1

Ch 4

ناج

Lecture (22)

The long run behavior of Markov chains

تفسير

* Interpretation of the limitting dist $\underline{n}$

$$\Pi = (\Pi_0, \Pi_1, \Pi_2, \dots, \Pi_N)$$

- Π_j , $j = 0, 1, 2, \dots, N$
  - It is the prob. of finding the process in state j بغض النظر irrespective of the starting state.
  - **OR**
  - It is the long run (mean) fraction of (time) that the process $\{X_n\}$ is in state j.

- The long run mean cost per unit time

$$= \sum_{j=0}^{N} \Pi_j \, c_j$$

where c_j is the cost of the visit to state j

pb 4.1.4 p. 173

$$P = \begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & 0.3 & 0.2 & 0.5 \\ 1 & 0.5 & 0.1 & 0.4 \\ 2 & 0.5 & 0.2 & 0.3 \end{array}$$

$\downarrow \pi_0 \quad \downarrow \pi_1 \quad \downarrow \pi_2$

Given X_0, X_1, X_2

$c_0 = \$2$, $c_1 = \$5$, $c_2 = \$3$

Find the long run cost per unit period ??

Ans:

* The long run mean cost per unit period

$$= \sum_{j=0}^{2} \pi_j c_j$$

$$= \pi_0 c_0 + \pi_1 c_1 + \pi_2 c_2 \qquad (I)$$

$$= 2\pi_0 + 5\pi_1 + 3\pi_2$$

$$, \pi_j = \sum_{k=0}^{2} \pi_k P_{kj}$$

at $j=0 \Rightarrow \pi_0 = 0.3\pi_0 + 0.5\pi_1 + 0.5\pi_2$ ①

at $j=1 \Rightarrow \pi_1 = 0.2\pi_0 + 0.1\pi_1 + 0.2\pi_2$ ②

$\therefore \pi_0 + \pi_1 + \pi_2 = 1$ ③

①, ② and ③ $\Rightarrow$

$7\pi_0 - 5\pi_1 - 5\pi_2 = 0$ ①

$2\pi_0 - 9\pi_1 + 2\pi_2 = 0$ ②

$\pi_0 + \pi_1 + \pi_2 = 1$ ③

3 Solving ①, ② and ③ using Cramer rule

$$\pi_0 = \frac{\Delta_0}{\Delta}, \quad \pi_1 = \frac{\Delta_1}{\Delta}, \quad \pi_2 = \frac{\Delta_2}{\Delta}$$

$$\Delta = \begin{vmatrix} 7 & -5 & -5 \\ 2 & -9 & 2 \\ 1 & 1 & 1 \end{vmatrix} = -55 - 24 + (-53) = -132$$

$+ \quad - \quad +$

$$\Delta_0 = \begin{vmatrix} 0 & -5 & -5 \\ 0 & -9 & 2 \\ 1 & 1 & 1 \end{vmatrix} = -55$$

$$\Delta_1 = \begin{vmatrix} 7 & 0 & -5 \\ 2 & 0 & 2 \\ 1 & 1 & 1 \end{vmatrix} = -24$$

$$\Delta_2 = \begin{vmatrix} 7 & -5 & 0 \\ 2 & -9 & 0 \\ 1 & 1 & 1 \end{vmatrix} = -53$$

$$\therefore \pi_0 = \frac{-55}{-132} = \frac{5}{12}, \quad \pi_1 = \frac{-24}{-132} = \frac{2}{11}, \quad \pi_2 = \frac{-53}{-132} = \frac{53}{132} \quad (II)$$

Subs. (II) in (I)

the mean Cost per unit period

$$= 2\left(\frac{5}{12}\right) + 5\left(\frac{2}{11}\right) + 3\left(\frac{53}{132}\right)$$

$$= \frac{389}{132} \simeq \$2.95$$