D branes

- brief review of CFT_2.

action

energy-momentum tensor $T_{\mu\nu} = \frac{2\pi}{\sqrt{g}} \frac{\delta S}{\delta g^{\mu\nu}}$

$$T_{z\bar{z}} = 0, \quad \partial_{\bar{z}} T_{zz} = 0, \quad \partial_z T_{\bar{z}\bar{z}} = 0.$$

$$T(z)\,T(w) \sim \frac{c/2}{(z-w)^4} + \left(\frac{2}{(z-w)^2} + \frac{1}{z-w}\partial_w \right) T(w) + \cdots$$

c : central charge.

$\phi(z,\bar{z})\,(dz)^h (d\bar{z})^{\bar{h}}$: primary field.

$$\Rightarrow \quad T(z)\,\phi(w,\bar{w}) \sim \left(\frac{h}{(z-w)^2} + \frac{1}{z-w}\partial_w \right) \phi(w,\bar{w}) + \cdots$$

$$\text{Hilbert space} = \bigoplus_{h,\bar{h}} N_{h,\bar{h}}\; Vir^{(h)} \otimes Vir^{(\bar{h})}$$

$Vir^{(h)}$: highest weight rep of $L_n = \oint_0 \frac{dz}{2\pi i} z^{n+1} T(z)$

$$\left(T(z) = \sum_n L_n z^{-n-2} \right)$$

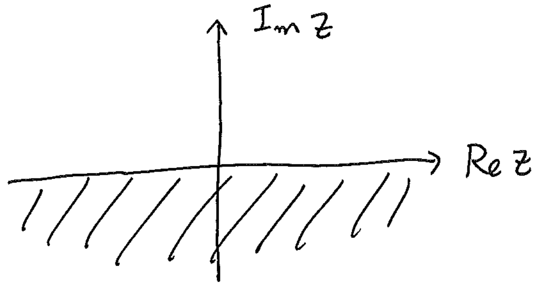
$L_0 |h\rangle = h|h\rangle$

$L_n |h\rangle = 0 \; (n \geq 1)$ $\qquad Vir^{(h)} = \{ L_{-n_1} \cdots L_{-n_k} |h\rangle \}$

state - operator correspondence :

$$\phi(z,\bar{z}) \longleftrightarrow |h,\bar{h}\rangle = \lim_{z\to 0} \phi(z,\bar{z})\,|0\rangle$$

- CFT_2 with boundary.



boundary = real axis

Conformal transf : $z \to z + \varepsilon_n z^n$

$\varepsilon_n \Leftrightarrow L_n$

$\bar{\varepsilon}_n \Leftrightarrow \bar{L}_n$

If we want to maintain conformal transf that keep the real axis, $\varepsilon_n \in \mathbb{R}$.

$\Rightarrow$ For the boundary condition compatible with this
$T(z) = \bar{T}(\bar{z})$ on the real axis.

method of images $\Rightarrow$ $T(z)$ can be extended over the entire $\mathbb{C}$.

<u>example</u> free massless scalar ϕ.

$$T = \frac{1}{2}(\partial\phi)^2, \quad \bar{T} = \frac{1}{2}(\bar{\partial}\phi)^2$$

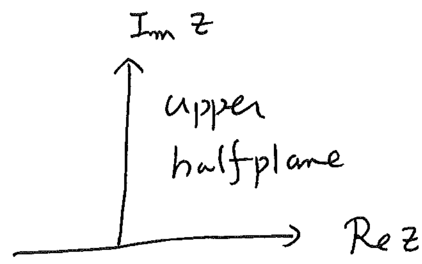
$T = \bar{T}$ on $z \in \mathbb{R}$ $\Leftrightarrow$ $\partial\phi = \bar{\partial}\phi$; Neumann

$\partial\phi = -\bar{\partial}\phi$: Dirichelet

($\Leftrightarrow$ ϕ : const on $\mathbb{R}$)

D branes

Consider X^I :



$\longrightarrow \mathbb{R}^N$

$I = 1, \cdots, N$.

Some X^μ may obey Neumann condition,

other X^i may obey Dirichelet condition.

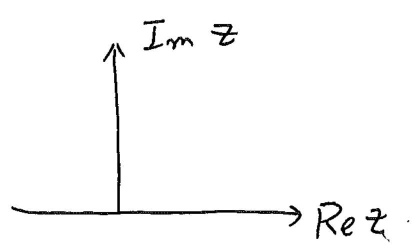
(There may also be some mix's of them.)

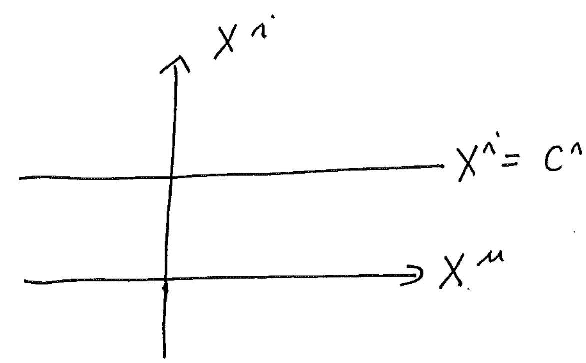
Suppose

$$\partial_\perp X^\mu = 0 \qquad \mu = 1, \cdots, n$$

$$X^i = c^i \qquad i = n+1, \cdots, N \qquad \text{on } z \in \mathbb{R}.$$

X^I :





The upper half plane

is mapped to $\mathbb{R}^N$ in such a way

that the Real Axis is on $\underline{X^i = c^i}$.

$\Uparrow$

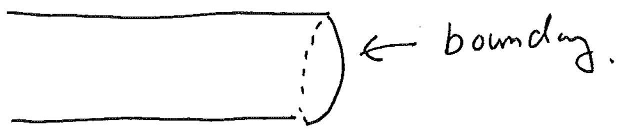
D brane.

geometric way to

think about the boundary condition

Ishibashi State / Cardy State

Consider periodically identifying the boundary



boundary condition $\Rightarrow$ a state in CFT_2. $|B\rangle\rangle$

$$T(z) = \bar{T}(\bar{z}) \quad \Rightarrow \quad L_n |B\rangle\rangle = \bar{L}_{-n} |B\rangle\rangle$$

There is a general solution to this.

Start with a highest weight state $|h\rangle$.

Choose an orthonormal basis $\{ |h, N : j\rangle \}$

for $\{ L_{-n_1} \cdots L_{-n_k} |h\rangle \}_{n_1 + \cdots + n_k = N}$.

Then $$|h\rangle\rangle = \sum_N \sum_j |h, N : j\rangle \otimes \overline{|h, N ; j\rangle}$$

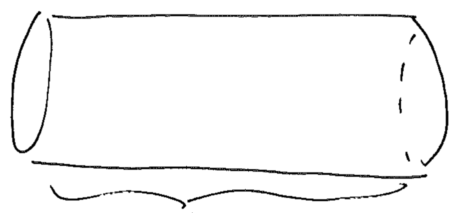
solves $L_n |h\rangle\rangle = \bar{L}_{-n} |h\rangle\rangle$

In general, $|B\rangle\rangle$ is a linear combination of $|h\rangle\rangle$ (Ishibashi states).

Candy condition.

Not all linear combinations of $|h\rangle\rangle$ represent consistent boundary conditions.

Suppose we have $|B_a\rangle\rangle$, $|B_b\rangle\rangle$

$\langle\langle B_a|$  $|B_b\rangle\rangle$

ℓ $\qquad \Uparrow \; e^{-\ell(L_0+\bar{L}_0)}$

One should be able to express this as $\mathrm{tr}\left(e^{-\frac{1}{\ell}H_{open}}\right)$ for CFT_2 on

the line segment $\underset{\ell}{\vdash\!\!-\!\!-\!\!\dashv}$ with boundary conditions a and b at the two ends.

In particular,

$\langle\langle B_a|\, e^{-\ell(L_0+\bar{L}_0)}\,|B_b\rangle\rangle$ should be

expanded by $e^{-\frac{\varepsilon}{\ell}}$ with <u>integer</u> coefficients

<u>Candy states</u> $\Leftarrow$ For the minimal model, one can construct such $|B\rangle\rangle$ as linear combinations of $|h\rangle\rangle$ and classify them.

D branes for SUSY sigma model on CY_n

$$\mathcal{L} = \frac{1}{2} g_{i\bar{j}} (\partial X^i \bar{\partial} X^{\bar{j}} + \partial X^{\bar{j}} \bar{\partial} X^i)$$

$$+ \frac{i}{2} \bar{\Psi}_L^{\bar{j}} \bar{D} \Psi_L^j g_{i\bar{j}} + \frac{i}{2} \bar{\Psi}_R^{\bar{j}} D \Psi_R^i g_{i\bar{j}} + \cdots$$

$$T = \frac{1}{2} g_{i\bar{j}} \partial X^i \partial X^{\bar{j}} + \text{fermions}$$

$$G_L^+ = g_{i\bar{j}} \Psi_L^i \partial X^{\bar{j}}, \quad G_L^- = g_{i\bar{j}} \Psi_L^{\bar{j}} \partial X^i$$

G_R^+, G_R^-

There are two types of D branes:

$$T = \bar{T}$$

A branes : $G_L^+ = \pm G_R^+$, $G_L^- = \pm G_R^-$

B branes : $G_L^+ = \pm G_R^-$, $G_L^- = \pm G_R^+$

- A brane = γ in CY_m such that
  - $\dim_{\mathbb{R}} \gamma = m$ ... Lagrangian
  - $k|_\gamma = 0$ $\quad k = i g_{i\bar{j}} dx^i \wedge dx^{\bar{j}}$

- B brane = γ in CY_m

  such that $\gamma \subset CY_n$

  is a holomorphic submanifold.

Large N duality

$F_{g;\, n_1 \cdots n_k}$: open string amplitudes

$$F_g(t_1, \ldots, t_k) = \sum_{n_1 \cdots n_k} F_{g;\, n_1 \cdots n_k} (t_1)^{n_1} \cdots (t_k)^{n_k}$$

Can we interpret $F_g(t)$ as

a closed string amplitude on genus g

for some CFT_2 with parameters $t_1 \cdots t_k$?

example 1 AdS / CFT correspondence.

$F_{g,n} \Leftarrow$ $N=4$ super Yang-Mills theory on $\mathbb{R}^4$ with gauge group $SU(N)$ $(t = g_{YM}^2 N)$

$F_g(t) \Leftarrow$ Type IIB superstring on $AdS_5 \times S^5$

$t \sim$ curvature radius of AdS_5 and S^5.

<u>example 2</u>

$F_{g,m} \Leftarrow$ Chern-Simons gauge theory on S^3
with gauge group $SU(N)$

$$S_{CS} = \frac{k}{4\pi} \int_{S^3} \mathrm{tr}\left(A dA + \frac{2}{3} A^3\right)$$

partition function

$$Z = \frac{e^{i\frac{\pi}{8} N(N-1)}}{(k+N)^{N/2}} \sqrt{\frac{k+N}{N}} \prod_{s=1}^{N-1} \left(2 \sin\left(\frac{s\pi}{k+N}\right)\right)^{N-s}$$

$$= \exp\left(-\sum_g \sum_m F_{g,m} \lambda^{2g-2} t^m\right)$$

$$\lambda = \frac{2\pi}{k+N}, \qquad t = \frac{2\pi i N}{k+N}$$

Then

$$F_0(t) = \frac{i}{12} t^3 + \sum_{m=1}^{\infty} m^{-3} e^{-mt}$$

$$F_1(t) = \frac{1}{24} t + \frac{1}{12} \log(1-e^{-t})$$

$$F_{g\geq 2}(t) = \frac{2 B_g \zeta(2g-2)}{(2\pi)^{2g-2} 2g(2g-2)} - \frac{(-1)^{g-1}}{2g(2g-2)!} B_g \sum_m m^{2g-3} e^{-mt}$$

Close topological string on the conifold