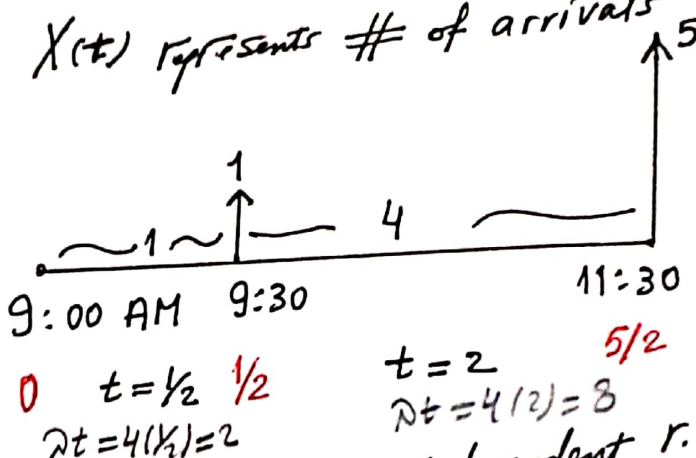
1 //

Poisson process تابع

Ex (2) p.226

In poisson process, $X(t)$ represents # of arrivals

معدل عملية بواسطة $\boxed{\lambda = 4}$ rate per hour



Note that:
$X(1/2) - X(0)$, $X(5/2) - X(1/2)$ are 2 independent r.v.s

$pr\{X(1/2) = 1, X(5/2) = 5\}$?? مطلوب

Ans:
$$pr\{X(1/2) = 1, X(5/2) = 5\}$$
$$= pr\{\underbrace{X(1/2) - X(0) = 1}, \underbrace{X(5/2) - X(1/2) = 4}\}$$

2 indep. r.v.s

$$= \frac{[4(1/2)]^1 e^{-2}}{1!} \cdot \frac{[4(2)]^4 e^{-8}}{4!}$$
$$= 2e^{-2}\left(\frac{512}{3} e^{-8}\right)$$
$$\simeq \boxed{0.015}$$

$p(x, y) = p(x)\,p(y)$ for indep. r.v.s X and Y

For poisson process
$$pr\{X(s+t) - X(s) = k\} = \frac{(\lambda t)^k e^{-\lambda t}}{k!}, \quad k = 0, 1, 2, \ldots$$

- For non-homogeneous Poisson process

If $\lambda = \lambda(t)$

then $X(s+t) - X(s) \sim \text{poisson}(\mu)$

where $\mu = \int_{s}^{t} \lambda(u)\,du$

Ex p.227

$X(t)$ represents demands عدد الطلبات على موقع الإسعاف في منطقة ما (الإسعافات الأولية)

معدل عملية بواسون $\lambda(t) = \begin{cases} 2t, & 0 \le t < 1 \\ 2, & 1 \le t < 2 \\ 4-t, & 2 \le t \le 4 \end{cases}$

0	μ_1	2	μ_2	4
$X(0)$		$X(2)$		$X(4)$

a) $pr\{X(2) = 2\}$

prob. that two demands occur in the first 2h of operation

b) $pr\{X(4) - \overset{\text{المطلوب}}{\overline{\overline{X(2)}}} = 2\}$

prob. that two demands occur in the second 2h

3

a) $$\mu_1 = \int_0^2 \lambda(u)\,du$$

$$= \int_0^1 2t\,dt + \int_1^2 2\,dt$$

$$= \left[\frac{\cancel{2}t^2}{\cancel{2}}\right]_0^1 + 2\left[t\right]_1^2$$

$$= 1 + 2(2-1) = 1 + 2 = 3$$

$$pr\{X(2) = 2\}$$

$$= pr\{X(2) - X(0) = 2\}$$

$$= \frac{e^{-\mu_1}\mu_1^k}{k!} = \frac{e^{-3}\cdot 3^2}{2!} = \boxed{0.2240}$$

b) $$\mu_2 = \int_2^4 \lambda(u)\,du = \int_2^4 (4-t)\,dt$$

$$\mu_2 = \left[4t - \frac{t^2}{2}\right]_2^4 = 8 - 6 = 2$$

$$\therefore\ pr\{X(4) - X(2) = 2\} = \frac{e^{-\mu_2}\mu_2^k}{k!}$$

$$= \frac{e^{-2}\cdot 2^2}{2!}$$

$$= 0.2707$$

Hw pb 5.1.1 p. 228