lecture (12)

Ch 3 Markov Chains

P. 79 Textbook

- A discrete-time Markov chain is a stochastic process

$\{X_t\}, t = 0, 1, 2, \dots, n$

defined by

$$pr\{\underbrace{X_{n+1} = j}_{\text{المستقبل}} \mid \underbrace{X_0 = i_0, X_1 = i_1, \dots, X_{n-1} = i_{n-1}}_{\text{الماضي}}, \underbrace{X_n = i}_{\text{الحاضر}}\}$$

$\forall$ time points n and states $i_0, i_1, i_2, \dots, i_{n-1}, i, j$

$$= pr\{X_{n+1} = j \mid X_n = i\}$$ Conditional prob.

prob. of X_{n+1} being in state j given that X_n is in state i

$$= P_{ij}^{n,n+1}$$ one step transition probability

واحتمال الانتقال في خطوة واحدة

or simply

$$= P_{ij}$$ for stationary transition prob.

واحتمال الانتقال المستقر

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$\ast \quad P = [P_{ij}]$ or $\| P_{ij} \|$

$i, j = 0, 1, 2, \dots, n$

الانتقال

$$= \begin{array}{c|cccccc} & 0 & 1 & 2 & 3 & \cdots \\ \hline 0 & P_{00} & P_{01} & P_{02} & P_{03} & \cdots \\ 1 & P_{10} & P_{11} & P_{12} & P_{13} & \cdots \\ 2 & P_{20} & P_{21} & P_{22} & P_{23} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{array}$$

الحاضر

is called Markov Matrix or transition prob. Mx

where $P_{ij} \geq 0$

$, \sum_{j=0}^{\infty} P_{ij} = 1$ for $i, j = 0, 1, 2, \dots$

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Theorem p. 80

$pr\{X_0 = i_0, X_1 = i_1, X_2 = i_2, \dots, X_n = i_n\}$ Joint prob.

$= P_{i_0} P_{i_0 i_1} P_{i_1 i_2} \dots P_{i_{n-1} i_n}$

where $P_{i_0} = pr\{X_0 = i_0\}$ for initial distribution

(التوزيع الابتدائي) .

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proof

$\because pr\{X_0 = i_0, X_1 = i_1, X_2 = i_2, \dots, X_n = i_n\}$

Joint prob.

$= pr\{X_0 = i_0, X_1 = i_1, X_2 = i_2, \dots, X_{n-1} = i_{n-1}\}$

$\cdot\ pr\{X_n = i_n \mid X_0 = i_0, X_1 = i_1, X_2 = i_2, \dots, X_{n-1} = i_{n-1}\}$

المستقبل — الماضي — الحاضر

Conditional prob.

$= pr\{X_0 = i_0, X_1 = i_1, X_2 = i_2, \dots, X_{n-1} = i_{n-1}\}$

$\cdot\ pr\{X_n = i_n \mid X_{n-1} = i_{n-1}\}$

Markov Def$\underline{n}$

تذكر

Remember that

$pr(x, y)$

$= pr(y)\, pr(x \mid y)$

marginal — Cond. prob.

$= pr\{X_0 = i_0, X_1 = i_1, X_2 = i_2, \dots, X_{n-1} = i_{n-1}\} \cdot P_{i_{n-1}\, i_n}$

$\therefore$ By repeating this argument $n-1$ times, we get

$pr\{X_0 = i_0, X_1 = i_1, X_2 = i_2, \dots, X_n = i_n\}$

$= P_{i_0}\, P_{i_0 i_1}\, P_{i_1 i_2} \cdots P_{i_{n-2}\, i_{n-1}}\, P_{i_{n-1}\, i_n}$

where $P_{i_0} = pr\{X_0 = i_0\}$ for initial dist$\underline{n}$ of the process.

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