# Lecture ④ Some Continuous probability distributions

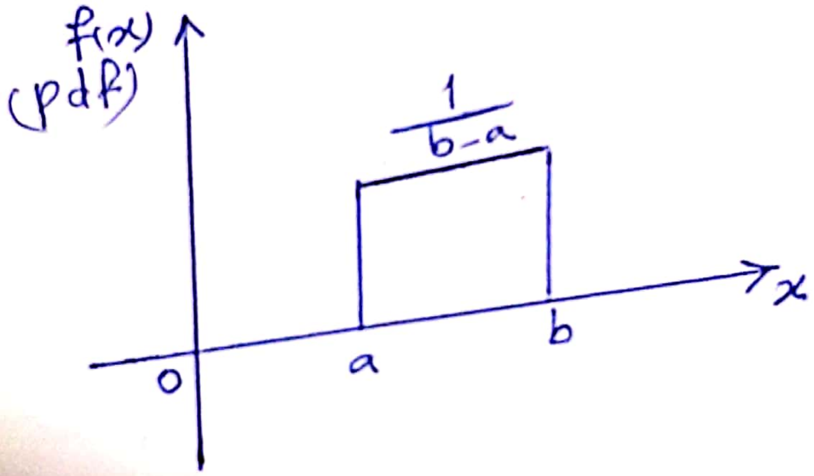
## [I] Uniform distn توزيع منتظم

$X \sim$ uniform $(a, b)$

If its pdf is

$$f(x|a,b) = \begin{cases} \dfrac{1}{b-a}, & a \leq x \leq b \\ 0, & \text{elsewhere} \end{cases}$$

prob. density fn




and CDF is

$$F(x) = \int_a^x \left(\frac{1}{b-a}\right) dx = \frac{1}{b-a}(x-a)$$

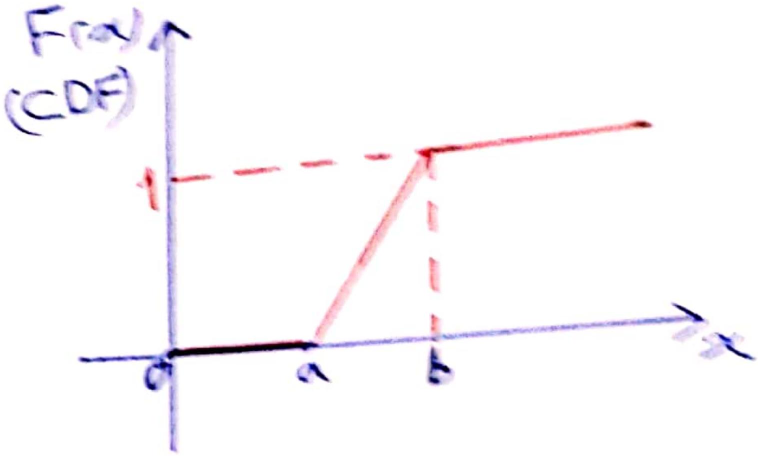
$$\therefore F(x|a,b) = \frac{x-a}{b-a}$$

i.e

$$F(x|a,b) = \begin{cases} 0, & x < a \\ \dfrac{x-a}{b-a}, & a \leq x \leq b \\ 1, & x > b \end{cases}$$

Cumulative distn fn

3)




Props

* $\boxed{1}$ mean $= \mu = E(X)$

$$= \int_a^b x\, f(x)\, dx \quad , \quad f(x) = \frac{1}{b-a}$$

$$= \frac{1}{b-a} \int_a^b x\, dx = \frac{1}{b-a} \left[ \frac{x^2}{2} \right]_a^b$$

$$= \frac{1}{2} \left( \frac{1}{b-a} \right) (b^2 - a^2)$$

$$\therefore \mu = \frac{1}{2}(a+b)$$ المتوسط

$\boxed{2}$ Varianc $= \sigma^2$

$$= E(X^2) - \mu^2$$

$$E(X^2) = \int_a^b \frac{x^2}{b-a}\, dx = \frac{1}{b-a} \left[ \frac{x^3}{3} \right]_a^b$$

$$= \frac{1}{3(b-a)} (b^3 - a^3)$$

$$\therefore E(X^2) = \frac{1}{3}(a^2 + ab + b^2)$$

$$\therefore \sigma^2 = \frac{1}{3}(a^2 + ab + b^2) - \left( \frac{a+b}{2} \right)^2$$

$$\therefore \sigma^2 = \frac{1}{12}(b-a)^2 , \quad b > a$$ والتباين

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$\boxed{3}$ Moment generating fn الدالة مولدة للعزوم

$$M_X(t) = E(e^{tX}), \quad t \in \mathbb{R}$$

$$M_X(t) = \int_a^b e^{tx} \cdot \left(\frac{1}{b-a}\right) dx$$

$$M_X(t) = \frac{1}{b-a} \int_a^b e^{tx}\, dx$$

$$= \frac{1}{b-a} \left[\frac{e^{tx}}{t}\right]_a^b$$

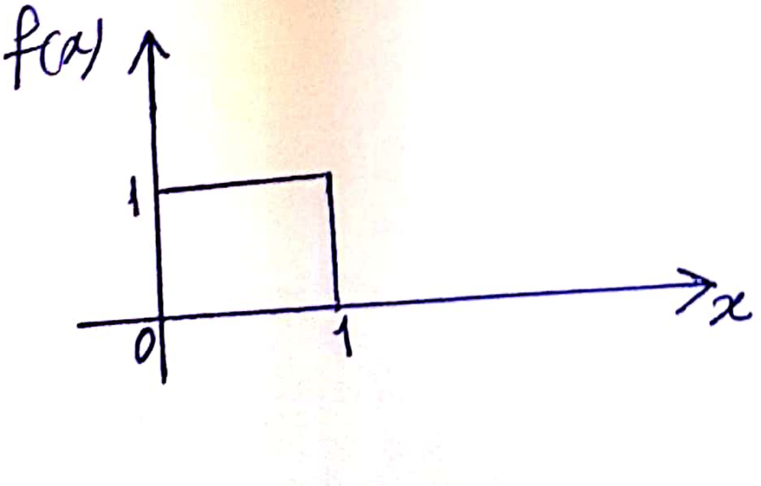
$$= \frac{1}{b-a} \left[\frac{e^{bt} - e^{at}}{t}\right]$$

$$\therefore M_X(t) = \frac{1}{t(b-a)} \left(e^{bt} - e^{at}\right), \quad t \neq 0$$

Note that

If $X \sim$ uniform $(a, b)$ then

$$Y = \frac{X - a}{b - a} \sim \text{Uniform } (0, 1)$$



## II Exponential distn

التوزيع الأسي

$$f(x|\lambda) = \begin{cases} \lambda e^{-\lambda x} & , x \geq 0 \\ 0 & , \text{else} \end{cases}$$

pdf

$$F(x|\lambda) = \begin{cases} 1 - e^{-\lambda x} & , x \geq 0 \\ 0 & , \text{else} \end{cases}$$

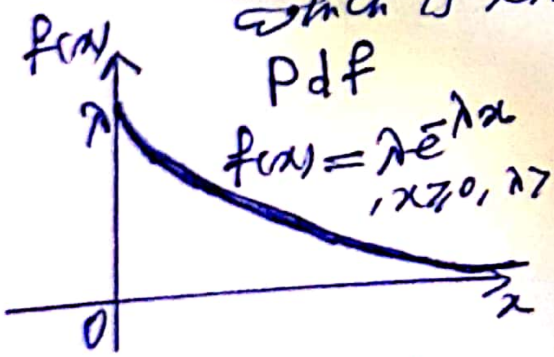
CDF

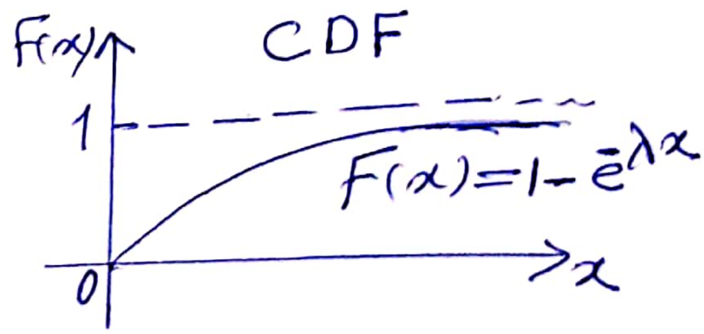
and the Reliability fn is $R(x|\lambda) = e^{-\lambda x}, x \geq 0$

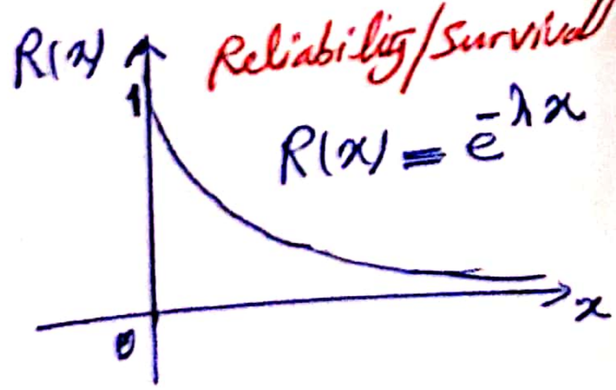
Not that: $R(x) = pr(X > x)$

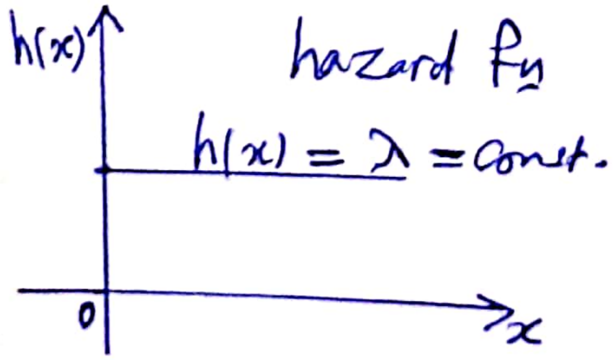
and $h(x|\lambda) = \lambda$ is the hazard fn (دالة الخطورة)

which is known also as failure rate.



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$r^{th}$ Moment for $X$ about origin

$$E(X^r) = \int_0^\infty x^r \lambda e^{-\lambda x} dx$$

let $u = \lambda x \Rightarrow \lambda dx = du$

$\Rightarrow x = \left(\frac{u}{\lambda}\right), \quad dx = \frac{du}{\lambda}$

when $x: 0 \to \infty \Rightarrow u: 0 \to \infty$

$$E(X^r) = \cancel{\lambda} \int_0^\infty \left(\frac{u}{\lambda}\right)^r e^{-u} \frac{du}{\cancel{\lambda}}$$

$$E(X^r) = \frac{1}{\lambda^r} \int_0^\infty u^r e^{-u} du$$

Gamma fn

remember

$$\int_0^\infty x^n e^{-x} dx = \Gamma(n+1)$$

$$\therefore E(X^r) = \frac{\Gamma(r+1)}{\lambda^r}$$

$$\therefore E(X^r) = \frac{r!}{\lambda^r}, \quad r = 0, 1, 2, \ldots$$

at $r = 1 \Rightarrow \mu = E(X) = \frac{1!}{\lambda} = \boxed{\frac{1}{\lambda}}$

at $r = 2 \Rightarrow E(X^2) = \frac{2!}{\lambda^2} = \frac{2}{\lambda^2}$

$\therefore$ التباين Variance

$$\sigma^2 = E(X^2) - \mu^2 = \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \boxed{\frac{1}{\lambda^2}}, \quad \sigma = \boxed{\frac{1}{\lambda}}$$

$\sigma$ [illegible] Standard deviation

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Moment generating fn for exp. distn (الدالة المولدة للعزوم)

$$M_X(t) = E(e^{tX}), \quad t \in \mathbb{R}$$

$$= \int_{-\infty}^{\infty} e^{tx} f_X(x)\, dx$$

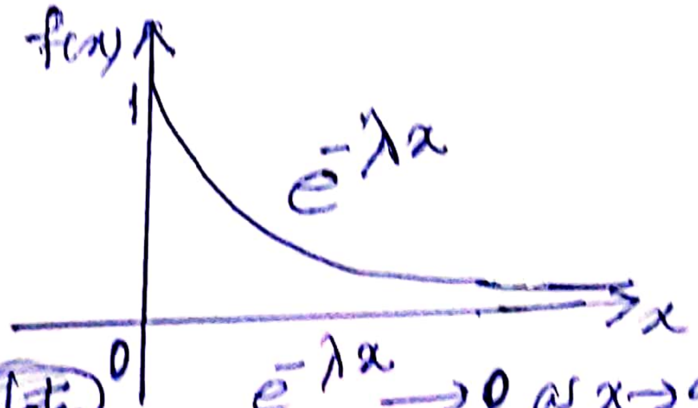
$$= \int_{0}^{\infty} e^{tx} \lambda e^{-\lambda x}\, dx$$

$$= \lambda \int_{0}^{\infty} e^{-(\lambda - t)x}\, dx \quad , t < \lambda$$

$$= \lambda \left[ \frac{e^{-(\lambda-t)x}}{-(\lambda - t)} \right]_0^{\infty}$$

$$\therefore M_X(t) = \frac{\lambda}{\lambda - t} \quad , t < \lambda \qquad //$$

where $\lim_{x \to \infty} e^{-(\lambda - t)x} = 0$




Note: $e^{-\lambda x} \to 0$ as $x \to \infty$

$$\Rightarrow \lim_{x \to \infty} e^{-\lambda x} = 0$$