

CHAPTER 29

MAGNETIC FIELD

29.1 MAGNETIC FIELDS AND FORCES

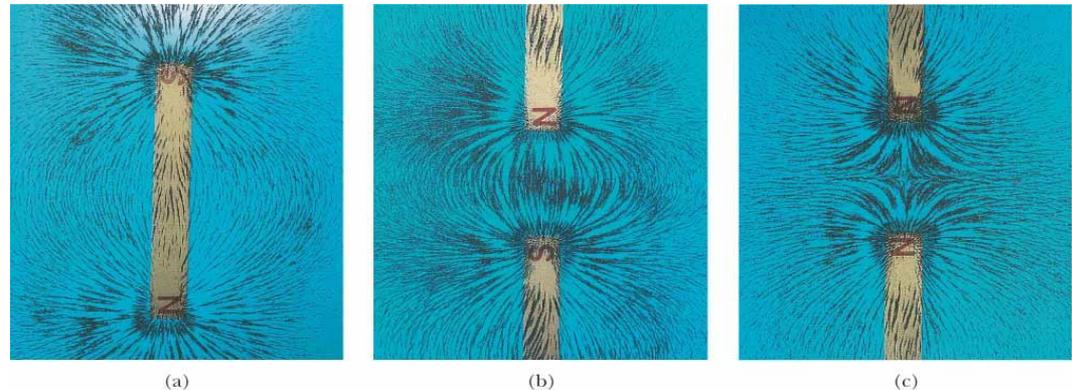
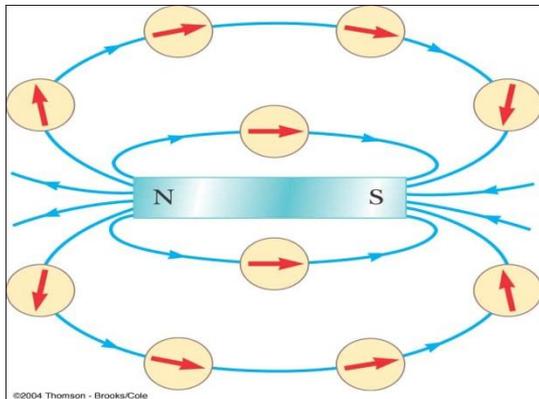
29.2 MAGNETIC FORCE ACTING ON A CURRENT-CARRYING CONDUCTOR

29.4 MOTION OF A CHARGED PARTICLE IN A UNIFORM MAGNETIC FIELD

29.5 APPLICATIONS INVOLVING CHARGED PARTICLES MOVING IN A MAGNETIC FIELD

29.1 MAGNETIC FIELDS AND FORCES

- The direction of the magnetic field \mathbf{B} at any location is the direction in which a compass needle points at that location.
- the magnetic field lines outside the magnet point away from north poles and toward south poles.

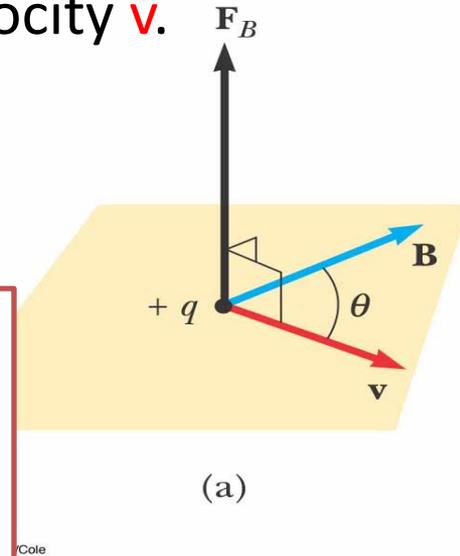


Active Figure 29.1 Compass needles can be used to trace the magnetic field lines in the region outside a bar magnet

Active Figure 29.1 Compass needles can be used to trace the magnetic field lines in the region outside a bar magnet

- We can define a magnetic field \mathbf{B} at some point in space in terms of the magnetic force \mathbf{F}_B that the field exerts on a test object, for which we use a charged particle moving with a velocity \mathbf{v} .
- ❖ assuming that no electric (\mathbf{E}) or gravitational (\mathbf{g}) fields are present at the location of the test object.

Figure 29.3 The direction of the magnetic force \mathbf{F}_B acting on a charged particle moving with a velocity \mathbf{v} in the presence of a magnetic field \mathbf{B} . (a) The magnetic force is perpendicular to both \mathbf{v} and \mathbf{B} . *Henry Leap and Jim Lehman*



The formula:

$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

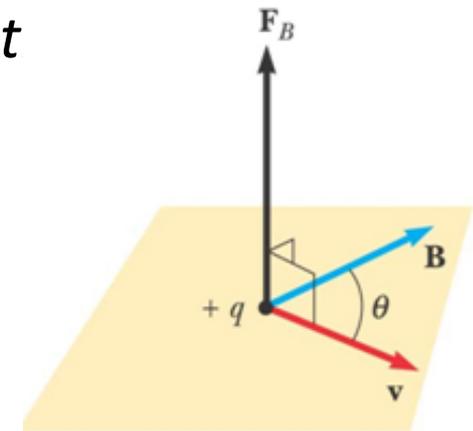
q is the charge

$\vec{\mathbf{v}}$ is velocity of the charge

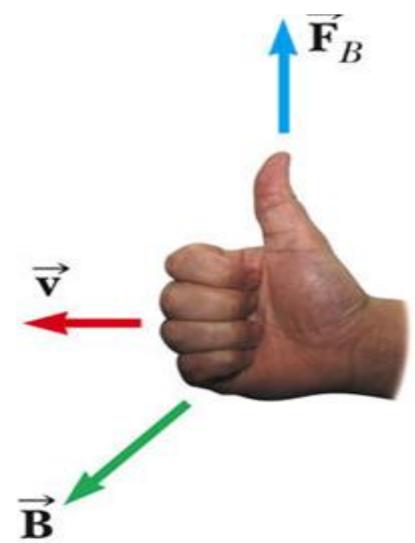
$\vec{\mathbf{F}}_B$ is the magnetic force

$\vec{\mathbf{B}}$ is the magnetic field

- The magnitude F_B of the magnetic force exerted on the particle is proportional to the charge q and to the speed v of t
- The magnitude and direction of F_B depend on the velocity of the particle V and on the magnitude and direction of the magnetic field B .
- When a charged particle moves parallel to the magnetic field vector (i.e., $\theta = 0$), the magnetic force acting on the particle is zero.
- When the particle's velocity vector makes any angle with the magnetic field, the magnetic force acts in a direction perpendicular to both v and B ; F_B is perpendicular to the plane formed by v and B



(a)

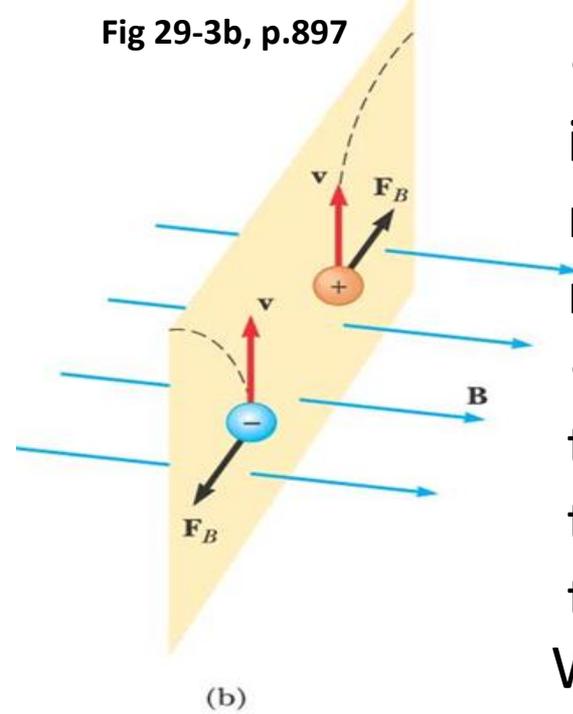


The magnitude:

For a positive charge

$$F_B = qvB \sin \theta$$

or $F_B = qvB$ When the velocity and the field are perpendicular to each other.



- The magnetic force exerted on a positive charge is in the direction opposite the direction of the magnetic force exerted on a negative charge moving in the same direction.
- The magnitude of the magnetic force exerted on the moving particle is proportional to $\sin \theta$, where θ is the angle the particle's velocity vector makes with the direction of \mathbf{B} .

We can summarize these observations by writing the magnetic force in the form

$$\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$$

- The **SI unit** of magnetic field is the **tesla (T)**

$$T = \frac{Wb}{m^2} = \frac{N}{C \cdot (m/s)} = \frac{N}{A \cdot m}$$

– **Wb (Weber) is the unit for magnetic field flux.**

- A non-SI commonly used unit is a gauss (G)

– **1 T = 10⁴ G**

when a charged particle moves with a velocity \mathbf{v} through a magnetic field, the field can alter the direction of the velocity vector but cannot change the speed or kinetic energy of the particle.

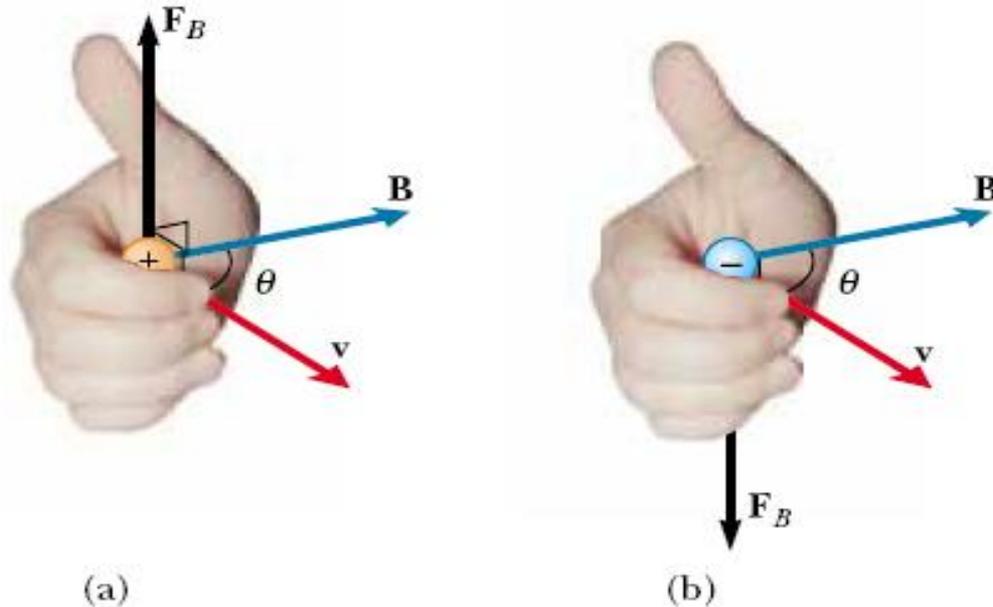


Figure 29.4 The right-hand rule for determining the direction of the magnetic force $\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$ acting on a particle with charge q moving with a velocity \mathbf{v} in a magnetic field \mathbf{B} . The direction of $\mathbf{v} \times \mathbf{B}$ is the direction in which the thumb points. (a) If q is positive, \mathbf{F}_B is upward. (b) If q is negative, \mathbf{F}_B is downward, antiparallel to the direction in which the thumb points.

$$\mathbf{i} \times \mathbf{j} = \mathbf{k},$$

$$\mathbf{j} \times \mathbf{i} = -\mathbf{k},$$

$$\mathbf{j} \times \mathbf{k} = \mathbf{i},$$

$$\mathbf{k} \times \mathbf{j} = -\mathbf{i},$$

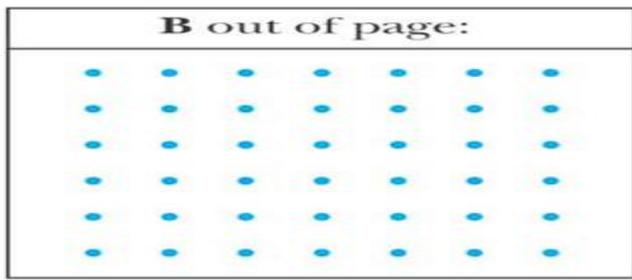
$$\mathbf{k} \times \mathbf{i} = \mathbf{j},$$

$$\mathbf{i} \times \mathbf{k} = -\mathbf{j}.$$

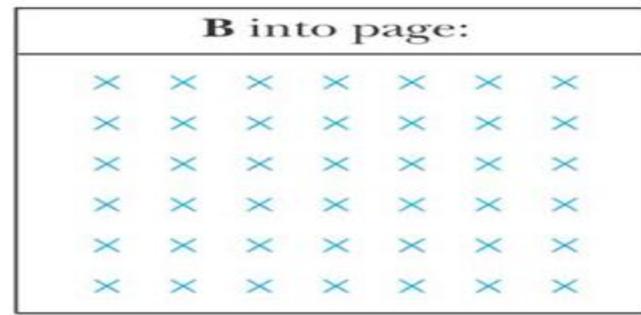
For positive charge

$$\mathbf{J} \times \mathbf{l} = \mathbf{k}$$

$$\mathbf{i} \times \mathbf{i} = \mathbf{j} \times \mathbf{j} = \mathbf{k} \times \mathbf{k} = \mathbf{0}$$



(a)



(b)

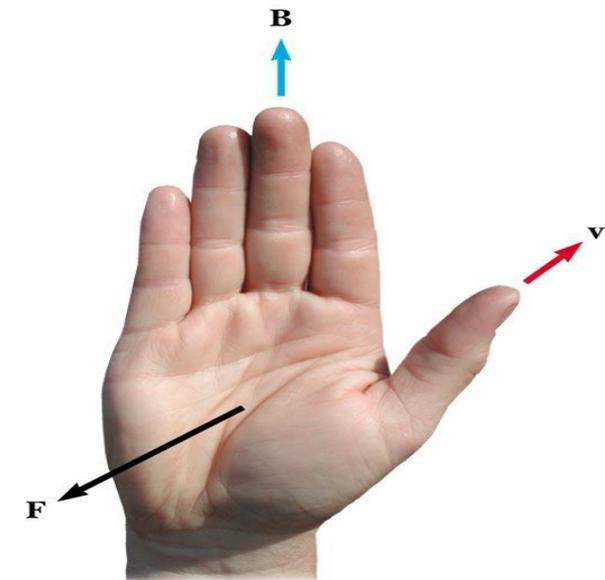
Figure 29.6 (a) Magnetic field lines coming out of the paper are indicated by dots, representing the tips of arrows coming outward. (b) Magnetic field lines going into the paper are indicated by crosses, representing the feathers of arrows going inward.

- If B is in the page, we use lines with arrow heads.



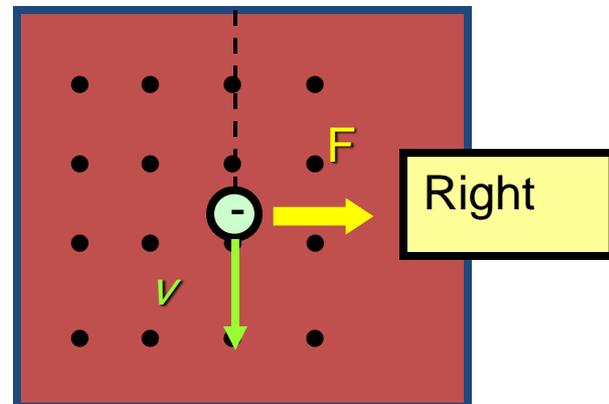
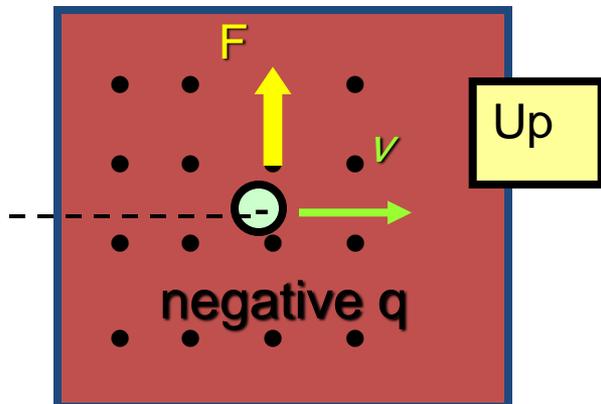
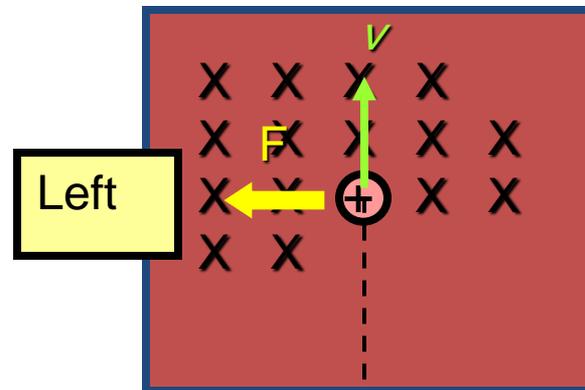
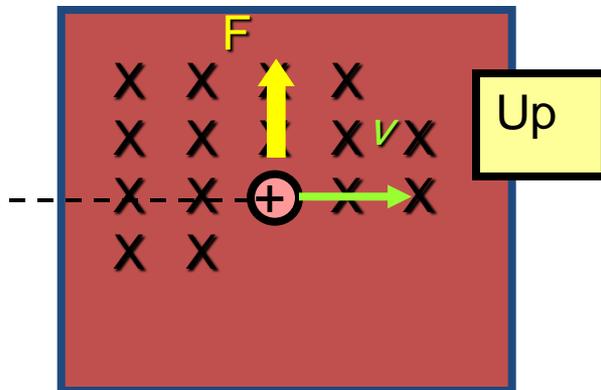
Right Hand Rule

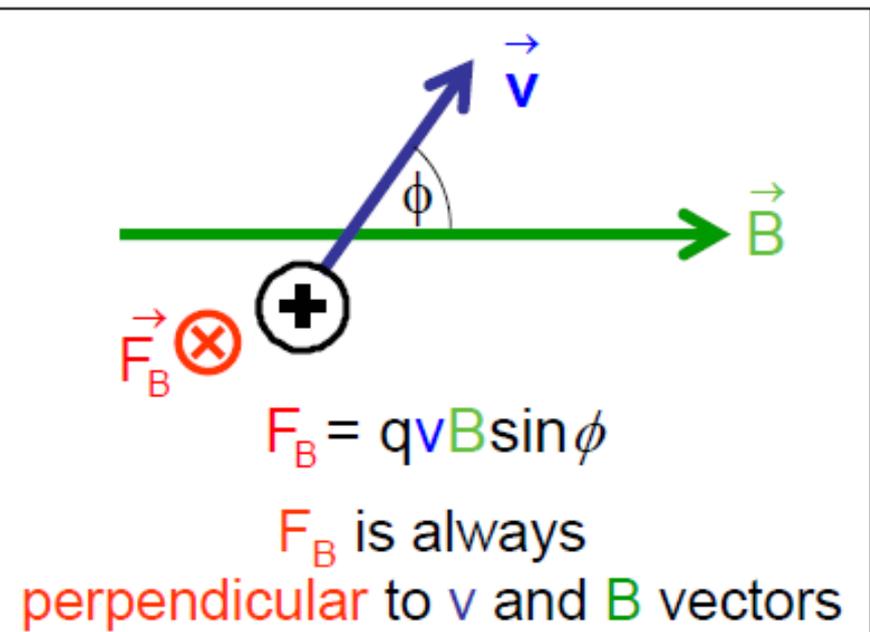
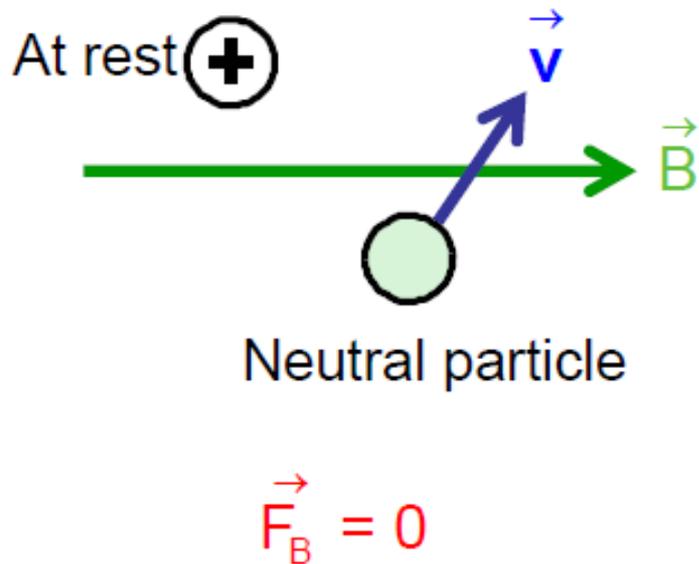
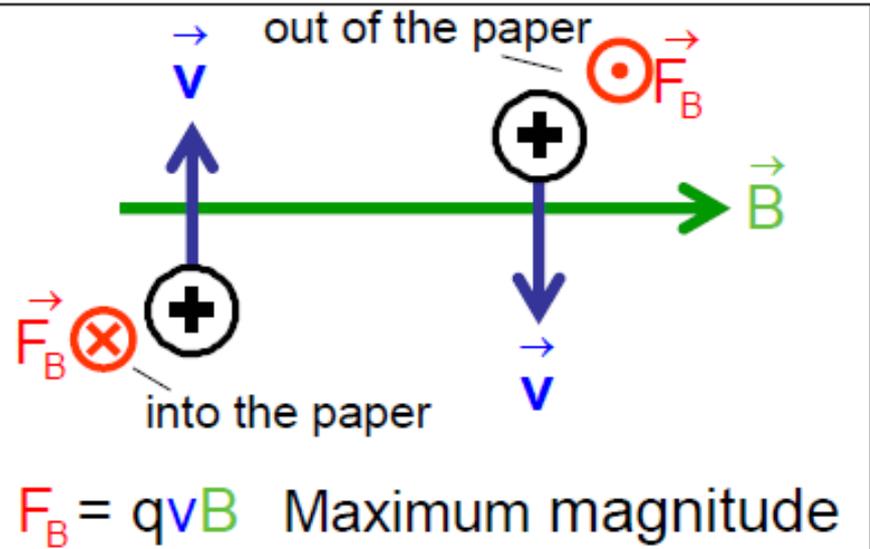
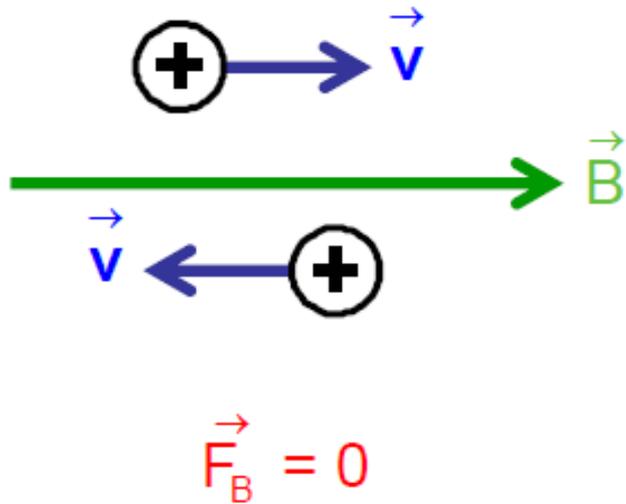
- Hold your right hand open
- Place your fingers in the direction of B
- Place your thumb in the direction of v
- The direction of the force on a positive charge is directed out of your palm
 - If the charge is negative, the force is opposite that determined by the right hand rule



© 2003 Thomson - Brooks Cole

What is the direction of the force F on the charge in each of the examples described below?





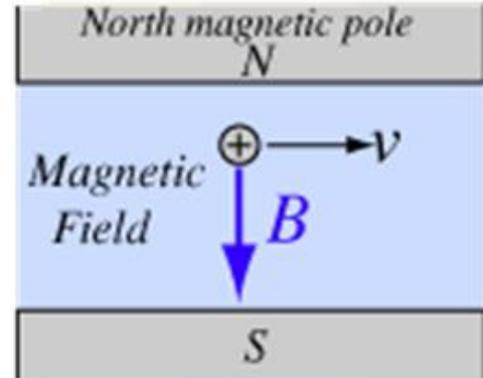
Differences Between Electric and Magnetic Fields

- **Direction of force**

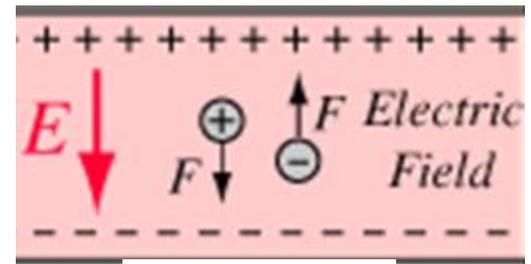
- The electric force acts parallel or antiparallel to the electric field ($F_E = qE$)
- The magnetic force acts perpendicular to the magnetic field

- $F_B = |q| v B \sin \theta$

- θ is the angle between the velocity and the field
- The force is zero when the velocity and the field are parallel or antiparallel
 $\theta = 0$ or $\theta = 180^\circ$
- The force is a maximum when the velocity and the field are perpendicular
 $\theta = 90^\circ$



Magnetic force of magnitude $qvB\sin\theta$ perpendicular to both v and B , away from viewer.



Electric force qE

Motion

The electric force acts on a charged particle regardless of its velocity

The magnetic force acts on a charged particle only when the particle is in motion and the force is proportional to the velocity

- Work

- The electric force does work in displacing a charged particle

- The magnetic force associated with a steady magnetic field does no work when a particle is displaced

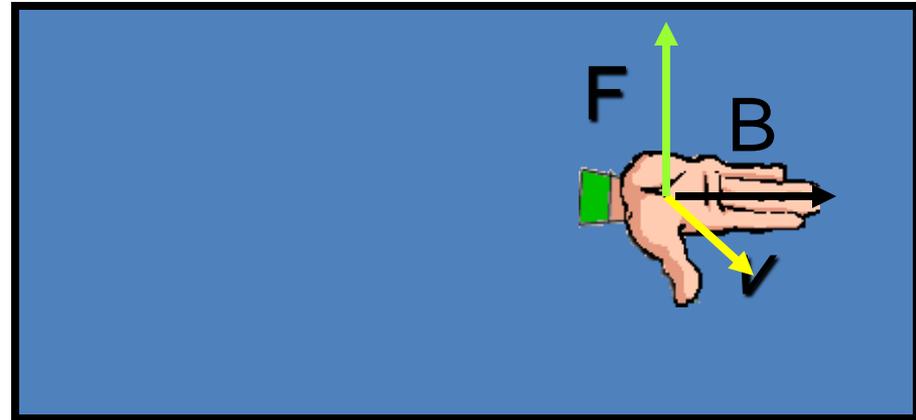
- This is because the force is perpendicular to the displacement

- The kinetic energy of a charged particle moving through a constant magnetic field cannot be altered by the magnetic field alone
- When a charged particle moves with a velocity through a magnetic field, the field can alter the *direction* of the velocity, but not the speed or the kinetic energy \vec{v}

Example.

A 2-nC charge is projected with velocity 5×10^4 m/s at an angle of 30° with a 3 mT magnetic field as shown. What are the magnitude and direction of the resulting force?

$$q = 2 \times 10^{-9} \text{ C} \quad v = 5 \times 10^4 \text{ m/s} \quad B = 3 \times 10^{-3} \text{ T} \quad \theta = 30^\circ$$



$$F = qvB \sin \theta = (2 \times 10^{-9} \text{ C})(5 \times 10^4 \text{ m/s})(3 \times 10^{-3} \text{ T}) \sin 30^\circ$$

Resultant Magnetic Force: $F = 1.50 \times 10^{-7}$ N, upward

Example

A proton moves with a speed of 1.0×10^5 m/s through the Earth's magnetic field, which has a value of $55 \mu\text{T}$ at a particular location. When the proton moves **eastward**, the magnetic force is a maximum, and when it moves **northward**, no magnetic force acts upon it. What is the magnitude and direction of the magnetic force acting on the proton?

$$\theta = 90^\circ \quad \sin 90^\circ = 1$$

$$F_B = qvB$$

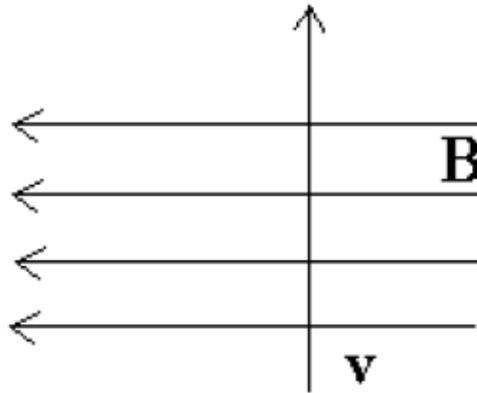
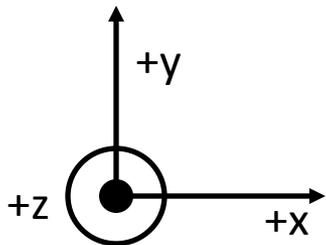
$$F_B = (1.6 \times 10^{-19})(1.0 \times 10^5)(55 \times 10^{-6})$$

$$F_B = 8.8 \times 10^{-19} \text{ N}$$

The direction cannot be determined precisely by the given information. Since no force acts on the proton when it moves northward (meaning the angle is equal to ZERO), we can infer that the magnetic field must either go northward or southward.

Example

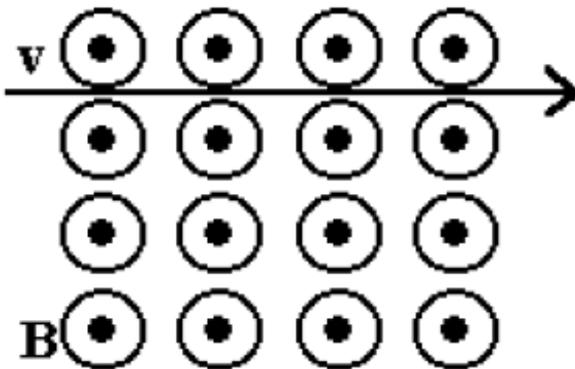
Determine the direction of the unknown variable for a proton moving in the field using the coordinate axis given



$$B = -x$$

$$v = +y$$

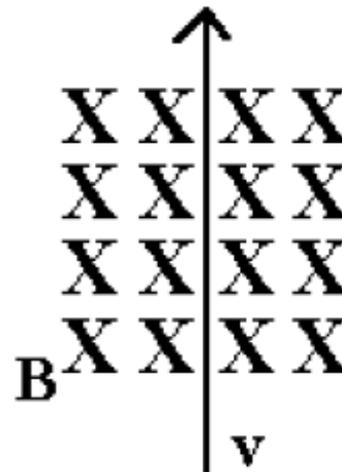
$$F = +z$$



$$B = +z$$

$$v = +x$$

$$F = -y$$



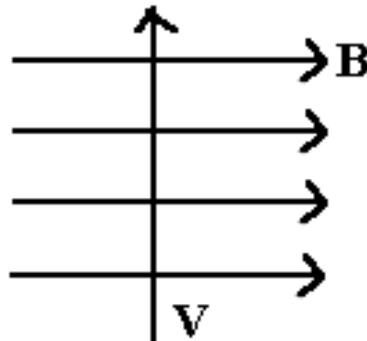
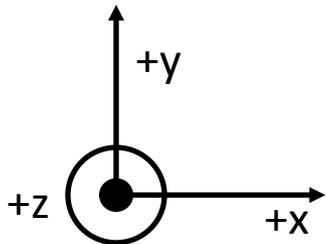
$$B = -z$$

$$v = +y$$

$$F = -x$$

Example

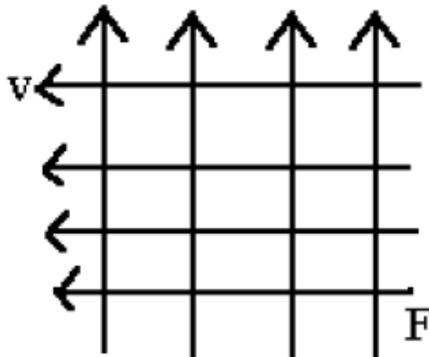
Determine the direction of the unknown variable for an electron using the coordinate axis given.



$$B = +x$$

$$v = +y$$

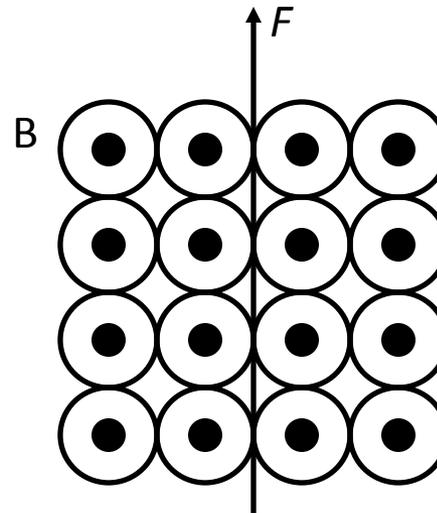
$$F = +z$$



$$B = -z$$

$$v = -x$$

$$F = +y$$



$$B = +z$$

$$v = +x$$

$$F = +y$$

Example

the force on a fast moving proton due to the earth's magnetic field. (Already we know we can neglect gravity, but can we neglect magnetism?)

Let $v = 10^7$ m/s moving North.

$E = 100$ V/m

What is the direction and magnitude of F ?

Take $B = 0.5 \times 10^{-4}$ T and $v \perp B$ to get maximum effect.

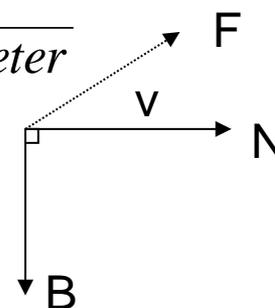
$$F = qvB = 1.6 \times 10^{-19} \text{ C} \cdot 10^7 \frac{\text{m}}{\text{s}} \cdot 0.5 \times 10^{-4} \text{ T}$$

$$F_M = 8 \times 10^{-17} \text{ N}$$

(a very fast-moving proton)

$$F_E = qE = 1.6 \times 10^{-19} \text{ C} \cdot 100 \frac{\text{volts}}{\text{meter}}$$

$$F_E = 1.6 \times 10^{-17} \text{ N}$$



$v \times B$ is into the paper (west).

Example

A charge $q_1 = 25.0 \mu\text{C}$ moves with a speed of $4.5 \times 10^3 \text{ m/s}$ perpendicularly to a uniform magnetic field. The charge experiences a magnetic force of $7.31 \times 10^{-3} \text{ N}$. A second charge $q_2 = 5.00 \mu\text{C}$ travels at an angle of 40.0° with respect to the same magnetic field and experiences a $1.90 \times 10^{-3} \text{ N}$ force. Determine

- (i) The magnitude of the **magnetic field** and
- (ii) The **speed of q_2** .

$$F = qvB \sin \theta$$

$q_1 = 25.0 \mu\text{C}$, $v_1 = 4.5 \times 10^3 \text{ m/s}$, $\Theta_1 = 90.0^\circ$
 $F_1 = 7.31 \times 10^{-3} \text{ N}$, $q_2 = 5.00 \mu\text{C}$, $\Theta_2 = 40.0^\circ$,
 $F_2 = 1.90 \times 10^{-3} \text{ N}$ force.

$$(i) \quad B = B_1 = \frac{F_1}{q_1 v_1} = 6.50 \times 10^{-2} \text{ T} = B_2$$

$$(ii) \quad v_2 = 9.10 \times 10^3 \text{ m/s}$$

Example

[6] A proton moves with a velocity of $v = (2\hat{i} - 4\hat{j} + \hat{k})$ m/s in a region in which the magnetic field is $B = (\hat{i} + 2\hat{j} - 3\hat{k})$ T. What is the magnitude of the magnetic force this charge experiences?

$$F = qvB \sin \theta = q(v \times B)$$

$$\text{Now, } v \times B = \begin{vmatrix} i & j & k \\ +2 & -4 & +1 \\ +1 & +2 & -3 \end{vmatrix} = 10i + 7j + 8k$$

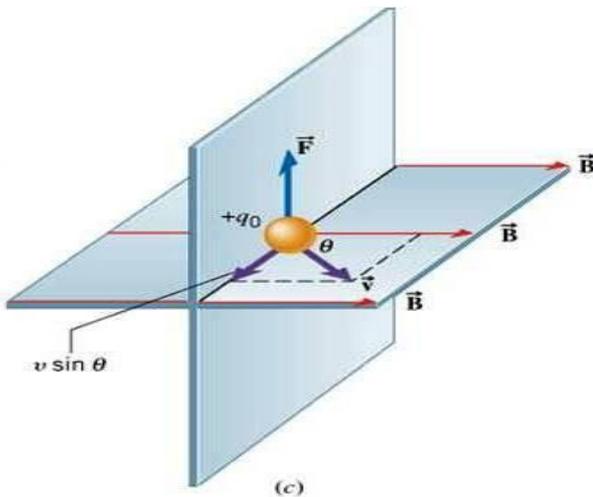
$$|v \times B| = 14.6 \text{ T} \cdot \text{m} / \text{s}$$

$$|F| = 1.6 \times 10^{-19} \times 14.6 = 2.34 \times 10^{-18} \text{ N}$$

Example Magnetic Forces on Charged Particles

A proton in a particle accelerator has a speed of 5.0×10^6 m/s. The proton encounters a magnetic field whose magnitude is 0.40 T and whose direction makes an angle of 30.0 degrees with respect to the proton's velocity (see part (c) of the figure). Find

- the magnitude and direction of the force on the proton and
- the acceleration of the proton.
- What would be the force and acceleration if the particle were an electron?



$$\begin{aligned}
 \text{(a)} \quad F &= |q|vB \sin \theta \\
 &= (1.60 \times 10^{-19} \text{ C})(5.0 \times 10^6 \text{ m/s})(0.40 \text{ T}) \sin(30.0^\circ) \\
 &= 1.6 \times 10^{-13} \text{ N}
 \end{aligned}$$

$$\text{(b)} \quad a = \frac{F}{m_p} = \frac{1.6 \times 10^{-13} \text{ N}}{1.67 \times 10^{-27} \text{ kg}} = 9.6 \times 10^{13} \text{ m/s}^2 \text{ upward}$$

(c) Magnitude of the force is the same, but direction is opposite. The mass is ~ 1800 times smaller

$$a = \frac{F}{m_e} = \frac{1.6 \times 10^{-13} \text{ N}}{9.11 \times 10^{-31} \text{ kg}} = 1.8 \times 10^{17} \text{ m/s}^2 \text{ downward}$$

EXAMPLE 29.1 An Electron Moving in a Magnetic Field

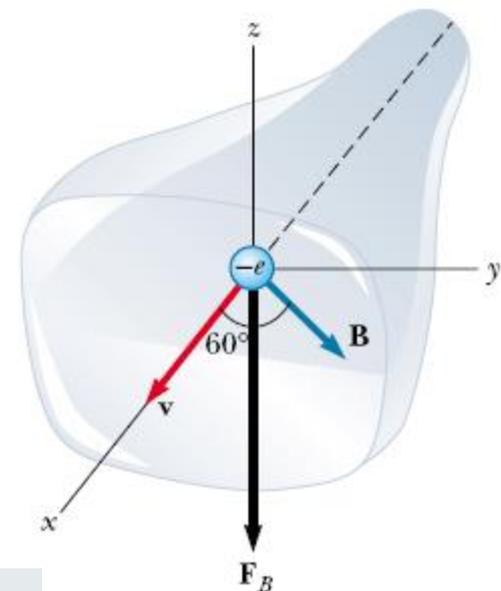
An electron in a television picture tube moves toward the front of the tube with a speed of 8.0×10^6 m/s **along the x axis**. Surrounding the neck of the tube are coils of wire that create a magnetic field of magnitude 0.025 T, directed at an angle of 60° to the *x axis* and **lying in the xy plane**. Calculate the magnetic force on and acceleration of the electron.

$$\begin{aligned} F_B &= |q|vB \sin \theta \\ &= (1.6 \times 10^{-19} \text{ C})(8.0 \times 10^6 \text{ m/s})(0.025 \text{ T})(\sin 60^\circ) \\ &= 2.8 \times 10^{-14} \text{ N} \end{aligned}$$

Because $\mathbf{v} \times \mathbf{B}$ is in the positive z direction (from the right-hand rule) and the charge is negative, \mathbf{F}_B is in the negative z direction.

$$a = \frac{F_B}{m_e} = \frac{2.8 \times 10^{-14} \text{ N}}{9.11 \times 10^{-31} \text{ kg}} = 3.1 \times 10^{16} \text{ m/s}^2$$

in the negative z direction.

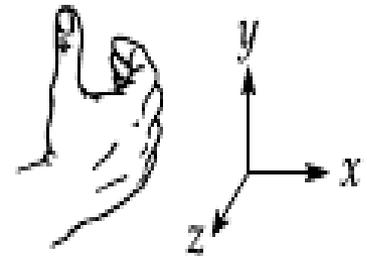


Example

3- **An electron** moving along the **positive x axis** perpendicular to a magnetic field experiences a magnetic deflection in the **negative y direction**. *What is the direction of the magnetic field?*

$$\mathbf{F}_B = q \mathbf{v} \times \mathbf{B}; \quad |\mathbf{F}_B|(-\mathbf{j}) = -e |\mathbf{v}| \mathbf{i} \times \mathbf{B}$$

Therefore, $B = |\mathbf{B}|(-\mathbf{k})$ which indicates the negative z direction

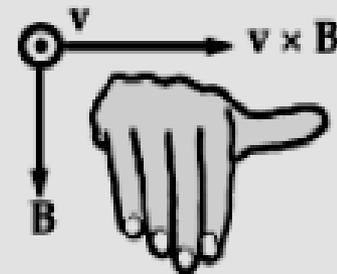


Example

5- A proton moves in a direction perpendicular to a uniform magnetic field B at 1.0×10^7 m/s and experiences an acceleration of 2.0×10^{13} m/s² in the **+ x direction** when its velocity is in the **+z direction**. Determine the magnitude and direction of the field.

$$F = ma = (1.67 \times 10^{-27} \text{ kg})(2.00 \times 10^{13} \text{ m/s}^2) = 3.34 \times 10^{-14} \text{ N} = qvB \sin 90^\circ$$

$$B = \frac{F}{qv} = \frac{3.34 \times 10^{-14} \text{ N}}{(1.60 \times 10^{-19} \text{ C})(1.00 \times 10^7 \text{ m/s})} = \boxed{2.09 \times 10^{-2} \text{ T}}$$



The right-hand rule shows that B must be in the $-y$ direction to yield a force in the $+x$ direction when v is in the z direction.

Example

$$F = qvB \sin \theta$$

1. Calculate the **magnitude of the force** on a proton travelling $3.1 \times 10^7 \text{ m s}^{-1}$ in the uniform magnetic flux density of 1.6 Wb m^{-2} , if :

(i) the velocity of the proton is **perpendicular** to the magnetic field.

$$F = 7.9 \times 10^{-12} \text{ N}$$

(ii) the velocity of the proton makes an angle **60°** with the magnetic field.

(charge of the proton = $+1.60 \times 10^{-19} \text{ C}$)

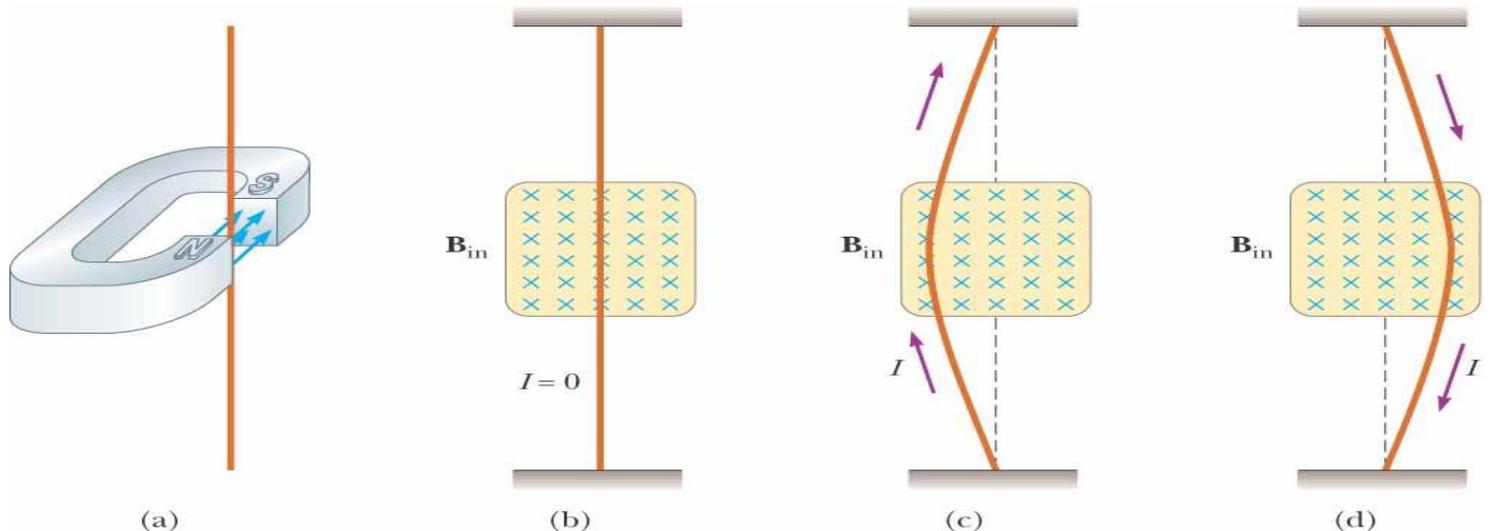
$$F = 6.9 \times 10^{-12} \text{ N}$$

29.2 MAGNETIC FORCE ACTING ON A CURRENT-CARRYING CONDUCTOR

If a magnetic force is exerted on a single charged particle when the particle moves through a magnetic field, it should not surprise you that a current-carrying wire also experiences a force when placed in a magnetic field.

the current is a collection of many charged particles in motion; hence, the resultant force exerted by the field on the wire is the vector sum of the individual forces exerted on all the charged particles making up the current.

The force exerted on the particles is transmitted to the wire when the particles collide with the atoms making up the wire.



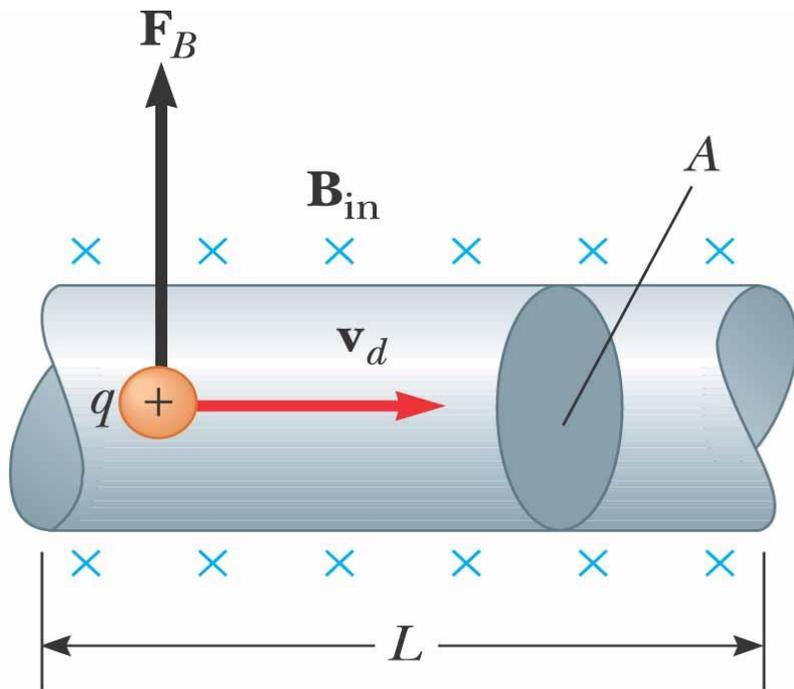


Figure 29.7 A segment of a current-carrying wire located in a magnetic field \mathbf{B} . The magnetic force exerted on each charge making up the current is $q\mathbf{v}_d \times \mathbf{B}$, and the net force on the segment of length L is $I\mathbf{L} \times \mathbf{B}$.

$$\mathbf{F}_B = (q\mathbf{v}_d \times \mathbf{B}) nAL$$

the current in the wire is $I = nqv_dA$. Therefore,

$$\mathbf{F}_B = I\mathbf{L} \times \mathbf{B}$$

Direction: Right-hand rule

$F=0$ when $\theta = 0^\circ$ $I \parallel B$

$F_{max}=IB$ when $\theta = 90^\circ$ $I \perp B$

- F is maximum when $\theta=90^\circ$

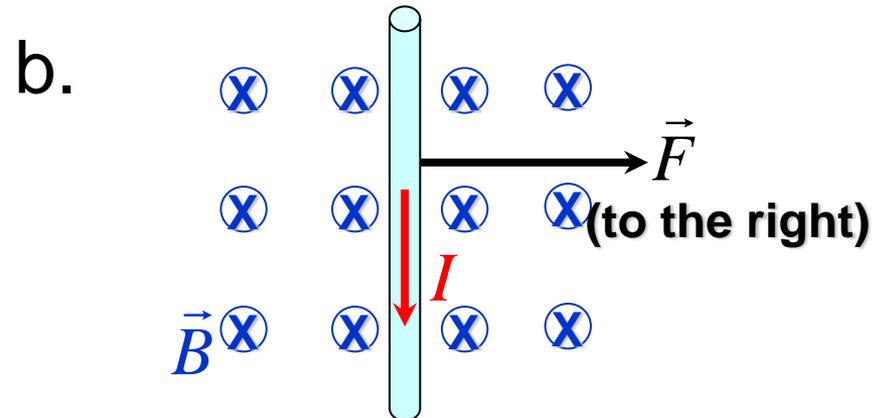
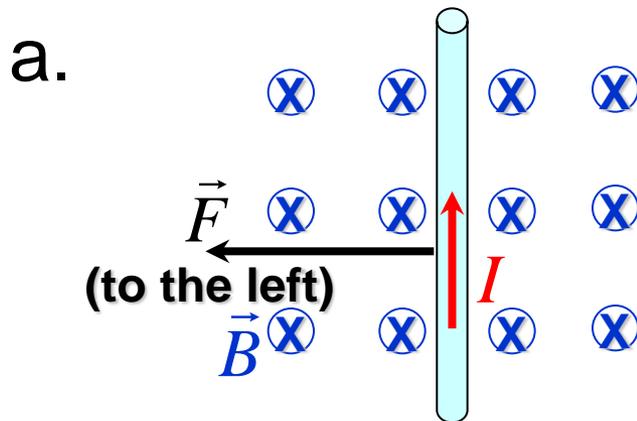
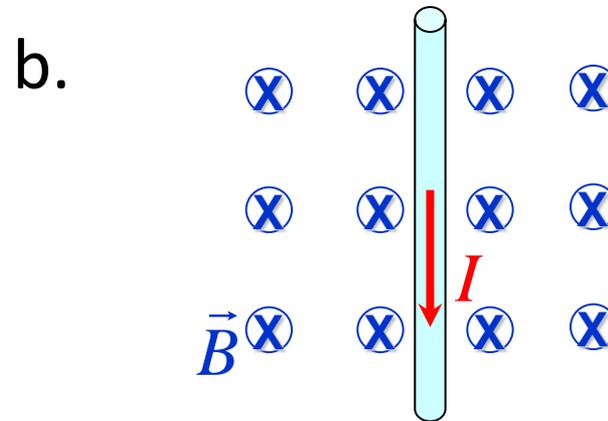
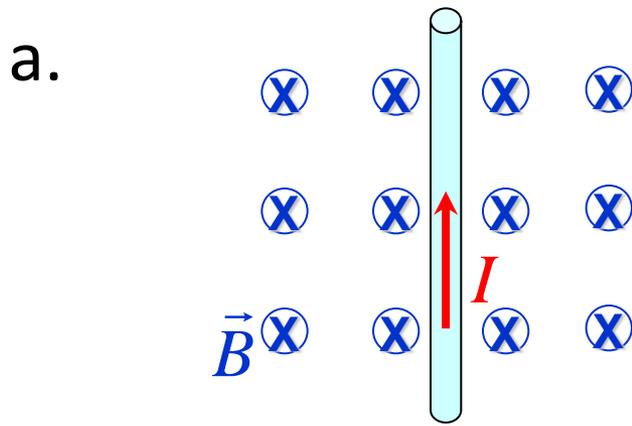
Example:

A wire carries a current of 22 A from east to west. Assume that at this location the magnetic field of the earth is horizontal and directed from south to north, and has a magnitude of 0.50×10^{-4} T. Find the magnetic force on a 36-m length of wire. What happens if the direction of the current is reversed?

$$\begin{aligned} F_{\max} &= BIl \\ &= (0.50 \times 10^{-4} T)(22 A)(36 m) \\ &= 4.0 \times 10^{-2} N \end{aligned}$$

Example

Determine the **direction of the magnetic force**, exerted on a conductor carrying current, I in each problem below.



Example

A wire of length 0.655 m carries a current of 21.0 A. In the presence of a 0.470 T magnetic field, the wire experiences a force of 5.46 N . What is the **angle** (less than 90°) between the wire and the magnetic field?

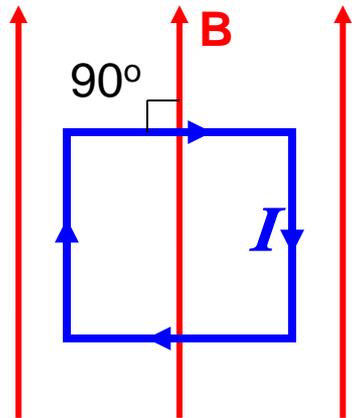
$$F = BIL \sin \theta$$

$$\theta = \sin^{-1} \frac{F}{BIL}$$

Example

$$F = BIL \sin \theta$$

1. A square coil of wire containing a single turn is placed in a uniform 0.25 T magnetic field. Each side has a length of 0.32 m, and the current in the coil is 12 A. Determine the magnitude of the **magnetic force** on each of the four sides.



0.96 N (top and bottom sides)
0 N (left and right sides)

Example

13- A wire having a mass per unit length of 0.500 g/cm carries a 2.00-A current horizontally to the south. What are the direction and magnitude of the minimum magnetic field needed to lift this wire vertically upward?

$$F_B = ILB \sin \theta$$

with

$$F_B = F_g = mg$$

$$mg = ILB \sin \theta$$

so

$$\frac{m}{L} g = IB \sin \theta$$

$$I = 2.00 \text{ A}$$

and

$$\frac{m}{L} = (0.500 \text{ g/cm}) \left(\frac{100 \text{ cm/m}}{1000 \text{ g/kg}} \right) = 5.00 \times 10^{-2} \text{ kg/m}$$

Thus

$$(5.00 \times 10^{-2})(9.80) = (2.00)B \sin 90.0^\circ$$

$B =$ 0.245 Tesla with the direction given by right-hand rule: eastward



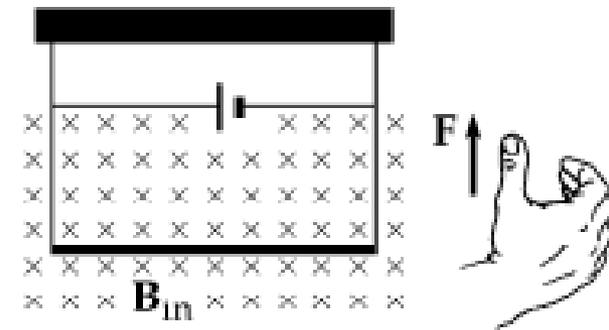
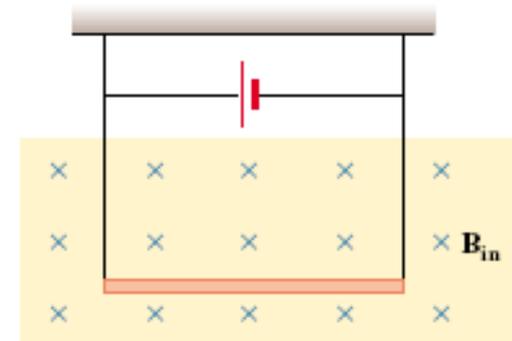
Example

16- A conductor suspended by two flexible wires as shown in Figure P29.16 has a mass per unit length of 0.0400 kg/m . What current must exist in the conductor for the tension in the supporting wires to be zero when the magnetic

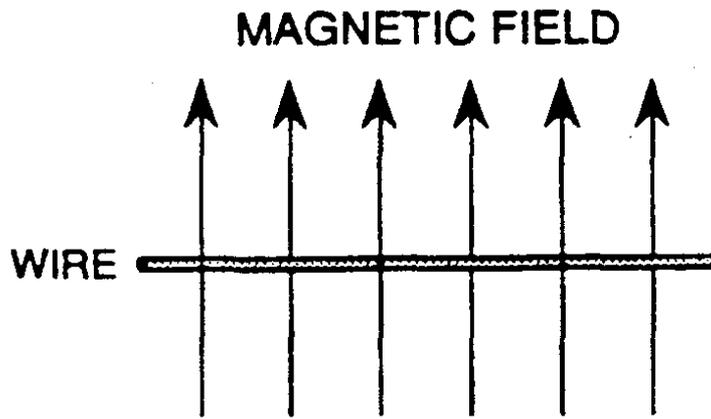
$$\frac{|F_B|}{L} = \frac{mg}{L} = \frac{I|\mathbf{L} \times \mathbf{B}|}{L}$$

$$I = \frac{mg}{BL} = \frac{(0.0400 \text{ kg/m})(9.80 \text{ m/s}^2)}{3.60 \text{ T}} = \boxed{0.109 \text{ A}}$$

The direction of I in the bar is to the right.



Example



A 36-m length wire carries a current of 22A running from right to left. Calculate the magnitude and direction of the magnetic force acting on the wire if it is placed in a magnetic field with a magnitude of 0.50×10^{-4} T and directed up the page.

$$F_B = ILB \sin \theta$$

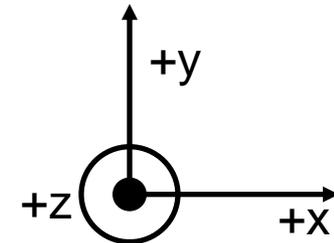
$$F_B = (22)(36)(0.50 \times 10^{-4}) \sin 90$$

$$F_B = \mathbf{0.0396 \text{ N}}$$

$$B = +y$$

$$I = -x$$

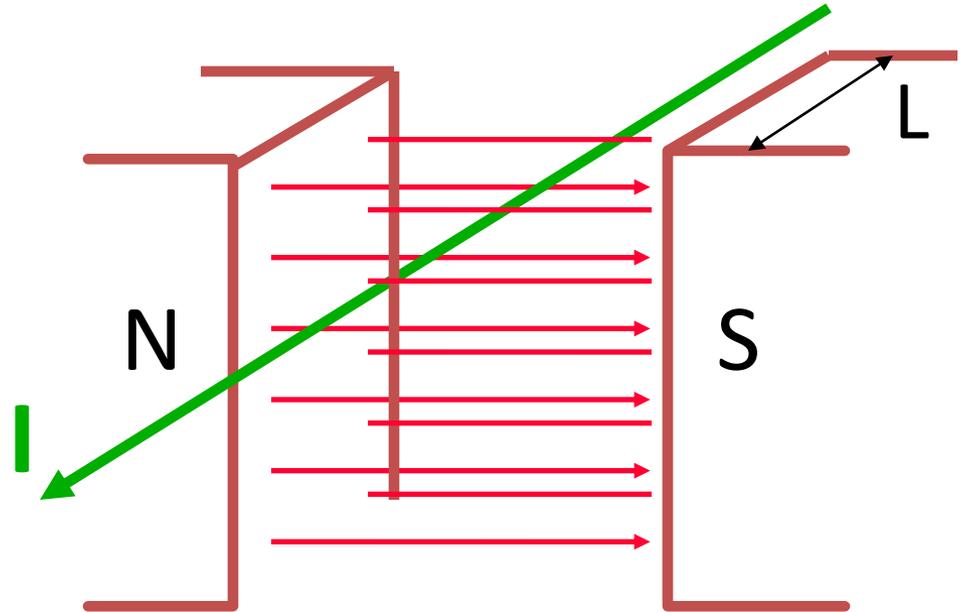
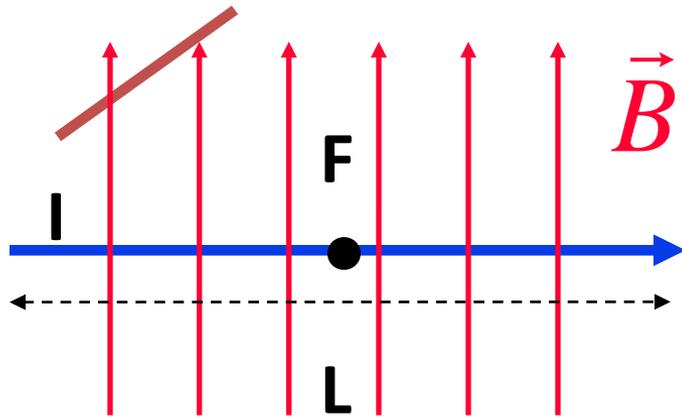
$$F = \mathbf{-z, \text{ into the page}}$$



Magnetic Force on a Current

Example: A current, $I=10\text{ A}$, flows through a wire, of length $L=20\text{ cm}$, between the poles of a 1000 Gauss magnet. The wire is at $\theta = 90^\circ$ to the field as shown. What is the *force* on the wire?

$$\begin{aligned}\vec{F} &= I\vec{L} \times \vec{B} \\ &= ILB \quad (\text{up}) \\ &= 10(\text{A}) \cdot 0.20(\text{m}) \cdot 0.1(\text{T}) = 0.2\text{N}\end{aligned}$$



[F points out of the page]

Example :The Force and Acceleration in a Loudspeaker

The voice coil of a speaker has a diameter of **0.0025 m**, contains **55 turns** of wire, and is placed in a 0.10-T magnetic field. The current in the voice coil is **2.0 A**. (a) Determine the magnetic force that acts on the coil and the cone. (b) The voice coil and cone have a combined mass of **0.0200 kg**. Find their acceleration.

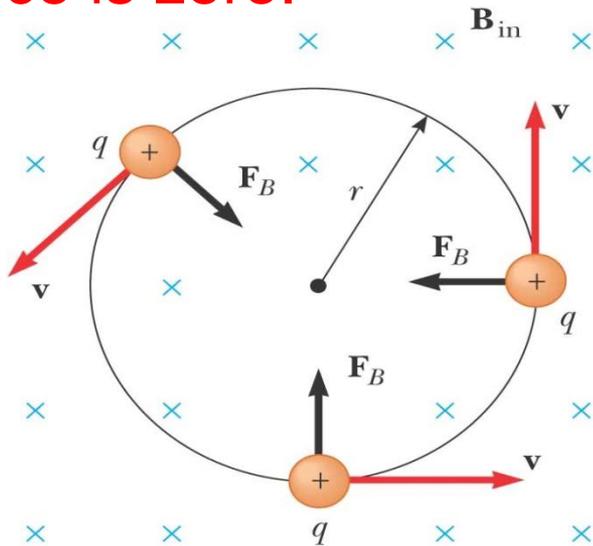
$$\begin{aligned} \text{(a)} \quad F &= ILB \sin \theta \\ &= (2.0 \text{ A}) [55\pi(0.0025 \text{ m})] (0.10 \text{ T}) \sin 90^\circ \\ &= 0.86 \text{ N} \end{aligned}$$

$$\text{(b)} \quad a = \frac{F}{m} = \frac{0.86 \text{ N}}{0.020 \text{ kg}} = 43 \text{ m/s}^2$$

29.4 MOTION OF A CHARGED PARTICLE IN A UNIFORM MAGNETIC FIELD

The magnetic force acting on a charged particle moving in a magnetic field is perpendicular to the velocity of the particle and that consequently **the work done on the particle by the magnetic force is zero.**

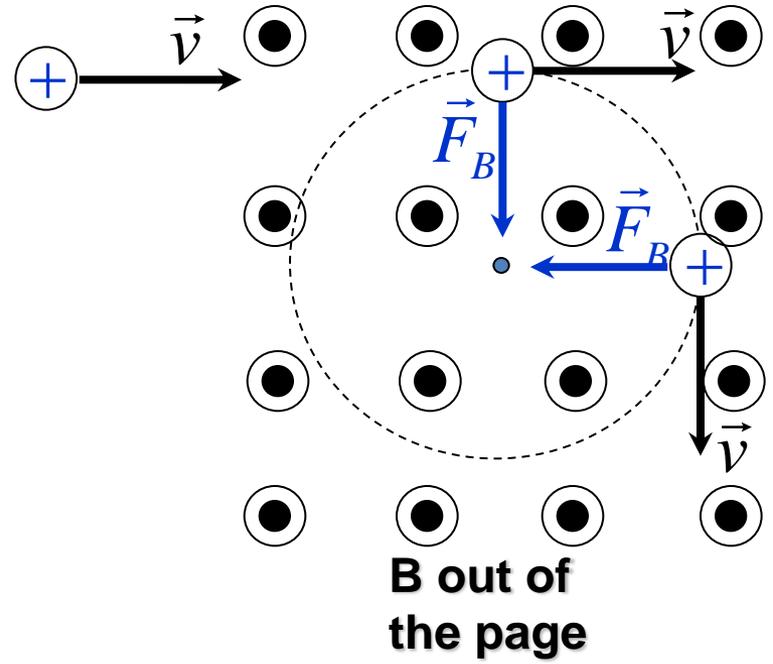
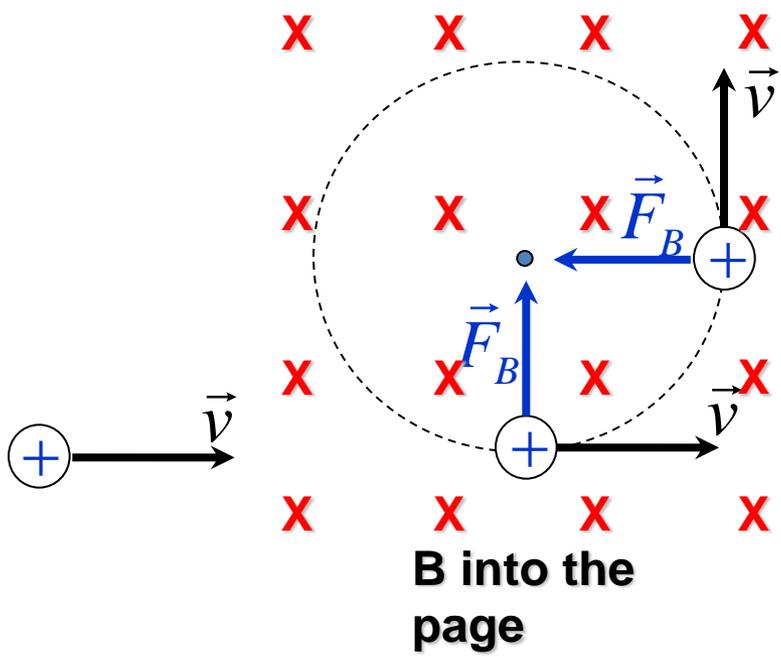
$$(W = F \cdot S = F s \cos \theta)$$



©2004 Thomson - Brooks/Cole

Because F_B always points toward the center of the circle, **it changes only the direction of v and not its magnitude.**

the rotation is counterclockwise for a **positive charge**. If q were **negative**, the rotation would be clockwise.



$$F_B = F_c$$

$$F_B = qvB = \frac{mv^2}{r}$$

$$r = \frac{mv}{qB}$$

r is proportional mass of the particle and inversely proportional to the magnetic field and the charge

The angular speed of the particle

$$\omega = \frac{v}{r} = \frac{qB}{m}$$

The period of the motion **T** (the time that the particle takes to complete one revolution) is equal to the circumference of the circle divided by the linear speed of the particle:

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$

$$T = \frac{1}{f}$$

$$f = \frac{Bq}{2\pi m}$$

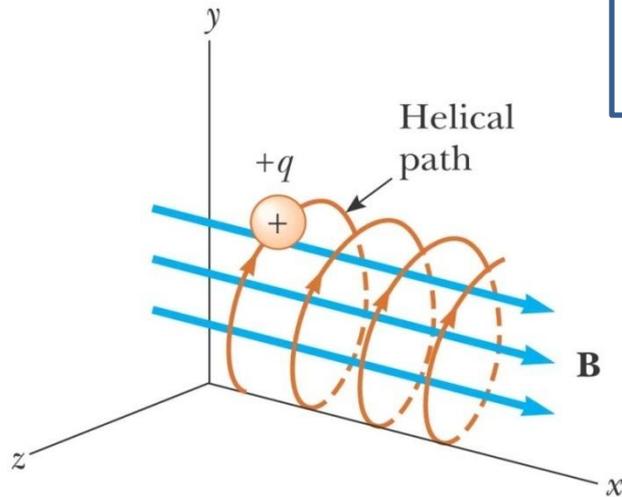


Figure 29.19 A charged particle having a velocity vector that has a component parallel to a uniform magnetic field moves in a helical path.

EXAMPLE 29.6 A Proton Moving Perpendicular to a Uniform Magnetic Field

A proton is moving in a circular orbit of radius 14 cm in a uniform 0.35-T magnetic field perpendicular to the velocity of the proton. Find the linear speed of the proton.

$$\begin{aligned}v &= \frac{qBr}{m_p} = \frac{(1.60 \times 10^{-19} \text{ C})(0.350 \text{ T})(14.0 \times 10^{-2} \text{ m})}{1.67 \times 10^{-27} \text{ kg}} \\ &= 4.69 \times 10^6 \text{ m/s}\end{aligned}$$

Find the period of the circular motion of the proton.

$$\begin{aligned}T &= \frac{2\pi m_p}{qB} = \frac{2\pi(1.67 \times 10^{-27} \text{ kg})}{(1.60 \times 10^{-19} \text{ C})(0.350 \text{ T})} \\ &= 1.87 \times 10^{-7} \text{ s}\end{aligned}$$

Exercise If an electron moves in a direction perpendicular to the same magnetic field with this same linear speed, what is the radius of its circular orbit?

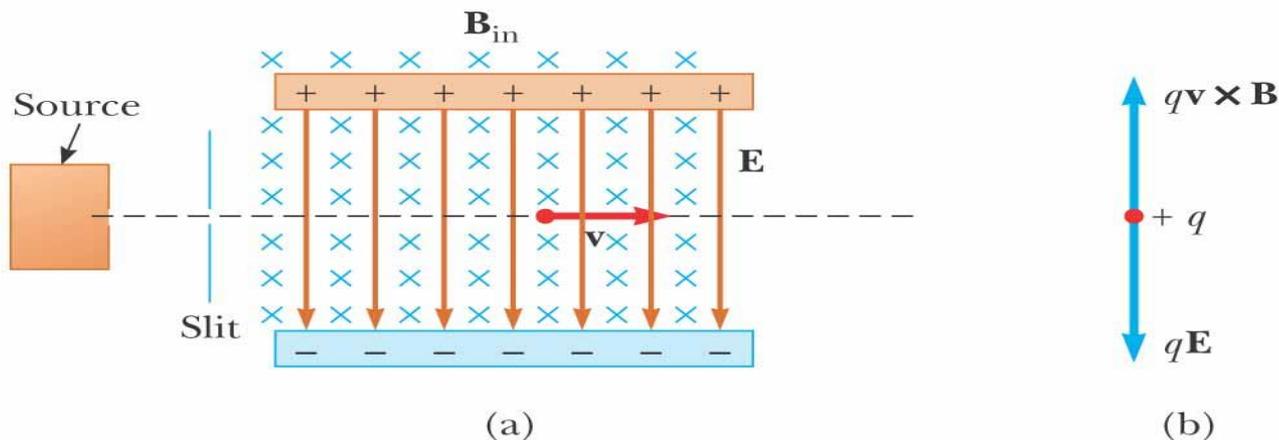
Answer $7.6 \times 10^{-5} \text{ m}$.

29.5 APPLICATIONS INVOLVING CHARGED PARTICLES MOVING IN A MAGNETIC FIELD

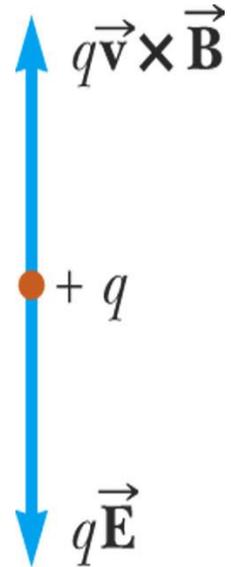
Velocity Selector:

- In many applications, the charged particle will move in the presence of both magnetic and electric fields
- In that case, the total force is the sum of the forces due to the individual fields
- In general:

- This force is called the Lorentz force $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$
- It is the vector sum of the electric force and the magnetic force



- Used when all the particles need to move with the same velocity
- A uniform electric field is perpendicular to a uniform magnetic field
- When the force due to the electric field is equal but opposite to the force due to the magnetic field, the particle moves in a straight line



- This occurs for velocities of value $\mathbf{v} = \mathbf{E} / \mathbf{B}$
- Only those particles with the given speed will pass through the two fields undeflected
- The magnetic force exerted on particles moving at speed greater than this is stronger than the electric field and the particles will be deflected upward
- Those moving more slowly will be deflected downward

(b)

Example

Suppose a cyclotron is operated at frequency $f=12$ MHz and has a dee radius of $R=53$ cm. What is the magnitude of the magnetic field needed for deuterons to be accelerated in the cyclotron ($m=3.34 \cdot 10^{-27}$ kg)?

$$f = \frac{qB}{2\pi m}$$

$$B = \frac{2\pi m f}{q} = \frac{(2\pi)(3.34 \cdot 10^{-27} \text{kg})(12 \cdot 10^6 \text{s}^{-1})}{1.6 \cdot 10^{-19} \text{C}} = 1.57 \text{T}$$

Example

• A **2.0-m** wire carrying a current of **0.60 A** is oriented **parallel** to a uniform magnetic field of **0.50 T**. What is the magnitude of the **force** it experiences?

a) zero

b) 0.60 N

c) 0.15 N

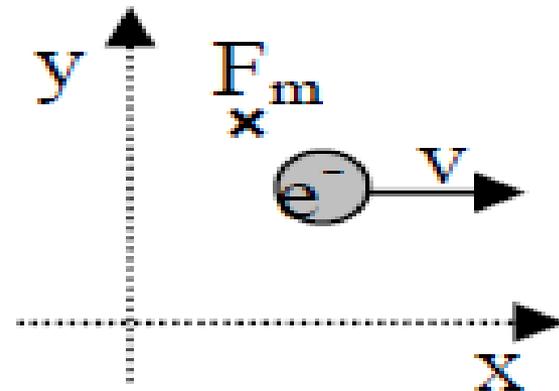
d) 0.30 N

A magnetic force acts on an electric charge (q) in a magnetic field (B) when:

- (A) q moves at 90° to B
- (B) q is at rest
- (C) q moves along B
- (D) q moves opposite to B
- (E) None of the above

An electron moving in the positive x direction experiences a magnetic force pointing into the plane of the page. The direction of magnetic field (B) that produced this magnetic force is along the:

- (A) Positive x axis
- (B) Negative x axis
- (C) Positive y axis
- (D) Negative y axis
- (E) out of the plane



Example

Suppose a cyclotron is operated at frequency $f=12$ MHz and has a dee radius of $R=53$ cm. What is the kinetic energy of the deuterons in this cyclotron when they travel on a circular trajectory with radius R ($m=3.34 \cdot 10^{-27}$ kg, $B=1.57$ T)?

A) $0.9 \cdot 10^{-14}$ J

B) $8.47 \cdot 10^{-13}$ J

C) $2.7 \cdot 10^{-12}$ J

D) $3.74 \cdot 10^{-13}$ J

$$r = \frac{mv}{qB} \quad \text{implies} \quad v = \frac{RqB}{m} = 3.99 \times 10^7 \text{ m/s}$$

$$K = \frac{1}{2}mv^2 = 2.7 \times 10^{-12} \text{ J}$$

Homework

1- Find the magnetic force on a proton moving in the $+x$ direction at a speed of 0.446 Mm/s in a uniform magnetic field of 1.75 T in the $+z$ direction.

2-A point particle has a charge equal to -3.64 nC and a velocity equal to $2.75 \times 10^3 \text{ m/s } \hat{i}$. Find the force on the charge if the magnetic field is:

(a) $0.38 \text{ T } \hat{j}$, (b) $0.75 \text{ T } \hat{i} + 0.75 \text{ T } \hat{j}$, (c) $0.65 \text{ T } \hat{i}$,
and (d) $0.75 \text{ T } \hat{i} + 0.75 \text{ T } \hat{k}$

3- A uniform magnetic field equal to **1.48 T** is in the **+z direction**. Find the force exerted by the field on a **proton** if the velocity of the proton is
(a) **2.7 km/s \hat{i}** , (b) **3.7 km/s \hat{j}** , (c) **6.8 km/s \hat{k}** , and
(d) **4.0 km/s \hat{i} + 3.0 km/s \hat{j}**

4- A straight segment of a current-carrying wire has a current element $I\vec{L}$, where $I = 2.7$ A and $\vec{L} = 3.0$ cm \hat{i} + 4.0 cm \hat{j} . The segment is in a region with a uniform magnetic field given by 1.3 T \hat{i} . Find the force on the segment of wire.

5- A straight wire segment that is 2.0 m long makes an angle of 30° with a uniform 0.37 T magnetic field. Find the magnitude of the force on the wire if the wire carries a current of 2.6 A.

6- What is the force on an electron that has a velocity equal to $2.0 \times 10^6 \text{ m/s } \hat{i} - 3.0 \times 10^6 \text{ } \hat{j}$ when it is in a region with a magnetic field given by $0.80 \text{ T } \hat{i} + 0.60 \text{ T } \hat{k}$?

7- A proton moves in a 65-cm-radius circular orbit that is perpendicular to a uniform magnetic field of magnitude 0.75 T. (a) What is the orbital period for the motion? (b) What is the speed of the proton? (c) What is the kinetic energy of the proton?

8- A 4.5-keV electron (an electron that has a kinetic energy equal to 4.5 keV) moves in a circular orbit that is perpendicular to a magnetic field of 0.325 T. (a) Find the radius of the orbit. (b) Find the frequency and period of the orbital motion.

9-An alpha particle has a charge of $+2e$ ($e = 1.6 \times 10^{-19}$ C) and is moving at right angles to a magnetic field $B = 0.27$ T with a speed $v = 6.15 \times 10^5$ m/s. The force acting on this charged particle is

10-A singly charged positive ion has a mass of 2.5×10^{-26} kg. After being accelerated through a potential difference of 250 V, the ion enters a magnetic field of 0.5 T, in a direction perpendicular to the field. Calculate the radius of the path of the ion in the field.

11- A proton is moving with velocity 3×10^5 m/s vertically across a magnetic field 0.02 T.

($m_p = 1.67 \times 10^{-27}$ kg)

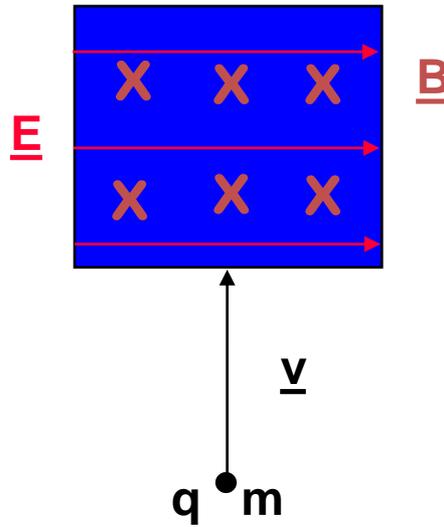
Calculate ;

- a) **kinetic energy** of the proton
- b) the **magnetic force** exerted on the proton
- c) the **radius** of the circular path of the proton.

12-An electron is projected from left to right into a magnetic field directed into the page. The velocity of the electron is 2×10^6 ms⁻¹ and the magnetic flux density of the field is 3.0 T. Find the magnitude and direction of the magnetic force on the electron.

(charge of electron = 1.6×10^{-19} C)

13-What is the relation between the intensities of the electric and magnetic fields for the particle to move in a straight line ?.



SUMMARY

The magnetic force that acts on a charge q moving with a velocity \mathbf{v} in a magnetic field \mathbf{B} is

$$\mathbf{F}_B = q\mathbf{v} \times \mathbf{B} \quad (29.1)$$

The direction of this magnetic force is perpendicular both to the velocity of the particle and to the magnetic field. The magnitude of this force is

$$F_B = |q|vB \sin \theta \quad (29.2)$$

where θ is the smaller angle between \mathbf{v} and \mathbf{B} . The SI unit of \mathbf{B} is the **tesla** (T), where $1 \text{ T} = 1 \text{ N/A} \cdot \text{m}$.

When a charged particle moves in a magnetic field, the work done by the magnetic force on the particle is zero because the displacement is always perpendicular to the direction of the force. The magnetic field can alter the direction of the particle's velocity vector, but it cannot change its speed.

If a straight conductor of length L carries a current I , the force exerted on that conductor when it is placed in a uniform magnetic field \mathbf{B} is

$$\mathbf{F}_B = I\mathbf{L} \times \mathbf{B} \quad (29.3)$$

If a charged particle moves in a uniform magnetic field so that its initial velocity is perpendicular to the field, the particle moves in a circle, the plane of which is perpendicular to the magnetic field. The radius of the circular path is

$$r = \frac{mv}{qB} \quad (29.13)$$

where m is the mass of the particle and q is its charge. The angular speed of the charged particle is

$$\omega = \frac{qB}{m} \quad (29.14)$$

Magnetic Field and Electric Field

Deflection of a charged particle in a magnetic field	Deflection of a charged particle in an electric field
The magnetic field can exert a magnetic force only on a moving charged particle moving perpendicular to the field . Stationary charged particles and charges moving parallel to the field experience no force.	The electric field always exerts an electric force on a charged particle , whether it is stationary or moving.
The magnetic force is perpendicular to the magnetic field and the direction of motion of the charged particle. Direction of force is deduced by Fleming's Left Hand Rule.	The electric force acts in the direction of the electric field . Deduced from the law of electrostatics (i.e. Like charges repel; unlike charges attract.)
Magnetic force is dependent on the speed and direction of motion of the charged particle: $F_B = Bqv \sin \theta$	Electric force is independent of the speed and direction of motion of the charged particle. $(F_E = qE)$
Uniform circular motion is obtained when a charged particle enters a magnetic field perpendicularly.	Parabolic motion is obtained when a charged particle enters an electric field perpendicularly.