

A discrete-time Markov chain is a stochastic process

$$\{X_t\}, t = 0, 1, 2, \dots, n$$

defined by

$$\text{pr} \left\{ X_{n+1} = j \mid X_0 = i_0, X_1 = i_1, \dots, X_{n-1} = i_{n-1}, X_n = i \right\}$$

at time points n and states $i_0, i_1, i_2, \dots, i_{n-1}, i, j$

$$= \text{pr} \left\{ X_{n+1} = j \mid X_n = i \right\}$$

Conditional prob.

prob. of X_{n+1} being in state j given that X_n is

in state i

$$= P_{ij}^{n, n+1}$$

one step transition probability
والانتقال في خطوة واحدة

or simply

$$= P_{ij}$$

for stationary transition prob.
والانتقال البسيط

* $P = [P_{ij}]$ or $\|P_{ij}\|$

$$= \begin{matrix} & \begin{matrix} i, j = 0, 1, 2, \dots, n \\ \text{الحالات} \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ \vdots \end{matrix} & \begin{bmatrix} P_{00} & P_{01} & P_{02} & P_{03} & \dots \\ P_{10} & P_{11} & P_{12} & P_{13} & \dots \\ P_{20} & P_{21} & P_{22} & P_{23} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} \end{matrix}$$

is called Markov Matrix or transition prob. M_X

where $P_{ij} \geq 0$

$\sum_{j=0}^{\infty} P_{ij} = 1$ for $i, j = 0, 1, 2, \dots$

جميع قيم الاحتمالات للانتقال لكل صف لا يوازي واحد.

Theorem p. 80

$pr \{ X_0 = i_0, X_1 = i_1, X_2 = i_2, \dots, X_n = i_n \}$
Joint prob.

$= P_{i_0 i_1} P_{i_1 i_2} \dots P_{i_{n-1} i_n}$

where $P_{i_0} = pr \{ X_0 = i_0 \}$ for initial distribution.
(التوزيع الحادي)

proof

$$\therefore \text{pr} \{X_0 = i_0, X_1 = i_1, X_2 = i_2, \dots, X_n = i_n\}$$

$$= \text{pr} \{X_0 = i_0, X_1 = i_1, X_2 = i_2, \dots, X_{n-1} = i_{n-1}\}$$

$$\cdot \text{pr} \{X_n = i_n \mid X_0 = i_0, X_1 = i_1, X_2 = i_2, \dots, X_{n-1} = i_{n-1}\}$$

joint prob.
Conditional prob.

$$= \text{pr} \{X_0 = i_0, X_1 = i_1, X_2 = i_2, \dots, X_{n-1} = i_{n-1}\}$$

$$\cdot \text{pr} \{X_n = i_n \mid X_{n-1} = i_{n-1}\}$$

Markov Defn

Remember that

$$\text{pr}(x, y) = \text{pr}(y) \text{pr}(x|y)$$

↑ marginal ← Cond. prob.

$$= \text{pr} \{X_0 = i_0, X_1 = i_1, X_2 = i_2, \dots, X_{n-1} = i_{n-1}\} \cdot P_{i_{n-1} i_n}$$

∴ By repeating this argument $n-1$ times, we get

$$\text{pr} \{X_0 = i_0, X_1 = i_1, X_2 = i_2, \dots, X_n = i_n\}$$

$$= P_{i_0} P_{i_0 i_1} P_{i_1 i_2} \dots P_{i_{n-2} i_{n-1}} P_{i_{n-1} i_n}$$

where $P_{i_0} = \text{pr}\{X_0 = i_0\}$ for initial distn of the process.

#