• Reliability and Redundancy p.179

Lecture (24)

Sec 4.2.2

State: (x, y) ← { 1 a day is ended without repairing; 0 otherwise

x = # of Computers in operating at the end of a day

, the prob. of Computer that break down is p

Form Markov prob. Mx

From state ↓ \ To state →	(2,0)	(1,0)	(1,1)	(0,1)
(2,0)	q	p	0	0
(1,0)	0	0	q	p
(1,1)	q	p	0	0
(0,1)	0	1	0	0
	↓ π_0	↓ π_1	↓ π_2	↓ π_3

π_3 ← is the prob. that neither Computer is operating

The limitting distn $= (\pi_0, \pi_1, \pi_2, \pi_3)$

2

العمود الأول $\Rightarrow q\pi_0 + q\pi_2 = \pi_0$

$$q\pi_2 = (1-q)\pi_0$$

$$q\pi_2 = p\pi_0 \Rightarrow \boxed{\pi_2 = \frac{p}{q}\pi_0} \quad (1)$$

العمود الثاني $\Rightarrow p\pi_0 + p\pi_2 + \pi_3 = \pi_1$

العمود الثالث $\Rightarrow q\pi_1 = \pi_2 \Rightarrow \pi_1 = \frac{1}{q}\pi_2$

$$(1) \Rightarrow \pi_1 = \frac{1}{q}\left(\frac{p}{q}\right)\pi_0$$

$$\therefore \boxed{\pi_1 = \frac{p}{q^2}\pi_0} \quad (2)$$

العمود الرابع $\Rightarrow p\pi_1 = \pi_3 \overset{(2)}{\Rightarrow} \pi_3 = p\left(\frac{p}{q^2}\right)\pi_0$

$$\therefore \boxed{\pi_3 = \frac{p^2}{q^2}\pi_0} \quad (3)$$

$$\because \boxed{\pi_0 + \pi_1 + \pi_2 + \pi_3 = 1} \quad (4)$$

Substitute (1), (2), (3) في (4)

$$\Rightarrow \pi_0 + \frac{p}{q^2}\pi_0 + \frac{p}{q}\pi_0 + \frac{p^2}{q^2}\pi_0 = 1$$

$$\therefore \left(1 + \frac{p}{q^2} + \frac{p}{q} + \frac{p^2}{q^2}\right)\pi_0 = 1$$

$$\therefore \left(\frac{q^2+p+pq+p^2}{q^2}\right)\pi_0 = 1$$

$$\therefore \left[\frac{q(q+1+p)+p+p^2}{q^2}\right]\pi_0 = 1$$

$$\left(\frac{q+p+p^2}{q^2}\right)\pi_0 = 1 \quad \therefore \left(\frac{1+p^2}{q^2}\right)\pi_0 = 1$$

$$\therefore \boxed{\pi_0 = \frac{q^2}{1+p^2}} \quad (5)$$

3.

Subs. ⑤ in ②, ①, ③

$$\textcircled{2}\Rightarrow \therefore \pi_1 = \frac{p}{q^2}\left(\frac{q^2}{1+p^2}\right) = \frac{p}{1+p^2}$$

$$\textcircled{1}\Rightarrow \pi_2 = \frac{p}{q}\left(\frac{q^2}{1+p^2}\right) = \frac{pq}{1+p^2}$$

$$\textcircled{3}\Rightarrow \pi_3 = \frac{p^2}{q^2}\left(\frac{q^2}{1+p^2}\right) = \frac{p^2}{1+p^2}$$

$$\therefore \pi = \left(\frac{q^2}{1+p^2} \;/\; \frac{p}{1+p^2} \;/\; \frac{pq}{1+p^2} \;/\; \frac{p^2}{1+p^2}\right)$$

π_0 ← π_1 ← π_2 ← π_3

* For long run π_3 means the prob. that neither Computer is operating

$$\text{Availability, } R_1 = 1 - \pi_3 = 1 - \frac{p^2}{1+p^2}$$

$$\therefore R_1 = \frac{1+p^2-p^2}{1+p^2} = \frac{1}{1+p^2}$$

* (Note) Availability is the prob. that at least one Computer is operating.

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4//

To increase system Availability, add a duplicate repair facility, so that both Computers can be repaired simultaneously

الحالة في نهاية اليوم	(2,0)	(1,0)	(1,1)	(0,1)
(2,0)	q	p	0	0
(1,0)	0	0	q	p
(1,1)	q	p	0	0
(0,1)	0	0	1	0
	π_0	π_1	π_2	π_3

Determine
1. The limiting dist^n ($\pi_0, \pi_1, \pi_2, \pi_3$)
2. The availability, P_2

H.W

* See p.181 Textbook.

⊕ H.w solve pb 4.2.4 p.189