1

Lecture ⑤ Continuous prob. distn$\underline{s}$

* pb 1.4.2 p.32

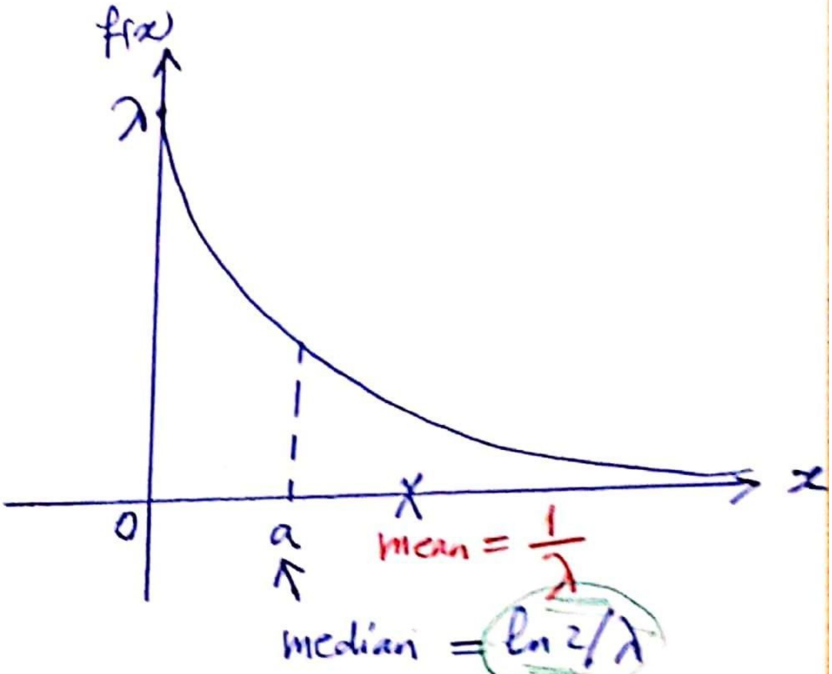
For exp. distn

pdf: $f(x) = \lambda e^{-\lambda x}, \quad x \geq 0$

CDF: $F(x) = 1 - e^{-\lambda x}$

$R(x) = e^{-\lambda x}$

Reliability/Survival fn



$pr(X \geq a) \geq \frac{1}{2}$

Reliability

$\Rightarrow e^{-\lambda a} \geq \frac{1}{2}$

$\therefore -\lambda a \geq \ln(1/2)$

$-\lambda a \geq -\ln 2 \quad (X-1)$

$\lambda a \leq \ln 2$

$\therefore \boxed{a \leq \frac{\ln 2}{\lambda}}$ ①

$pr(X \leq a) \geq \frac{1}{2}$

CDF

$1 - e^{-\lambda a} \geq \frac{1}{2}$

$-e^{-\lambda a} \geq -\frac{1}{2}$

$-\lambda a \leq \ln(\frac{1}{2})$

$-\lambda a \leq -\ln 2$

$\lambda a \geq \ln 2 \quad (X-1)$

$\therefore \boxed{a \geq \frac{\ln 2}{\lambda}}$ ②

$\therefore$ ① and ② $\Rightarrow a = \frac{\ln 2}{\lambda}$ which is called median for the r.v $X \sim exp(\lambda)$, where the mean $= \frac{1}{\lambda}$

$\therefore$ mean $>$ median, where $\ln 2 \simeq 0.7$

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• Memoryless property for exp. dist:

خاصية فقدان الذاكرة في التوزيع الأسي

If $T \sim exp(\lambda)$ See p. 29 Memoryless property

then $pr(T > t+s \mid T > s)$

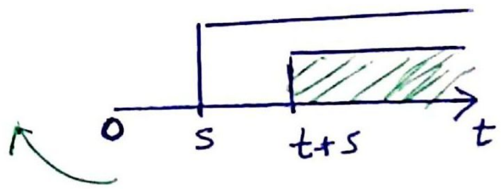
$= pr(T > t) \quad \forall\, t, s \geq 0$

كما لو كان المتغير العشوائي لم يعرف شيء عن s

proof $pr(T > t+s \mid T > s)$

$$pr(A|B) = \frac{pr(A \cap B)}{pr(B)}$$

$$= \frac{pr(T > t+s,\, T > s)}{pr(T > s)}$$



$$= \frac{pr(T > t+s)}{pr(T > s)}$$

$\because T \sim exp(\lambda)$

$\therefore pr(T > t+s \mid T > s)$

$$= \frac{e^{-\lambda(t+s)}}{e^{-\lambda s}} = e^{-\lambda t} = R(t)$$

$$= pr(T > t)$$

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pb Mid I Exam S'1 39/40 Memoryless prop.

If $T \sim \exp(0.02)$, find $pr(T \leq 110 | T > 100)$

Ans: $\because pr(T > t+s | T > s) = pr(T > t)$ where $s, t \geq 0$

$$pr(T \leq 110 | T > 100) = 1 - pr(T > 110 | T > 100)$$

$$= 1 - pr(T > \underset{s}{100} + \underset{t}{10} | T > \underset{s}{100})$$

$$= 1 - pr(T > 10 + 100 | T > 100)$$

$$= 1 - pr(T > 10)$$

$$= pr(T \leq 10) = F(10)$$

$$= 1 - e^{-0.02(10)} = 1 - e^{-0.2}$$

$$\therefore pr(T \leq 110 | T > 100) \simeq 0.18$$

• Another solution

$$pr(T \leq 110 | T > 100) = \frac{pr(T \leq 110, T > 100)}{pr(T > 100)}$$

$$= \frac{pr(100 < T \leq 110)}{pr(T > 100)}$$

$$= \frac{F(110) - F(100)}{1 - F(100)}$$

$$= \frac{\cancel{1} - e^{-0.02(110)} \cancel{- 1} + e^{-0.02(100)}}{\cancel{1 - 1} + e^{-0.02(100)}}$$

$$\therefore pr(T \leq 110 | T > 100) = \frac{e^{-2} - e^{-2.2}}{e^{-2}} \simeq 0.18$$

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Gamma distn

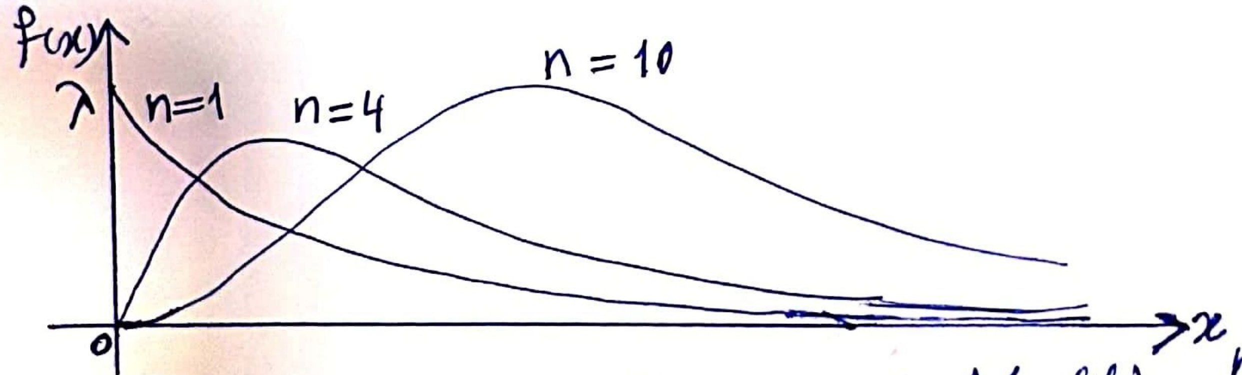
* توزیع گاما

$X \sim Gamma(n, \lambda)$ if its pdf is

$$f(x) = \frac{\lambda^n}{\Gamma(n)} x^{n-1} e^{-\lambda x}, \quad x \geq 0$$

for $n=1 \Rightarrow \boxed{f(x) = \lambda e^{-\lambda x}}, \quad x \geq 0$
(exp. distn)

Not: $\Gamma(n) = (n-1)!\,, \quad n = 1, 2, 3, \ldots$



Moments: $\mu = E(X) = \frac{n}{\lambda}\,, \quad \sigma^2 = Var(X) = \frac{n}{\lambda^2}$

* property sec 1.4.4 p.30

If $X_n = Y_1 + Y_2 + \cdots + Y_i + \cdots + Y_n$
and $Y_i \sim exp(\lambda)$, $i = 1, 2, \ldots, n$
are independent r.v's

then $X_n \sim Gamma(n, \lambda)$