MATH203 Calculus

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Outline

- Definition of sequences.
- Definition of convergent sequence.
- Definition of divergent sequence.
- Definition of constant sequence.
- Theorem 1.
- L' Hopital's rule.
- Theorem 2 (Properties of limits of sequences).
- Theorem 3 (Absolute value).

Definition of sequences

A sequence is a function whose domain is the set of positive integers. It is denoted by $\{a_n\} = a_1, a_2, a_3, \ldots, a_n, \ldots$ (entire seq) and $\{a_n\} = a_1, a_2, a_3, \ldots, a_n$ (finite seq).

Example: Find the first four terms and nth term of each:

(a) $\{\frac{n}{n+1}\}$ (b) $\{2 + (0.1)^n\}$ (c) $\{(-1)^{n+1}\frac{n^2}{3n-1}\}$

(d) $\{4\}$ (e) $a_1 = 3$ and $a_{k+1} = 2a_k$ for $k \geqslant 1$.

Definition of convergent sequence (c$'$gt)

A sequence is $\{a_n\}$ has a limit L, or converges to L denoted by either $\lim_{n\to\infty} a_n = L$ or $a_n \to L$ as $n \to \infty$.

Definition of divergent sequence (d$'$gt)

A sequence $\{a_n\}$ is called if

- $\lim_{n\to\infty} a_n$ does not exist.
- $\lim_{n\to\infty} a_n = +\infty$ or $\lim_{n\to\infty} a_n = -\infty$.

Definition of constant sequence

A $\{a_n\}$ is constant if $a_n = c$ for every n, $c \in \mathbb{R}$ and $\lim_{n\to\infty} a_n = \lim_{n\to\infty} c = c$.

Theorem 1

Let $\{a_n\}$ be a sequence and f be a function such that

- $f(n) = a_n$
- $f(x)$ exists for every real number $x \geqslant 1$

then

1. If $\lim_{x\to\infty} f(x) = L$, then $\lim_{n\to\infty} f(n) = L$
2. If $\lim_{x\to\infty} f(x) = \infty$ (or $-\infty$), then $\lim_{n\to\infty} f(n) = \infty$ (or $-\infty$).

Examples:

(1) If $a_n = 1 + (\frac{1}{n})$, determine whether $\{a_n\}$ converges or diverges.

(2) Determine whether $\{a_n\}$ converges or diverges

(a)$\{\frac{1}{4}n^2 - 1\}$ (b) $\{(-1)^{n-1}\}$

L' Hopital's rule

It is a method for computing a limit of form $\lim_{n\to\infty} \frac{f(n)}{g(n)}$ if $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{f(n)}{g(n)} = \frac{\infty}{\infty}$, then we can use L' Hopital's rule which is defined as $\lim_{n\to\infty} \frac{f'(n)}{g'(n)}$.

Theorem 2 (properties)

Let $\lim_{n\to\infty} a_n = L$ and $\lim_{n\to\infty} b_n = K$

- $\lim_{n\to\infty} (a_n \pm b_n) = L \pm K$.
- $\lim_{n\to\infty} (a_n . b_n) = L.K$.
- $\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{L}{K}, \; K \neq 0$.
- $\lim_{n\to\infty} Ca_n = CL$.

Theorem 3 (Absolute value)

For a seq $\{a_n\}$, $\lim_{n\to 0} |a_n| = 0 \Leftrightarrow \lim_{n\to\infty} a_n = 0$.

Theorem 4 (Geometric seq)

- $\lim_{n\to\infty} r^n = 0$ if $|r| < 1$
- $\lim_{n\to\infty} r^n = \infty$ if $|r| > 1$

Example: Determine whether the following sequences converge or diverge

(1) $\{\frac{5n}{e^{2n}}\}$, (2) $\{(\frac{-2}{3})^n\}$ (3) $\{(1.01)^n\}$ (4) $\{\frac{2n^2}{5n^2-3}\}$
(5) $\{6(\frac{-5}{6})^n\}$ (6) $\{8-(\frac{7}{8})^n\}$ (7) $\{1000-n\}$ (8) $\{\frac{4n^4+1}{2n^2-1}\}$
(9) $\{\frac{e^n}{4}\}$.