Chapter 9

Static Equilibrium and Elasticity

Conditions of Static Equilibrium

A rigid object is in **equilibrium** if and only if **the resultant external force acting on it is zero and the resultant external torque on it is zero about any axis:**

1. The resultant external force must equal zero:

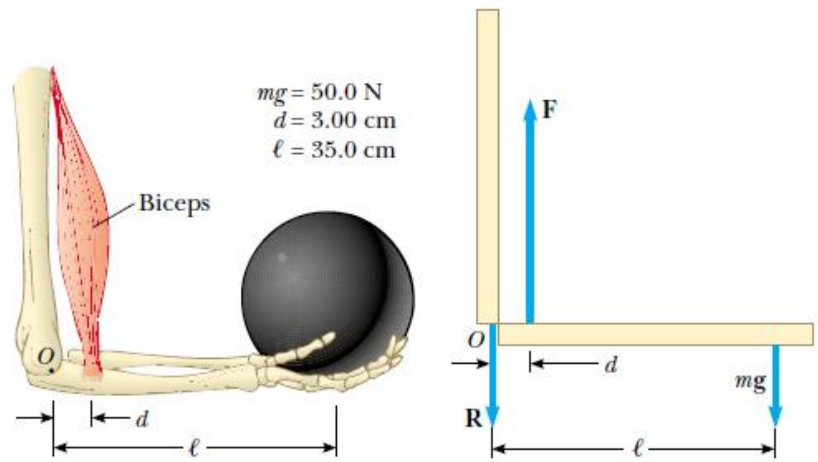
$$\sum \mathbf{F} = 0$$

2. The resultant external torque about *any* axis must be zero:

$$\sum \boldsymbol{\tau} = 0$$

A person holds a 50.0-N sphere in his hand. The forearm is horizontal, as shown in Figure 12.9a. The biceps muscle is attached 3.00 cm from the joint, and the sphere is 35.0 cm from the joint. Find the upward force exerted by the biceps on the forearm and the downward force exerted by the upper arm on the forearm and acting at the joint. Neglect the weight of the forearm.

(1) $$\sum F_y = F - R - 50.0\text{ N} = 0$$



(a) (b)

From the second condition for equilibrium, we know that the sum of the torques about any point must be zero. With the joint O as the axis, we have

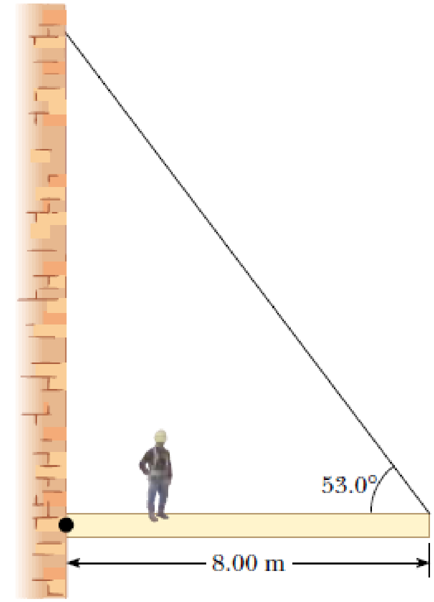
$$\sum \tau = Fd - mg\ell = 0$$

$$F(3.00\text{ cm}) - (50.0\text{ N})(35.0\text{ cm}) = 0$$

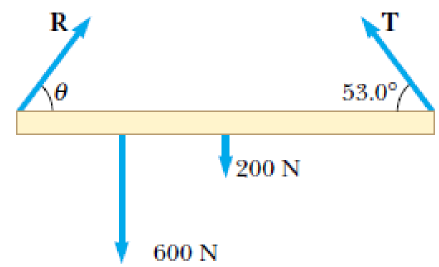
$$F = 583\text{ N}$$

This value for F can be substituted into Equation (1) to give $R = 533$ N. As this example shows, the forces at joints and in muscles can be extremely large.

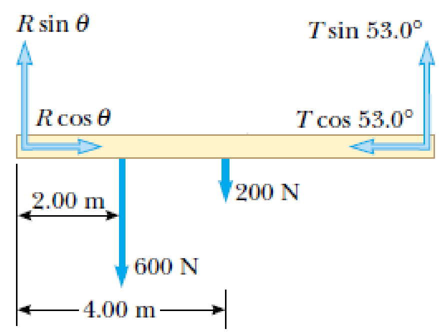
A uniform horizontal beam with a length of 8.00 m and a weight of 200 N is attached to a wall by a pin connection. Its far end is supported by a cable that makes an angle of 53.0° with the beam (Fig. 12.10a). If a 600-N person stands 2.00 m from the wall, find the tension in the cable as well as the magnitude and direction of the force exerted by the wall on the beam.



(a)



(b)



(c)

(1) $$\sum F_x = R\cos\theta - T\cos 53.0° = 0$$

(2) $$\sum F_y = R\sin\theta + T\sin 53.0° - 600\text{ N} - 200\text{ N} = 0$$

(3) $$\sum \tau = (T\sin 53.0°)(8.00\text{ m}) - (600\text{ N})(2.00\text{ m}) - (200\text{ N})(4.00\text{ m}) = 0$$

$$T = \boxed{313\text{ N}}$$

$$R\cos\theta = 188\text{ N}$$

$$R\sin\theta = 550\text{ N}$$

$$\tan\theta = \frac{550\text{ N}}{188\text{ N}} = 2.93$$

$$\theta = \boxed{71.1°}$$

This positive value indicates that our estimate of the direction of **R** was accurate.

Finally,

$$R = \frac{188\text{ N}}{\cos\theta} = \frac{188\text{ N}}{\cos 71.1°} = \boxed{580\text{ N}}$$

Elastic Properties of Solids

- The deformation of solids will be discussed in terms of the concepts of *stress* and *strain*.
- **Stress** is a quantity that is proportional to the force causing a deformation;
- more specifically, stress is the external force acting on an object per unit cross-sectional area. The result of a stress is **strain**, which is a measure of the degree of deformation.
- It is found that, for sufficiently small stresses, strain is proportional to stress; the constant of proportionality depends on the material being deformed and on the nature of the deformation. We call this proportionality constant the elastic modulus. The elastic modulus is therefore defined as the ratio of the stress to the resulting
- strain:

$$\text{Elastic modulus} \equiv \frac{\text{stress}}{\text{strain}}$$

1. **Young's modulus**, which measures the resistance of a solid to a change in its length
2. **Shear modulus**, which measures the resistance to motion of the planes within a solid parallel to each other
3. **Bulk modulus**, which measures the resistance of solids or liquids to changes in their volume

Young's Modulus

- We define the tensile stress as the ratio of the magnitude of the external force *F* to the cross-sectional area *A*. The tensile strain in this case is defined as the ratio of the change in length ΔL to the original length L_i. We define Young's modulus (Y) by a combination of these two ratios:

$$Y \equiv \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{F/A}{\Delta L / L_i}$$

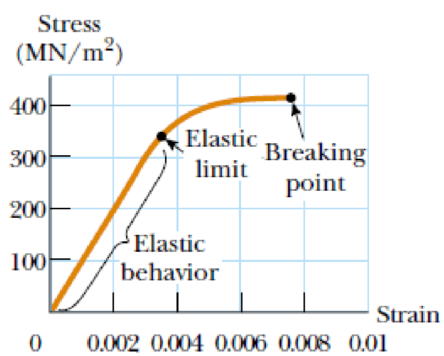
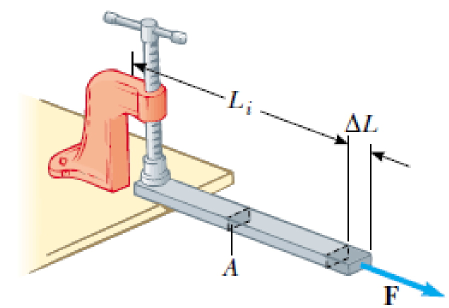


Figure 12.15 Stress-versus-strain curve for an elastic solid.

Typical Values for Elastic Moduli

Substance	Young's Modulus (N/m²)	Shear Modulus (N/m²)	Bulk Modulus (N/m²)
Tungsten	35×10^{10}	14×10^{10}	20×10^{10}
Steel	20×10^{10}	8.4×10^{10}	6×10^{10}
Copper	11×10^{10}	4.2×10^{10}	14×10^{10}
Brass	9.1×10^{10}	3.5×10^{10}	6.1×10^{10}
Aluminum	7.0×10^{10}	2.5×10^{10}	7.0×10^{10}
Glass	$6.5–7.8 \times 10^{10}$	$2.6–3.2 \times 10^{10}$	$5.0–5.5 \times 10^{10}$
Quartz	5.6×10^{10}	2.6×10^{10}	2.7×10^{10}
Water	—	—	0.21×10^{10}
Mercury	—	—	2.8×10^{10}

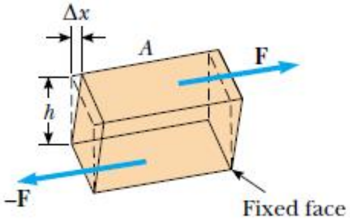


Active Figure 12.14 A long bar clamped at one end is stretched by an amount ΔL under the action of a force **F**.

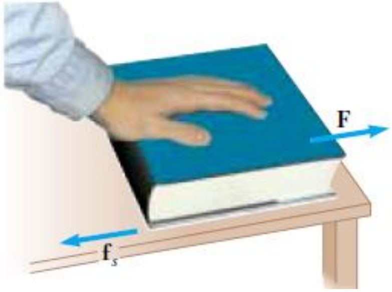
Shear Modulus

- We define the shear stress as *F*/*A*, the ratio of the tangential force to the area *A* of the face being sheared. The shear strain is defined as the ratio $\Delta x/h$, where Δx is the horizontal distance that the sheared face moves and *h* is the height of the object. In terms of these quantities, the shear modulus is:

$$S \equiv \frac{\text{shear stress}}{\text{shear strain}} = \frac{F/A}{\Delta x/h}$$



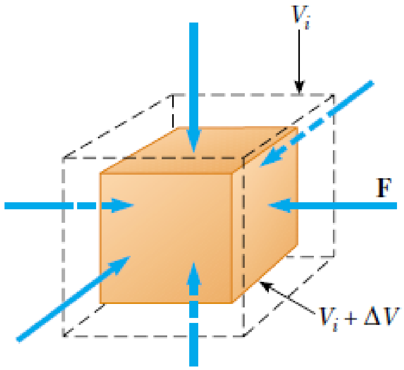
(a)



Bulk Modulus

- The volume stress is defined as the ratio of the magnitude of the total force *F* exerted on a surface to the area *A* of the surface. The quantity $P = F/A$ is called pressure. If the pressure on an object changes by an amount $\Delta P = \Delta F/A$, then the object will experience a volume change ΔV. The volume strain is equal to the change in volume ΔV divided by the initial volume V_i. Thus, we can characterize a volume ("bulk") compression in terms of the bulk modulus, which is defined as

$$B \equiv \frac{\text{volume stress}}{\text{volume strain}} = -\frac{\Delta F/A}{\Delta V/V_i} = -\frac{\Delta P}{\Delta V/V_i}$$



Active Figure 12.17 When a solid is under uniform pressure, it undergoes a change in volume but no change in shape. This cube is compressed on all sides by forces normal to its six faces.

Suppose that the tension in the cable is 940 N as the actor reaches the lowest point. What diameter should a 10-m-long steel wire have if we do not want it to stretch more than 0.5 cm under these conditions? (Y = 20 x 10^{10} N/m^2)

$$Y = \frac{F/A}{\Delta L/L_i}$$

$$A = \frac{FL_i}{Y\Delta L} = \frac{(940\text{ N})(10\text{ m})}{(20 \times 10^{10}\text{ N/m}^2)(0.005\text{ m})}$$

$$= 9.4 \times 10^{-6}\text{ m}^2$$

Because $A = \pi r^2$, the radius of the wire can be found from

$$r = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{9.4 \times 10^{-6}\text{ m}^2}{\pi}} = 1.7 \times 10^{-3}\text{ m} = 1.7\text{ mm}$$

$$d = 2r = 2(1.7\text{ mm}) = 3.4\text{ mm}$$

A solid brass ($B = 6.1 \times 10^{10}$ N/m^2) sphere is initially surrounded by air, and the air pressure exerted on it is 1×10^5 N/m^2 (normal atmospheric pressure). The sphere is lowered into the ocean to a depth where the pressure is 2×10^7 N/m^2. The volume of the sphere in air is 0.50 m^3 By how much does this volume change once the sphere is submerged?

$$B = -\frac{\Delta P}{\Delta V / V_i}$$

$$\Delta V = -\frac{V_i \Delta P}{B}$$

Substituting the numerical values, we obtain

$$\Delta V = -\frac{(0.50 \text{ m}^3)(2.0 \times 10^7 \text{ N/m}^2 - 1.0 \times 10^5 \text{ N/m}^2)}{6.1 \times 10^{10} \text{ N/m}^2}$$

$$= -1.6 \times 10^{-4} \text{ m}^3$$

The negative sign indicates that the volume of the sphere decreases.