Lecture 15

1

ch 3: Markov chains

تابع Sec 3.2 — Transition prob. Matrices of a Markov chain

p.83

سقوط الأمطار

EX For modelling daily rainfall phenomena, it's found that Markov chain with two states (0 and 1) 0 for dry day and 1 for rainy day. Consider the prob. that tomorrow will be rainy given that today is rainy is 0.8 and the prob. that tomorrow will rainy given that today is dry is 0.4.

Find (i) the transition probs Mx

(ii) If $\{X_n\}$ be a Markov chain for the daily rainfall, calculate

$$pr\{X_n = 0 \mid X_0 = 0\} \text{ for } n = 0, 1, 2$$

Ans:

(i)

$$P = \begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{bmatrix} \end{array}$$

Tomorrow; Today: 0 جاف, 1 ممطر

is the trans. probs Mx

(ii) For $n = 0 \Rightarrow$

$$pr\{X_0 = \overset{j}{0} \mid X_0 = \overset{i}{0}\} = P^0_{00} = 1$$

$$P^0_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

وهنا يعني أنه في البداية كان الطقس جافاً

, For $n = 1 \Rightarrow P_r\{X_1 = \overset{j}{0} \mid X_0 = \overset{i}{0}\}$

$$= P^1_{00} = 0.6$$

وهنا يعني أنه بشرط أن الطقس اليوم يكون جافاً فإنه سيكون جافاً غداً باحتمال يساوي 0.6.

2/

* For $n = 2 \Rightarrow P_r\{X_2 = \overset{j}{0} \mid X_0 = \overset{i}{0}\} = P^{2}_{00}$

$$\therefore P^2 = \begin{bmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{bmatrix} \begin{bmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{bmatrix}$$

$$= \begin{array}{c} \\ 0 \\ 1 \end{array} \begin{array}{c} \begin{array}{cc} 0 & \quad 1 \end{array} \\ \begin{bmatrix} 0.44 & 0.56 \\ 0.28 & 0.72 \end{bmatrix} \end{array}$$

$$\therefore Pr\{X_2 = 0 \mid X_0 = 0\} = P^{2}_{00} = 0.44$$

وهذا يعني أنه بشرط أن الطقس اليوم يكون جافاً فإنه سيكون جافاً خلال اليومين القادمين وذلك باحتمال يساوي 0.44

H.W
what's the probability for the weather to be dry today and rainy on the coming two days?

EX Let X_n denote the quality of nth item produced by a production system with $X_n = 0$ meaning "good" and $X_n = 1$ meaning "defective".

P.86 Text book
pb 3.2.3

Suppose that X_n evolves (يتطور) as a Markov chain whose transition probability matrix is

$$P = \begin{array}{c} \\ 0 \\ 1 \end{array} \begin{array}{c} \begin{array}{cc} 0 & \quad 1 \end{array} \\ \begin{bmatrix} 0.99 & 0.01 \\ 0.12 & 0.88 \end{bmatrix} \end{array}$$

what's the prob. that the fourth item is defective given that the first item is defective?

المطلوب $pr\{X_3 = 1 \mid X_0 = 1\}$

X_0, X_1, X_2, X_3
1st ... 4th

3/ Ans:

$$P^2 = \begin{bmatrix} 0.99 & 0.01 \\ 0.12 & 0.88 \end{bmatrix} \begin{bmatrix} 0.99 & 0.01 \\ 0.12 & 0.88 \end{bmatrix}$$

$$= \begin{bmatrix} 0.9813 & 0.0187 \\ 0.2244 & 0.7756 \end{bmatrix}$$

$$P^3 = P \cdot P^2$$

$$= \begin{bmatrix} 0.99 & 0.01 \\ 0.12 & 0.88 \end{bmatrix} \begin{bmatrix} 0.9813 & 0.0187 \\ 0.2244 & 0.7756 \end{bmatrix}$$

$$= \begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 0.9737 & 0.0263 \\ 0.3152 & 0.6848 \end{bmatrix} \end{array}$$

$\therefore \Pr\{X_3 = 1 \mid X_0 = 1\} = P^3_{11} = 0.6848 = 68.48\%$

is the prob. that the fourth item is defective given that the first item is defective. #

H.W what is the prob. that the fourth item is defective given that the first item is good.

4 // pb 3.2.4 Text p. 85

Given

$$P = \begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & 0.6 & 0.3 & 0.1 \\ 1 & 0.3 & 0.3 & 0.4 \\ 2 & 0.4 & 0.1 & 0.5 \end{array}$$

$pr\{X_0=1\}=1 \rightarrow p_1=1$

Find $pr\{X_2=2\}$

Ans: $\because pr\{X_2=2\} = pr\{X_2=2 \mid X_0=1\} \cdot pr\{X_0=1\}$

$\therefore pr\{X_2=2\} = P^2_{12}\, p_1 = P^2_{12}, \; p_1=1$

, where $p_1 = pr\{X_0=1\} = 1$

$$P^2 = \begin{bmatrix} 0.6 & 0.3 & 0.1 \\ 0.3 & 0.3 & 0.4 \\ 0.4 & 0.1 & 0.5 \end{bmatrix} \begin{bmatrix} 0.6 & 0.3 & 0.1 \\ 0.3 & 0.3 & 0.4 \\ 0.4 & 0.1 & 0.5 \end{bmatrix}$$

$$P^2 = \begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & 0.49 & 0.28 & 0.23 \\ 1 & 0.43 & 0.22 & 0.35 \\ 2 & 0.47 & 0.2 & 0.33 \end{array}$$

$\Rightarrow$

$\therefore Pr\{X_2=2\} = P^2_{12} = 0.35$