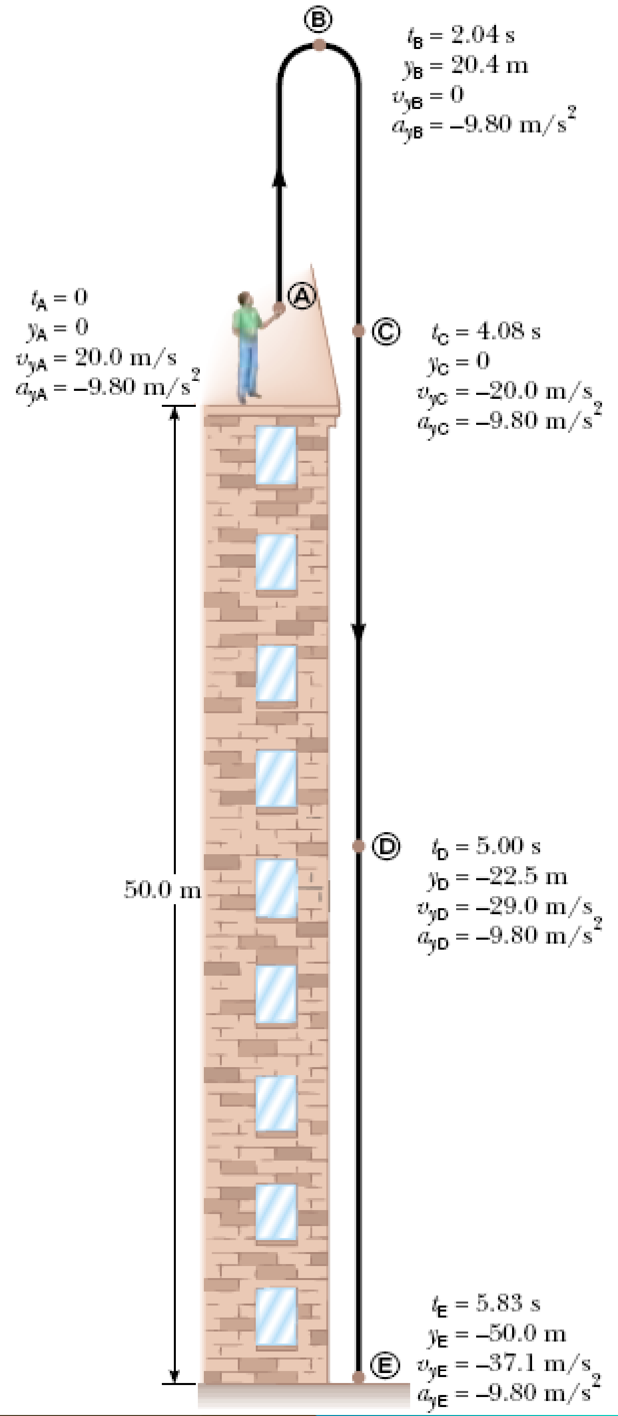
(B) What due you expect (v, t, y)

$$y_{max} = y_B = y_A + v_{xA}t + \frac{1}{2}a_y t^2$$

$$y_B = 0 + (20.0\ \text{m/s})(2.04\ \text{s}) + \tfrac{1}{2}(-9.80\ \text{m/s}^2)(2.04\ \text{s})^2$$
$$= 20.4\ \text{m}$$



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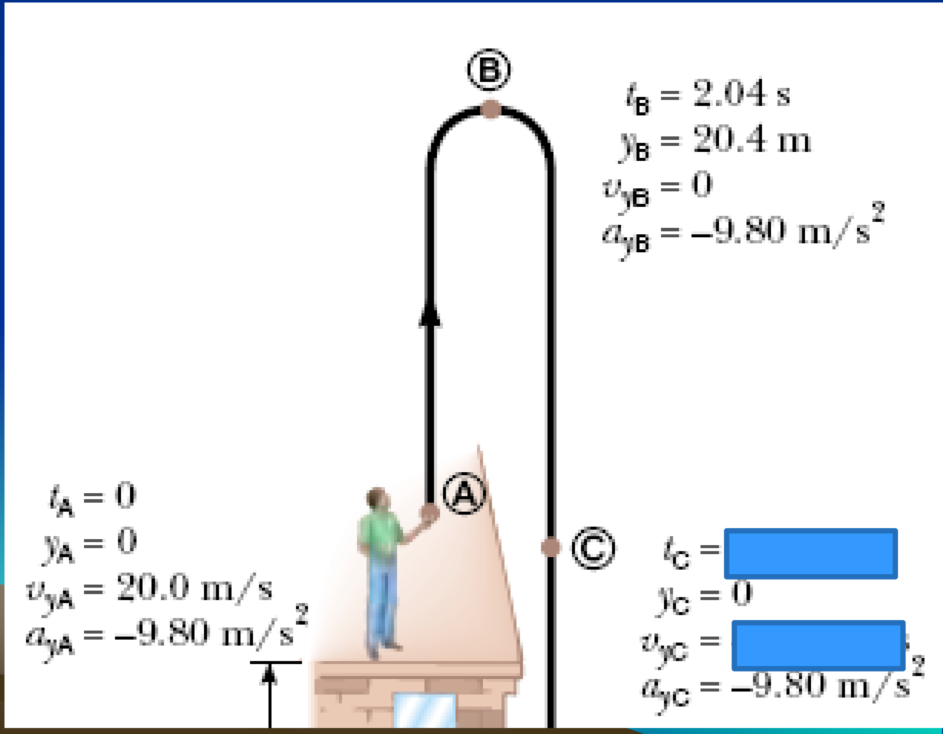
(C) What do you think about the points A and C re time, position

$$y_C = y_A + v_{yA}t + \frac{1}{2}a_y t^2$$

$$0 = 0 + 20.0t - 4.90t^2$$

$$t(20.0 - 4.90t) = 0$$

$$t = 4.08 \text{ s}$$



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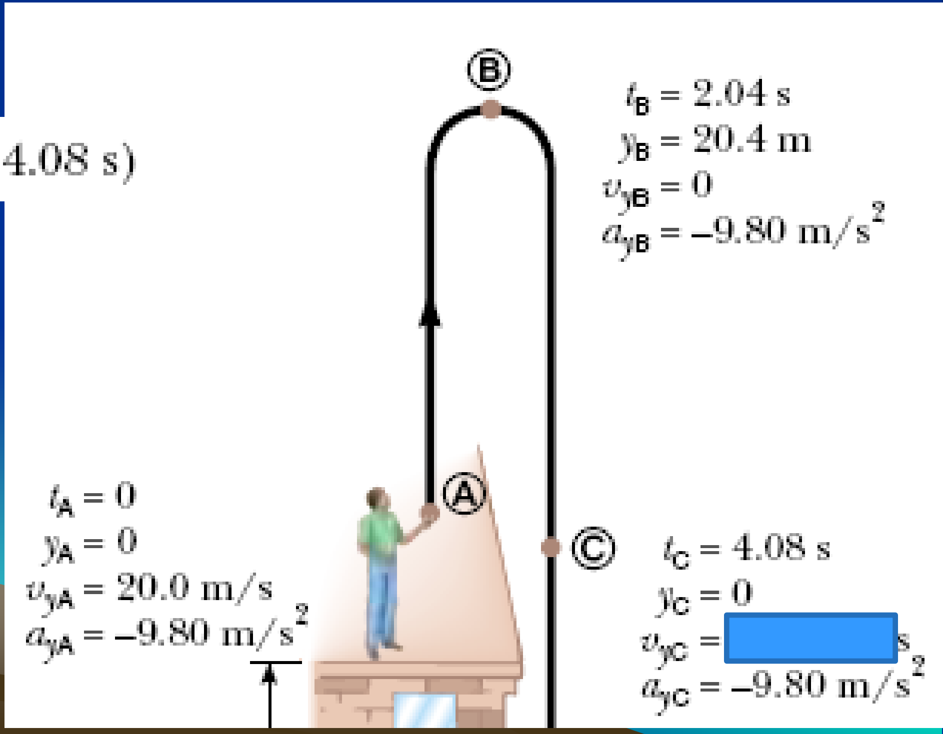
(D) What do you think about the velocity

$$v_{yC} = v_{yA} + a_y t = 20.0\ \mathrm{m/s} + (-9.80\ \mathrm{m/s^2})(4.08\ \mathrm{s})$$

$$= -20.0\ \mathrm{m/s}$$



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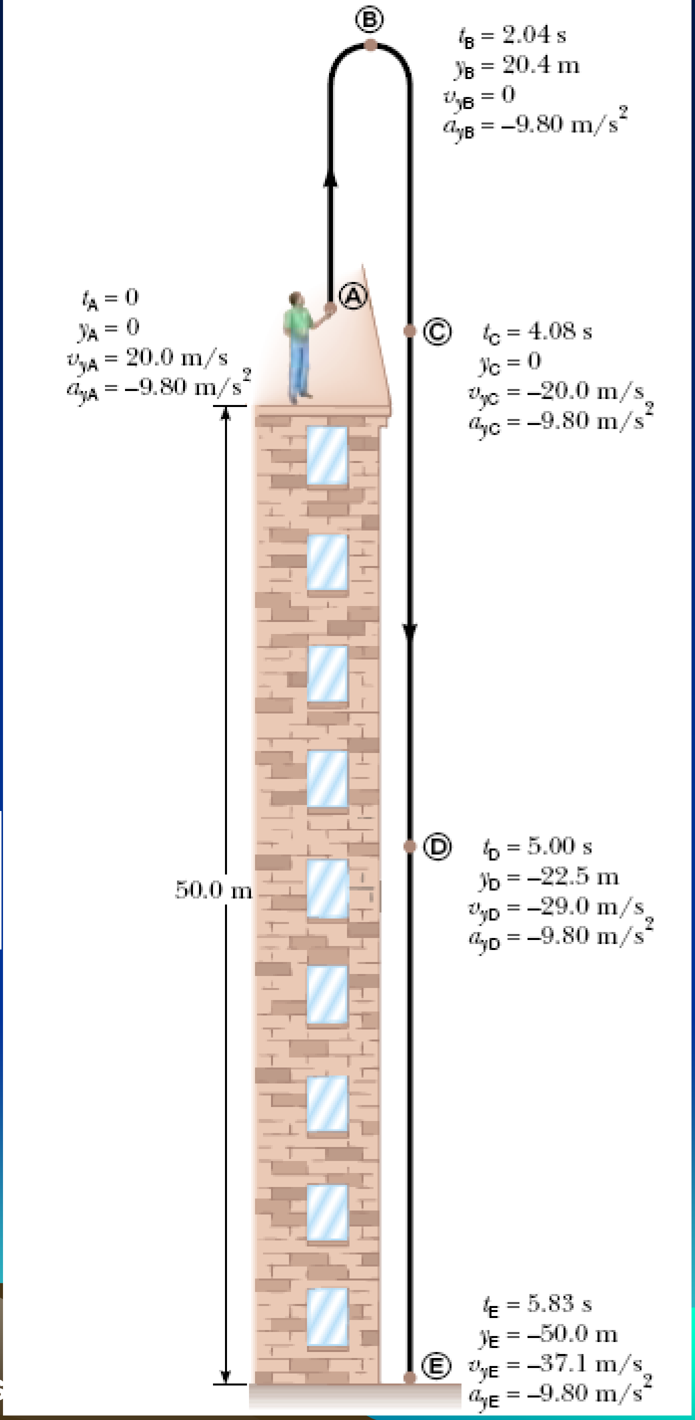
(E) What do you think about the velocity

$$v_{yD} = v_{yA} + a_y t = 20.0\ \text{m/s} + (-9.80\ \text{m/s}^2)(5.00\ \text{s})$$
$$= -29.0\ \text{m/s}$$

What if the building is 30 m high? What will Change?



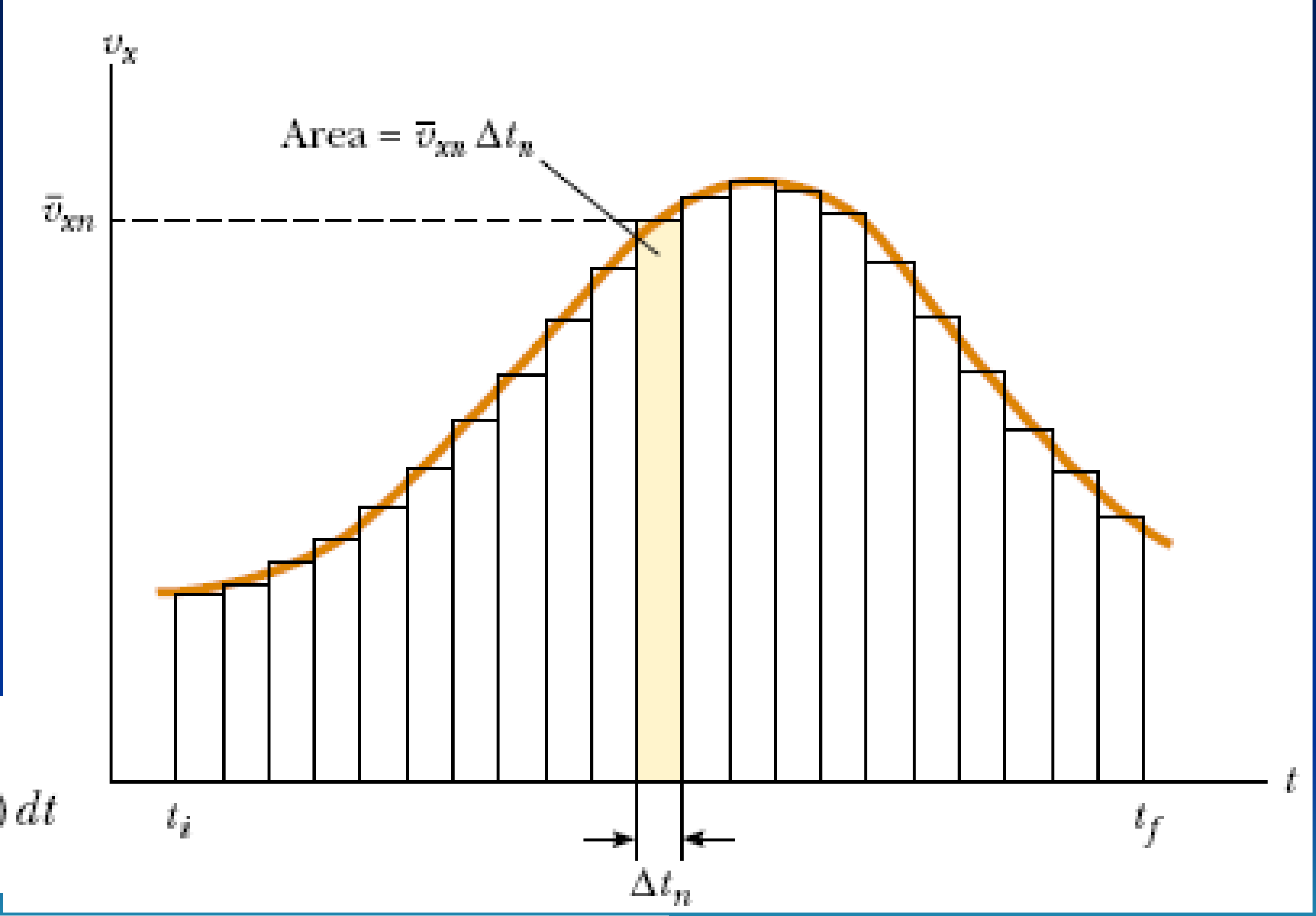
Kinematics equations derived from Calculus

How to find the position from the velocity graph?

$$\Delta x = \sum_n \bar{v}_{xn} \Delta t_n$$

$$\Delta x = \lim_{\Delta t_n \to 0} \sum_n v_{xn} \Delta t_n$$

$$\lim_{\Delta t_n \to 0} \sum_n v_{xn} \Delta t_n = \int_{t_i}^{t_f} v_x(t)\, dt$$

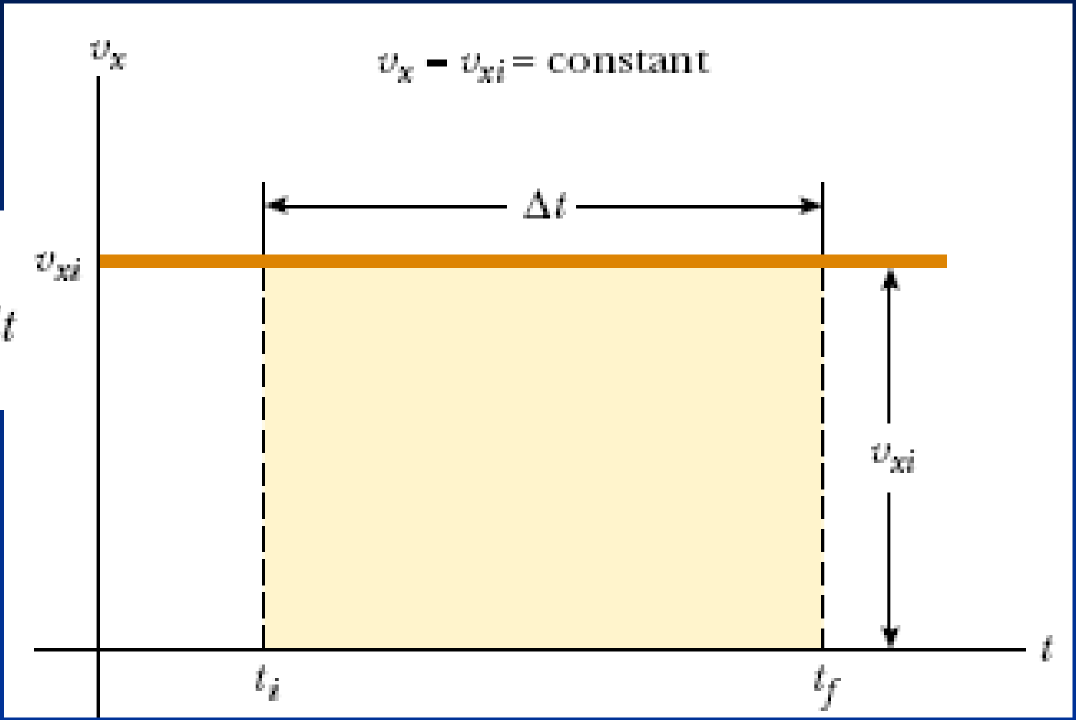


Displacement = area under the v_x-t graph

Special case 1: Constant velocity

$$\Delta x = \lim_{\Delta t_n \to 0} \sum_n v_{xn} \Delta t_n = \int_{t_i}^{t_f} v_x(t)\,dt$$

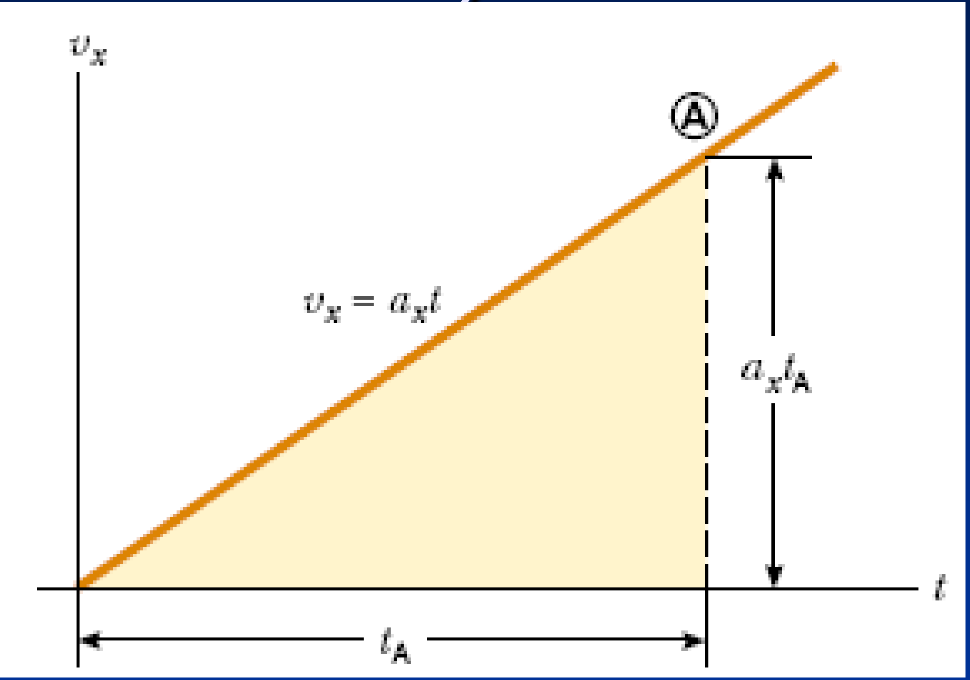
$$\Delta x = v_{xi}\,\Delta t$$



Special case 1: linear velocity (Constant acceleration)

$$\Delta x = \lim_{\Delta t_n \to 0} \sum_n v_{xn} \Delta t_n = \int_{t_i}^{t_f} v_x(t)\, dt$$

$$\Delta x = \frac{1}{2}(t_A)(a_x t_A) = \frac{1}{2} a_x t_A{}^2$$



Kinematics equations

$$a_x = \frac{dv_x}{dt}$$

How to find the position, velocity from the acceleration mathematically?

$$v_{xf} - v_{xi} = \int_0^t a_x dt$$

here a_x is constant

$$v_{xf} - v_{xi} = a_x \int_0^t dt = a_x(t - 0) = a_x t$$

Kinematics equations

$$v_x = \frac{dx}{dt}$$

How to find the position, velocity from the acceleration mathematically?

$$x_f - x_i = \int_0^t v_x dt$$

If a_x is constant

$$x_f - x_i = \int_0^t (v_{xi} + a_x t)\, dt = \int_0^t v_{xi} dt + a_x \int_0^t t dt = v_{xi}(t - 0) + a_x\left(\frac{t^2}{2} - 0\right)$$
$$= v_{xi} t + \tfrac{1}{2} a_x t^2$$

Example

The speed of a bullet as it travels down the barrel of a rifle toward the opening is given by $v = (-5 \times 10^7)\, t^2 + (3 \times 10^5)\, t$, *where v is in meters per second and t is in* seconds. The acceleration of the bullet just as it leaves the barrel is zero. (a) Determine the acceleration and position of the bullet as a function of time when the bullet is in the barrel. (b) Determine how long the bullet is accelerated.
(c) Find the speed at which the bullet leaves the barrel. (d) What is the length of the barrel?

a

$$a = \frac{dv}{dt}$$

$$a = -\left(10.0 \times 10^7 \ \mathrm{m/s^3}\right)t + 3.00 \times 10^5 \ \mathrm{m/s^2}$$

$$x = -5.00 \times 10^7 \frac{t^3}{3} + 3.00 \times 10^5 \frac{t^2}{2}$$

Take $x_i = 0$ at $t = 0$. Then $v = \frac{dx}{dt}$

$$x = -\left(1.67 \times 10^7 \ \mathrm{m/s^3}\right)t^3 + \left(1.50 \times 10^5 \ \mathrm{m/s^2}\right)t^2$$

$$x - 0 = \int_0^t v\,dt = \int_0^t \left(-5.00 \times 10^7 t^2 + 3.00 \times 10^5 t\right)dt$$

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(c) Find the speed at which the bullet leaves the barrel. (d) What is the length of the barrel?

b

The bullet escapes when $a = 0$, at $-(10.0 \times 10^7 \text{ m/s}^3)t + 3.00 \times 10^5 \text{ m/s}^2 = 0$

$$t = \frac{3.00 \times 10^5 \text{ s}}{10.0 \times 10^7} = \boxed{3.00 \times 10^{-3} \text{ s}}.$$

c

$$\text{New } v = (-5.00 \times 10^7)(3.00 \times 10^{-3})^2 + (3.00 \times 10^5)(3.00 \times 10^{-3})$$

$$v = -450 \text{ m/s} + 900 \text{ m/s} = \boxed{450 \text{ m/s}}.$$

d

$$x = -(1.67 \times 10^7)(3.00 \times 10^{-3})^3 + (1.50 \times 10^5)(3.00 \times 10^{-3})^2$$

$$x = -0.450 \text{ m} + 1.35 \text{ m} = \boxed{0.900 \text{ m}}$$