Lecture 25

Ch 5: Poisson Processes

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* poisson dist$\underline{n}$

$$P_k = \frac{e^{-\mu}\mu^k}{k!} \quad \text{for } k = 0, 1, \dots$$

with parameter $\mu > 0$

If $X \sim \text{poisson}(\mu)$

$$\Rightarrow \underset{\text{mean}}{E(X) = \mu}, \quad \underset{\text{Variance}}{\sigma_X^2 = \mu}$$

* Poisson process

Counts # "arrivals"

e.g.s (1) Telephone calls, emails, defects, typos, ...

(2) people arrive a certain mole

(3) patients visit a hospital / clinic

(4) people waitting in a queue for a certain service

, ...

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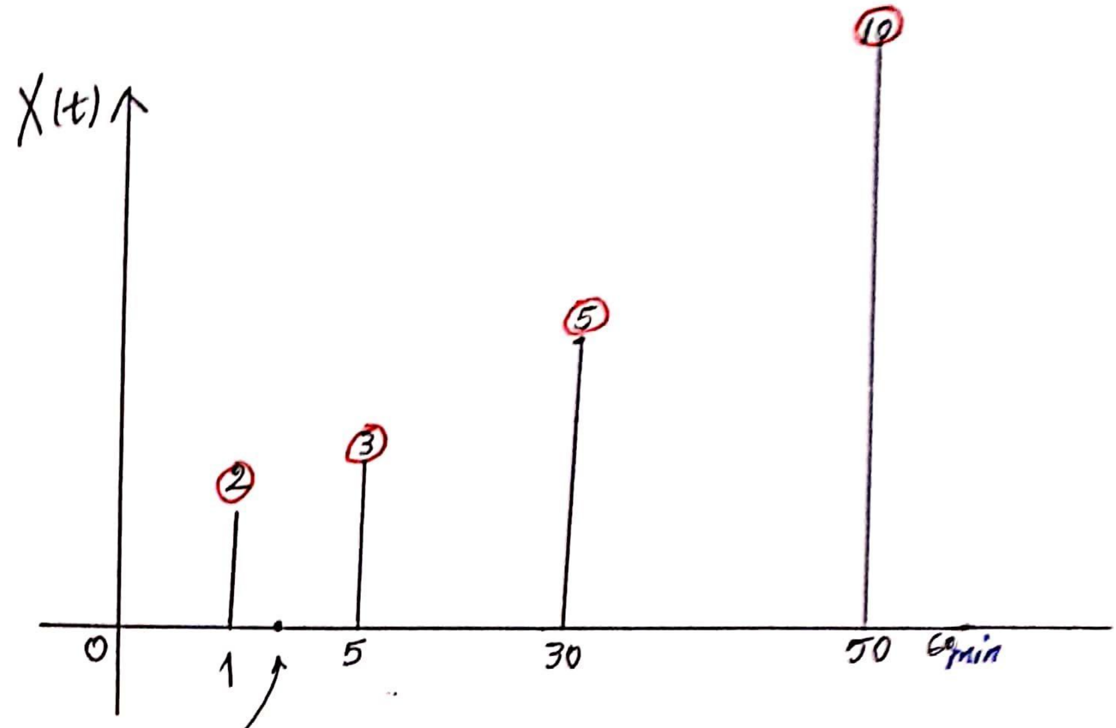
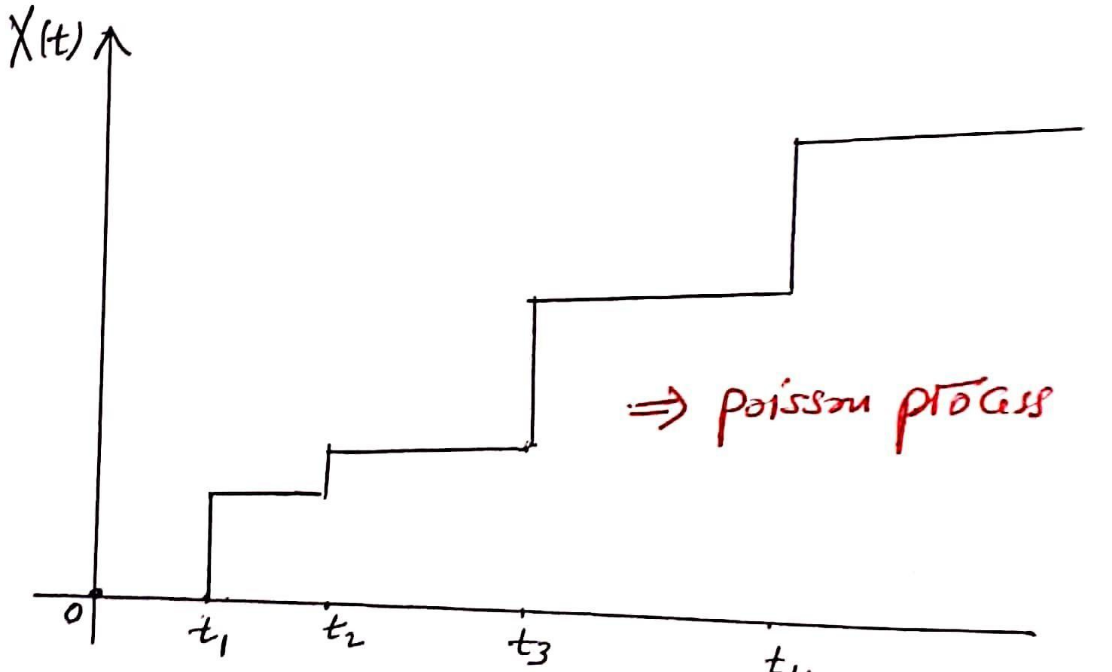


arrivals = 20 during 60 min



Continuous time t and discrete valued $X(t)$

Def$^{\underline{n}}$ poisson process

A poisson process of rate $\lambda > 0$ is an integer-valued stochastic process

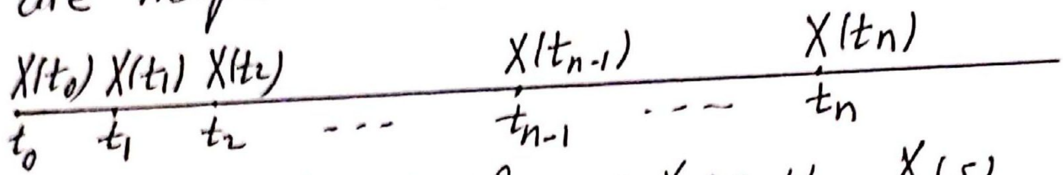
$$\{X(t) : t \geq 0\} \text{ for which}$$

① for any time points $t_0 = 0 < t_1 < t_2 < \dots < t_n$ (orderd values)

the process increments

$X(t_1) - X(t_0),\ X(t_2) - X(t_1), \dots,\ X(t_n) - X(t_{n-1})$

are independent r.v's



② for $s \geq 0$ and $t > 0$ the r.v $X(s+t) - X(s)$ has poisson dist$\underline{n}$, its mass prob. f$\underline{n}$ is

$$pr\{X(s+t) - X(s) = k\} = \frac{(\lambda t)^k e^{-\lambda t}}{k!}, \quad k = 0, 1, 2, \dots$$

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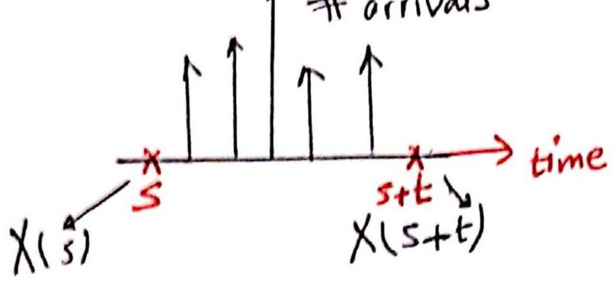

③ $X(0) = 0$

* In particular, If $X(t)$ is a poisson process of rate $\lambda > 0$ then the moments are $E[X(t)] = \lambda t$ and

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$$Var[X(t)] = \sigma^2_{X(t)} = \lambda t.$$

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$\lambda = 0.1$, $X(t)$ is the # of defects

For poisson process

$$pr\{X(s+t) - X(s) = k\} = \frac{(\lambda t)^k e^{-\lambda t}}{k!} \quad , k = 0, 1, 2, \ldots$$

s $\quad s+t$

a) $pr\{X(2) = 0\}$

$$= pr\{X(\underset{s+t}{2}) - X(\underset{s}{0}) = \underset{k}{0}\}$$

$$= \frac{(\lambda t)^k e^{-\lambda t}}{k!} \quad , \lambda t = 0.1(2) = 0.2$$

$$= \frac{(0.2)^0 e^{-0.2}}{0!} = e^{-0.2} \simeq \boxed{0.8187}$$

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0 $\quad$ 2 $\quad$ 3

$X(0)$ $\quad X(2)$ $\quad X(3)$

b) $pr\{X(\underset{s+t}{3}) - X(\underset{s}{2}) = \underset{k}{0}\}$ $\quad \lambda t = 0.1(1) = 0.1$

$$= \frac{(\lambda t)^k e^{-\lambda t}}{k!} = \frac{(0.1)^0 e^{-0.1}}{0!} = e^{-0.1}$$

$$\simeq \boxed{0.9048}$$

Note that: $X(2) - X(0)$ and $X(3) - X(2)$ are 2 indep. r.v's

Note: $pr(A|B) = pr(A)$ when A, B are independent