

1

ch 4

75

The long run behavior of Markov chains

Lecture (22)

\* Interpretation of the limiting dist<sup>تفسير</sup>  $\pi$

$$\pi = (\pi_0, \pi_1, \pi_2, \dots, \pi_N)$$

•  $\pi_j$

$j = 0, 1, 2, \dots, N$

It is the prob. of finding the process in state  $j$  irrespective of the starting state.  
بغض النظر

OR

It is the long run mean fraction of time that the process  $\{X_n\}$  is in state  $j$ .

• The long run mean cost per unit time

$$= \sum_{j=0}^N \pi_j c_j$$

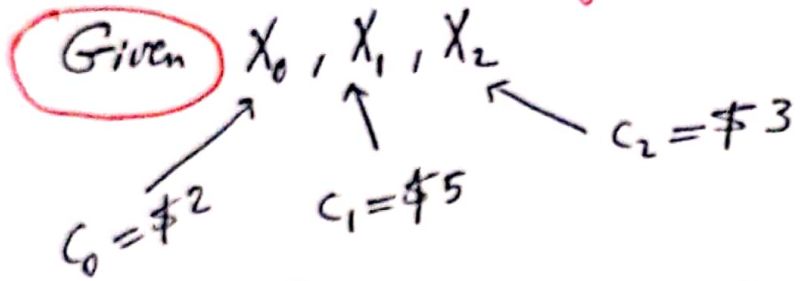
where  $c_j$  is the cost of the visit to state  $j$

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pb 4.1.4 p. 173

$$P = \begin{matrix} & \begin{matrix} 0 & 1 & 2 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \end{matrix} & \begin{bmatrix} 0.3 & 0.2 & 0.5 \\ 0.5 & 0.1 & 0.4 \\ 0.5 & 0.2 & 0.3 \end{bmatrix} \end{matrix}$$

$\downarrow$   $\pi_0$       $\downarrow$   $\pi_1$       $\downarrow$   $\pi_2$



Find the long run cost per unit period ??

Ans:

\* The long run mean cost per unit period

$$= \sum_{j=0}^2 \pi_j c_j$$

$$= \pi_0 c_0 + \pi_1 c_1 + \pi_2 c_2$$

$$= 2\pi_0 + 5\pi_1 + 3\pi_2$$

(I)

$$\pi_j = \sum_{k=0}^2 \pi_k P_{kj}$$

at  $j=0 \Rightarrow \pi_0 = 0.3\pi_0 + 0.5\pi_1 + 0.5\pi_2$  (1)

at  $j=1 \Rightarrow \pi_1 = 0.2\pi_0 + 0.1\pi_1 + 0.2\pi_2$  (2)

$\therefore \pi_0 + \pi_1 + \pi_2 = 1$  (3)

(1), (2) and (3)  $\Rightarrow$

$$\begin{aligned} 7\pi_0 - 5\pi_1 - 5\pi_2 &= 0 & (1) \\ 2\pi_0 - 9\pi_1 + 2\pi_2 &= 0 & (2) \\ \pi_0 + \pi_1 + \pi_2 &= 1 & (3) \end{aligned}$$

3 Solving (1), (2) and (3) using Cramer rule

$$\pi_0 = \frac{\Delta_0}{\Delta}, \pi_1 = \frac{\Delta_1}{\Delta}, \pi_2 = \frac{\Delta_2}{\Delta}$$

$$\Delta = \begin{vmatrix} 7 & -5 & -5 \\ 2 & -9 & 2 \\ 1 & 1 & 1 \end{vmatrix} = -55 - 24 + (-53) = -132$$

$$\Delta_0 = \begin{vmatrix} 0 & -5 & -5 \\ 0 & -9 & 2 \\ 1 & 1 & 1 \end{vmatrix} = -55$$

$$\Delta_1 = \begin{vmatrix} 7 & 0 & -5 \\ 2 & 0 & 2 \\ 1 & 1 & 1 \end{vmatrix} = -24$$

$$\Delta_2 = \begin{vmatrix} 7 & -5 & 0 \\ 2 & -9 & 0 \\ 1 & 1 & 1 \end{vmatrix} = -53$$

$$\therefore \pi_0 = \frac{-55}{-132} = \frac{5}{12}, \pi_1 = \frac{-24}{-132} = \frac{2}{11}, \pi_2 = \frac{-53}{-132} = \frac{53}{132}$$

Subst. (II) in (I)

The mean cost per unit period

$$= 2\left(\frac{5}{12}\right) + 5\left(\frac{2}{11}\right) + 3\left(\frac{53}{132}\right)$$

$$= \frac{389}{132} \approx \$2.95$$