

Defn

The transition from state n to state $n-1$ is called death.

Defn

The death process starts from state N and moves successively until to reach state 1

$N, N-1, N-2, \dots, 2, 1$

and then it's absorbed in state 0.

Note that pure birth process increasing through states $0, 1, 2, \dots, n$

but pure death process decreasing through states $N, N-1, N-2, \dots, 2, 1$

* postulates for death process

- ① $pr \{X(t+h) = k-1 | X(t) = k\} = \mu_k h + o(h)$
 is the prob. of only one death at $(t, t+h)$
 where μ_k is the death rate, $k=1, 2, \dots, N$
زمن واحد وفاة في تلك الفترة
- ② $pr \{X(t+h) = k | X(t) = k\} = 1 - \mu_k h + o(h)$
 is the prob. of no death at $(t, t+h)$, $k=1, 2, \dots, N$
زمن واحد لم يوفى
- ③ $pr \{X(t+h) > k | X(t) = k\} = 0$, $k=0, 1, 2, \dots, N$
 is the prob. of more than one death at $(t, t+h)$.
زمن واحد وفاة أكثر من شخص في الفترة $(t, t+h)$ صفر

2/ The transition prob. $M \times P = [P_{ij}]$ for death process is given by

$$P = \begin{matrix} & \begin{matrix} 0 & 1 & 2 & \dots & N-1 & N \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ \vdots \\ N-1 \\ N \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & \dots & 0 & 0 \\ \mu_1 h & 1 - \mu_1 h & 0 & \dots & 0 & 0 \\ 0 & \mu_2 h & 1 - \mu_2 h & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 - \mu_{N-1} h & 0 \\ 0 & 0 & 0 & \dots & \mu_N h & 1 - \mu_N h \end{bmatrix} \end{matrix}$$

where $\mu_k > 0$ for $k=1, 2, \dots, N$ are the death parameters.

* Continuous Markov chains for death process

For pure death process

$$P_N(t) = e^{-\mu_N t}, \quad n = N$$

and for $n < N$

$$P_n(t) = \text{pr} \{ X(t) = n \mid X(0) = N \}$$

$$= \mu_{n+1} \mu_{n+2} \dots \mu_N \left[A_{n,n} e^{-\mu_n t} + \dots + A_{N,n} e^{-\mu_N t} \right]$$

where

$$A_{k,n} = \frac{1}{(\mu_N - \mu_k) \dots (\mu_{k+1} - \mu_k) (\mu_{k-1} - \mu_k) \dots (\mu_n - \mu_k)}$$